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Extraction of the s-wave πN scattering lengths from data on pionic atoms

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For many years a combined analysis of pionic hydrogen and deuterium atoms has been known as a good tool to extract information on the isovector and especially on the isoscalar s-wave πN scattering length. However, given the smallness of the isoscalar scattering length, the analysis becomes useful only if the pion–deuteron scattering length is controlled theoretically to a high accuracy comparable to the experimental precision. To achieve the required few-percent accuracy one needs theoretical control over all isospin-conserving three-body $\pi NN \to \pi NN$ operators up to one order before the contribution of the dominant unknown $(N^\dagger N)^2\pi\pi$ contact term. This term appears at next-to-next-to-leading order in Weinberg counting. In addition, one needs to include isospin-violating effects in both two-body (πN) and three-body (πNN) operators. In this talk we discuss the results of the recent analysis where these isospin-conserving and -violating effects have been carefully taken into account. Based on this analysis, we present the up-to-date values of the s-wave πN scattering lengths.

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1. Introduction

The πN scattering lengths are fundamental quantities of pion physics. As low-energy parameters they are subjects for an investigation in Chiral Perturbation Theory (ChPT), the low-energy effective field theory of QCD, see e.g. Ref. [1] for a recent review. In fact, the leading effect of chiral symmetry on the πN scattering lengths, derived by Weinberg [2], shows a strong suppression of the isoscalar scattering length (a^+) with respect to its isovector counterpart (a^-)

$$a^- = \frac{M_\pi}{8\pi(1+M_\pi/m_{\rm p})F_\pi^2} \approx 80\cdot 10^{-3} M_\pi^{-1}, \qquad\qquad a^+ = 0, \tag{1.1}$$

where M_π (m_p) is the mass of the charged pion (proton), F_π is the pion decay constant, and the scattering lengths are defined in the isospin symmetric world. Higher-order corrections to a^+ and a^- involve low-energy constants (LECs) [3]. Unfortunately, the lack of information about the LECs and large cancellations between individual contributions strongly restrict the predictive power of the chiral expansion for a^+. Meanwhile, a precise determination of πN scattering lengths is of high importance. The smallness of the isoscalar scattering length leaves a^+ as a measure of the explicit breaking of chiral symmetry. The scattering lengths appear as input to a determination of the pion–nucleon coupling constant (via the Goldberger–Miyazawa–Oehme sum rule [4]) and to a dispersive analysis of the πN σ-term [5].

The most promising way to get access to the πN scattering lengths is the experimental investigation of pionic atoms, the simplest of which are pionic hydrogen and pionic deuterium. Pionic atoms are loosely bound states of pions and light nuclei, which are formed mainly by a static Coulomb potential. However, the energy spectrum measured experimentally to a very high precision exhibits a small deviation from the Coulombic spectrum. This shift is predominantly due to strong πN interactions. Deser *et al.* derived the expression which connects the shift of the ground (1s) state level of a hadronic atom due to strong interactions to the scattering length in the elastic channel (Re a) [6]. Further, if there are open channels below threshold, the scattering length is complex and the ground state level of a hadronic atom has a non-zero width. The energy shift ε_{1s} and the line width Γ_{1s} of the ground state are given by [6, 7, 8]

$$\varepsilon_{1s} - \frac{i}{2}\Gamma_{1s} = -2\alpha^3\mu_H^2 a\left(1 - 2\alpha\mu_H(\ln\alpha - 1)a\right), \tag{1.2}$$

where $\alpha = e^2/4\pi$, μ_H is the reduced mass of the hadronic system and the term $\propto \mathscr{O}(\alpha^4)$ stands for the corrections at higher order in α [7, 8]. In principle, the measurement of the shift and the width of the πH atom [9] is already sufficient to extract both a^+ and a^-. Indeed, the level shift of πH in the isospin limit is sensitive to $a^+ + a^-$, whereas the width is solely determined by a^-. However, due to the chiral suppression of a^+ and a relatively large experimental uncertainty in Γ_{1s} [9] additional experimental information stemming from the shift of the πD atom [10] is extremely useful. Schematically, the system of three equations to determine a^+ and a^- can be written as

