

# Orbit and gravity field: common versus sequential analysis

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*Gravity field mapping methodology from GRACE  
and future gravity missions*

Hotine Marussi 2013, Roma

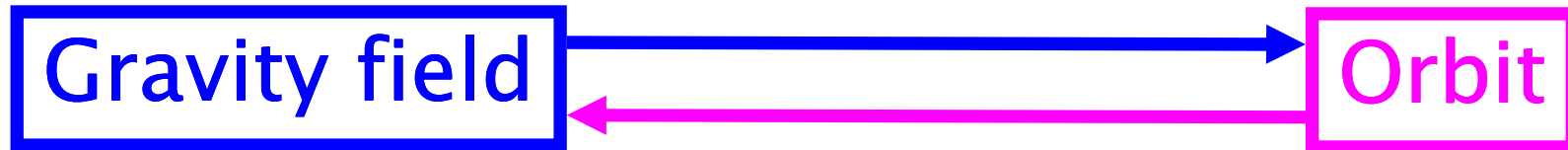
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- Gravity field and Orbit
- Signal and Noise in monthly models (GRACE)
- Timewise analysis: the concept of Lumped Coefficients
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# Gravity field and Orbit

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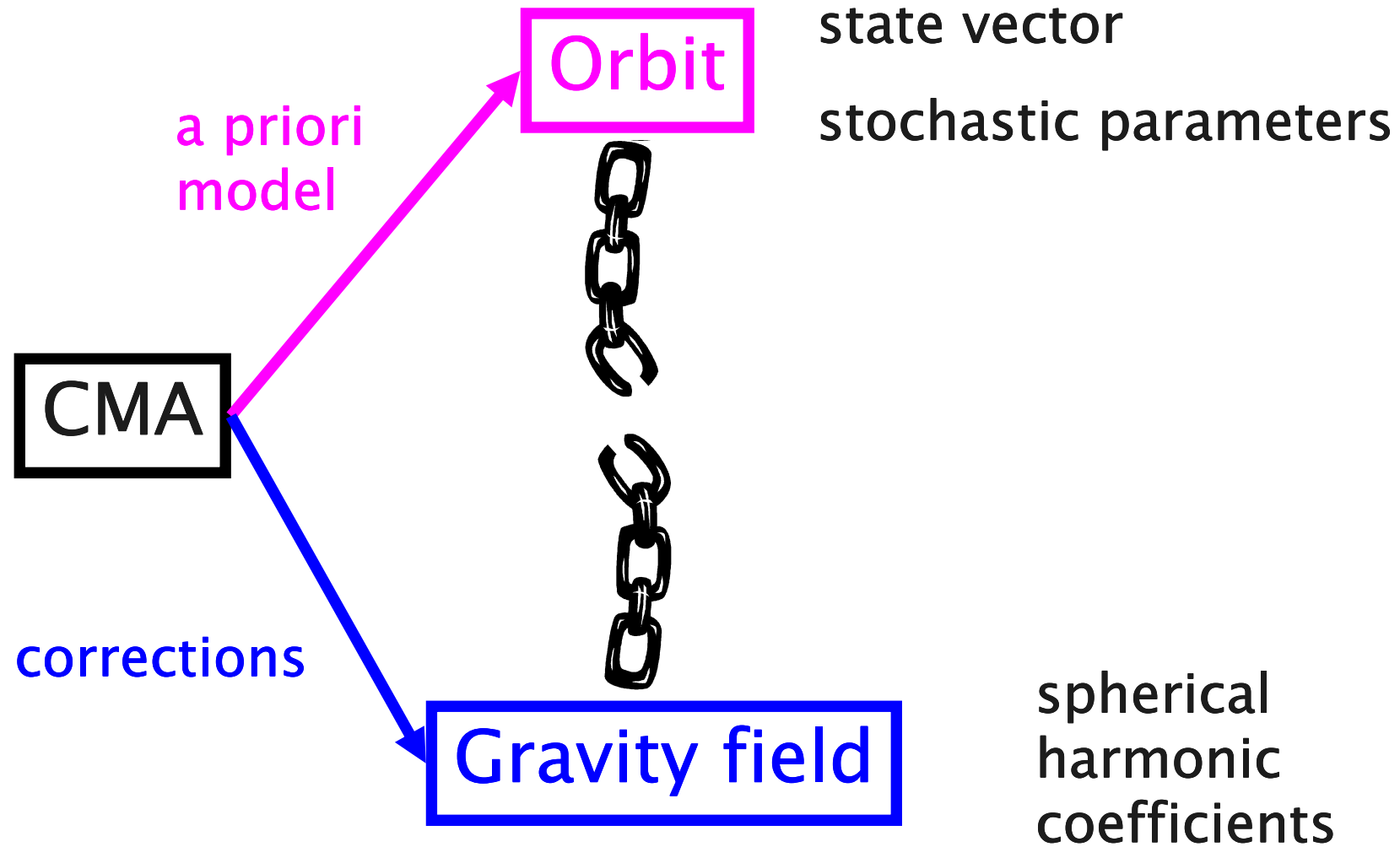