$$\begin{aligned}
a_{\pi^- p} &= \tilde a^+ + a^- + \Delta\tilde a_{\pi^- p}, \\
a_{\pi^- p\to\pi^0 n} &= -\sqrt{2}\,a^- + \Delta a_{\pi^- p\to\pi^0 n}, \\
\text{Re}\, a_{\pi d} &= 2\frac{1+\frac{M_\pi}{m_p}}{1+\frac{M_\pi}{2m_p}}\left(\tilde a^+ + \Delta\tilde a^+\right) + a^{\rm IC}_{(3-{\rm body})} + a^{\rm IV}_{(3-{\rm body})},
\end{aligned} \tag{1.3}$$

where the first two equations stem from the data on πH,[1] whereas the last line corresponds to the πd scattering length from the shift of the πD atom [10]. Here, $\tilde{a}^+$ includes isospin-violating (IV) corrections to a^+ at leading order (LO) [11, 12]

$$\tilde{a}^+ = a^+ + \frac{1}{4\pi(1+\frac{M_\pi}{m_p})}\left(\frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2}c_1 - 2e^2 f_1\right), \tag{1.4}$$

with c_1 and f_1 being low-energy constants, M_{π^0} the mass of the neutral pion, and $\Delta\tilde{a}^+$, $\Delta a_{\pi^-p\to\pi^0 n}$, $\Delta\tilde{a}_{\pi^-p}$ stand for IV corrections to the scattering lengths at next-to-leading (NLO) order [13]. In addition to isospin violation in the two-body sector, a precise determination of $\tilde{a}^+$ and a^-, that means with an accuracy better than $1\cdot 10^{-3} M_\pi^{-1}$ [14], requires a careful analysis of three-body (nuclear) contributions in the isospin-conserving (IC) and isospin-violating case. The terms $a^{\rm IC}_{(3-\rm body)}$ and $a^{\rm IV}_{(3-\rm body)}$ in Eq. (1.3) take these corrections into account. We discuss them in Secs. 2 and 3.

2. Isospin-conserving three-body contributions to $a_{\pi d}$

As usual in EFT, in order to provide an estimate of the theoretical uncertainty one needs to classify the diagrams according to some counting scheme. In particular, it was shown in Ref. [15] that an application of the Weinberg scheme allows one to systematically account for IC three-body contributions to $a_{\pi^- d}$ to very high accuracy. The hierarchy of the isospin-conserving three-body diagrams in the Weinberg counting scheme is illustrated in Table 1 with the order quoted as the predicted size in the counting relative to the leading, double-scattering term. The goal is to include all three-body operators up to one order lower than the contribution of the leading unknown $(N^\dagger N)^2\pi\pi$ contact term, which appears at next-to-next-to-leading order (N^{2}LO). Its contribution cannot easily be determined from data, and is a key source of uncertainty in our result. Given that $\mathscr{O}(p) \sim \chi = M_\pi/m_{\rm p}$, we anticipate an accuracy of a few per cent for threshold $\pi^- d$ scattering.

A detailed discussion of the power counting, relevant scales and the role of the individual diagrams can be found in Ref. [14]. Here we briefly sketch the main results. By far the largest contribution to the πd scattering length stems from the double scattering diagram at LO. At this order the nucleons in the deuteron are treated as being static. This contribution is comparable with the experimental value of the scattering length. The contribution of the other diagrams at LO (with $3\pi NN$ and 4π vertices) is numerically negligible, see [16, 17, 15] for more details.

The operators at NLO (the first line of NLO diagrams in Table 1) involve sub-leading vertices and were shown to cancel amongst themselves in [18]. In addition, at NLO there is a triple-scattering term. The actual size of this diagram is enhanced as compared to the estimate based on dimensional analysis, which predicts that this diagram contributes only at N^{2}LO. The origin of this enhancement was associated in [15] with the special topology of the diagram consisting of two consecutive pion exchanges with Coulombic-type pion propagators. Numerically the contribution of the triple-scattering diagram is around 10% of the double-scattering term. The other (higher-order) contributions of multiple-scattering topology appear to be negligible [23, 14].

In addition, starting from NLO, nucleon recoil effects to the leading double-scattering operator have to be taken into account. This means the nucleon kinetic energies enter and the static

[1] the shift (width) is proportional to the elastic $\pi^- p$ (charge-exchange $\pi^- p \to \pi^0 n$) scattering length

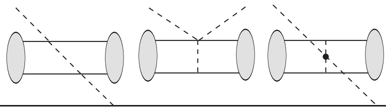
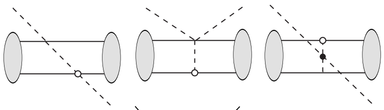
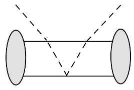
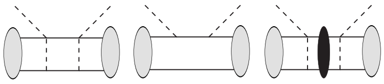
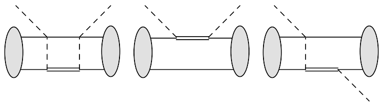
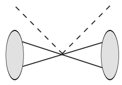
Chiral order	Three-body operator	Reference
LO $= \mathscr{O}(1)$		[16, 17]
NLO $= \mathscr{O}(p)$		[18]
		[18, 15]
	Effect of nucleon recoil in LO diagrams	[19, 20]
N$^{3/2}$LO $= \mathscr{O}(p^{3/2})$		[21]
		[22]
	Effect of nucleon recoil in LO diagrams	[19, 20]
N^{2}LO $= \mathscr{O}(p^2)$	 $+\cdots$	

Table 1: Hierarchy of isospin-conserving three-body operators within Weinberg power counting. Solid (open) circles correspond to leading (sub-leading) vertices, grey blobs indicate the deuteron wave functions, and the black ellipse corresponds to NN interactions in the intermediate state. Solid single, solid double, and dashed lines correspond to nucleons, $\Delta(1232)$-isobars, and pions, respectively.

pion propagator needs to be replaced by the full propagator corresponding to the three-body πNN intermediate state. Due to a non-perturbative regime in which the three-body propagator goes to zero the expansion of the double-scattering diagram contains half-integer powers of M_π/m_p. Note that the largest isovector recoil correction at order $\mathscr{O}(p^{1/2})$ fully determined by the small scales vanishes exactly as a consequence of the Pauli principle [19, 20], see also [24]. Furthermore, it was demonstrated in [20] that the recoil effect for $\pi^- d$ scattering is relevant only at orders $\mathscr{O}(p)$ and $\mathscr{O}(p^{3/2})$, which partially cancel each other.

At order $\mathscr{O}(p^{3/2})$ there are two additional contributions to the $\pi^- d$ scattering length. First, diagrams with pure NN or $NN\gamma$ intermediate states yield so-called dispersive corrections. Also diagrams with explicit Δ degrees of freedom enter at the same order. Both classes were computed in [21, 22] using a calculation for $NN \to d\pi$ up to NLO in ChPT [25]. The combined effect of the dispersive corrections and the $\Delta(1232)$ contributions at $\mathscr{O}(p^{3/2})$ does not exceed a few per cent due to significant cancellations between these corrections.

3. Isospin-violating three-body contributions to $a_{\pi d}$

In analogy to Sec. 2, in Table 2 we present the hierarchy of IV three-body operators relative

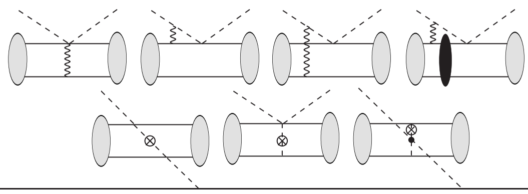
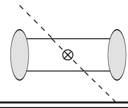
Chiral order	Three-body operator
LO $= \mathscr{O}(e^2)$	
$\mathrm{N}^{1/2}\mathrm{LO} = \mathscr{O}(e^2 p^{1/2})$	

Table 2: Hierarchy of isospin-violating three-body operators relevant for the study. Isospin violation appears either due to the inclusion of virtual photons or due to the pion mass difference marked by crossed circles.

to each other, and their relative suppression compared to the IC operators at LO. At leading order in isospin violation diagrams that involve a virtual photon and one insertion of the IC πN vertex occur. These are represented by the first row of diagrams in Table 2, which form a gauge-invariant set of diagrams at order $\mathscr{O}(e^2)$. Due to the presence of photon and pion propagators these diagrams are potentially infrared enhanced. The non-perturbative (singularity) regime of the πNN propagator leads to an enhancement of the individual diagrams from momenta of order $\sqrt{M_\pi \varepsilon}$, with ε the deuteron binding energy. However, these ostensibly enhanced contributions vanish for both isovector and isoscalar πN scattering [14], where the cancellation can be traced back to the Pauli principle and the orthogonality of deuteron and continuum wave functions, respectively. The ultimate result for this class of diagrams is 4% of the double scattering term. At the same order effects due to the pion mass difference in the leading-order IC diagrams enter (the second row of diagrams in Table 2). Next in importance is the higher-order correction due to the inclusion of the pion mass difference in the πNN propagator of the double-scattering diagram. This results in the appearance of the non-analytic $\mathrm{N}^{1/2}\mathrm{LO}$ contribution in Table 2, as soon as the propagator is expanded. The net effect caused by the pion mass difference is about 2% of the double-scattering term.

The operators at NLO are suppressed by $\mathscr{O}(e^2 p)$ compared to the three-body isospin-conserving operators at LO, and, given the smallness of the expansion parameter, are irrelevant for our present purposes. Therefore up to the order we are working isospin violation in three-body operators is purely of electromagnetic origin.

4. Concluding remarks

We have demonstrated that all isospin-conserving three-body corrections can be reliably calculated up to $\mathscr{O}(p^{3/2})$. This is half an order lower than the contribution of the leading unknown $(N^\dagger N)^2\pi\pi$ contact term, which is $\mathscr{O}(p^2)$. The uncertainty anticipated due to the truncation of higher-order terms is a few per cent, as follows from naive dimensional analysis. Convolving the operators of Tables 1 and 2 with different wave functions derived from chiral and phenomenological NN potentials we find a variation in the results of about 5%: an independent estimate of the contact term's effect. However, to achieve this accuracy one also has to account for two-body and three-body isospin-violating corrections. The two-body corrections were derived in Ref. [13] up to NLO. A complete calculation of the isospin-violating three-body corrections up to $\mathscr{O}(e^2 p^{1/2})$ was presented here, see also Refs. [14] for more details. Solving the system of Eqs. (1.3) we find

$$\tilde{a}^+ = (1.9 \pm 0.8) \cdot 10^{-3} M_\pi^{-1}, \quad a^- = (86.1 \pm 0.9) \cdot 10^{-3} M_\pi^{-1}. \tag{4.1}$$

Using $c_1 = (-1.0 \pm 0.3)\,\mathrm{GeV}^{-1}$ [14], the rough estimate $|f_1| \leq 1.4\,\mathrm{GeV}^{-1}$ [7], and Eq. (4.1) yields a non-zero a^+ at better than the 95% confidence level

$$a^+ = (7.6 \pm 3.1) \cdot 10^{-3} M_\pi^{-1}. \tag{4.2}$$

A reduction of the uncertainty beyond that of the present analysis will be hard to achieve without additional QCD input that helps pin down the unknown contact terms in the πN and πNN sectors.

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