Non-linear parameter estimation problem

- A priori model (linearization)
- Observations
- Regularization (a priori knowledge via pseudo-observations)

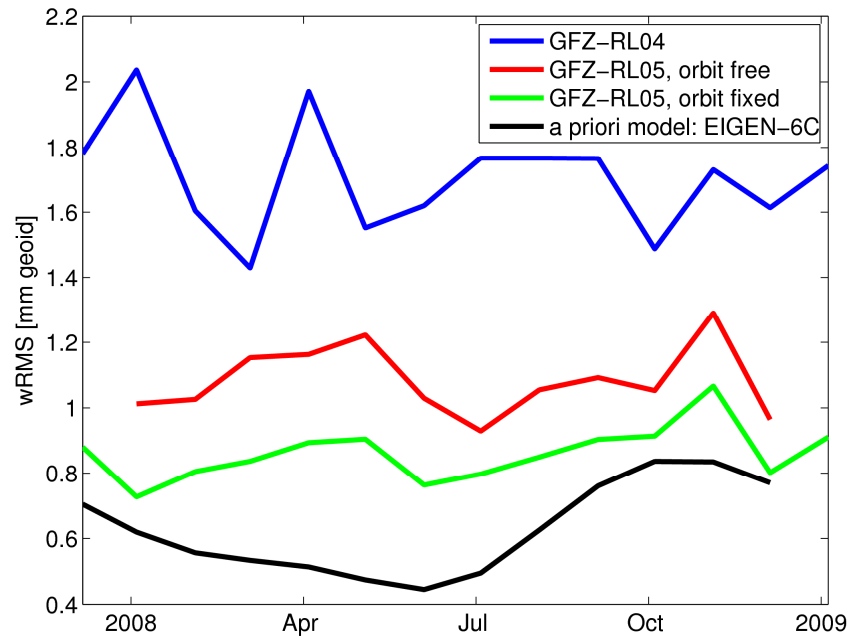
# A generalized orbit determination problem

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# Signal and Noise in monthly models (GRACE)

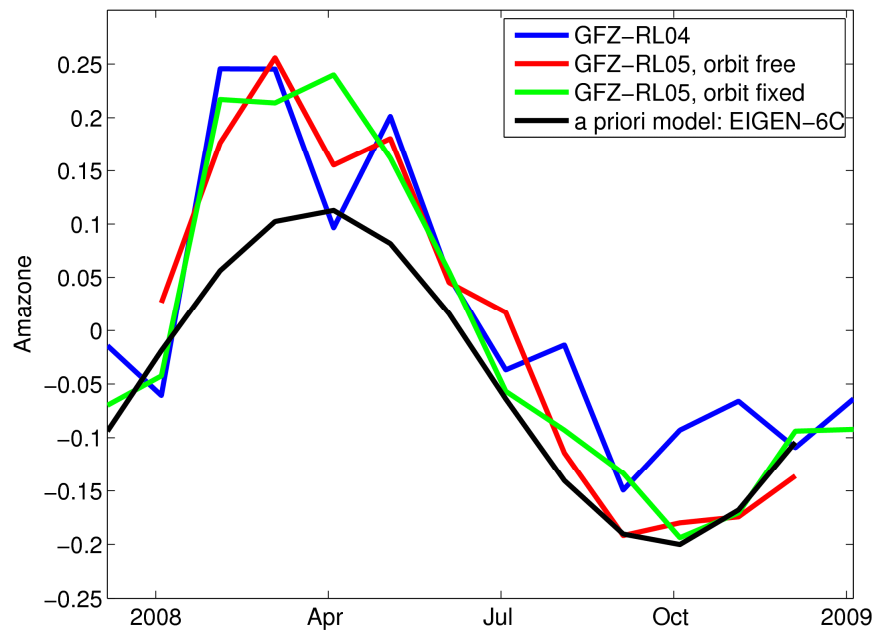
## Noise



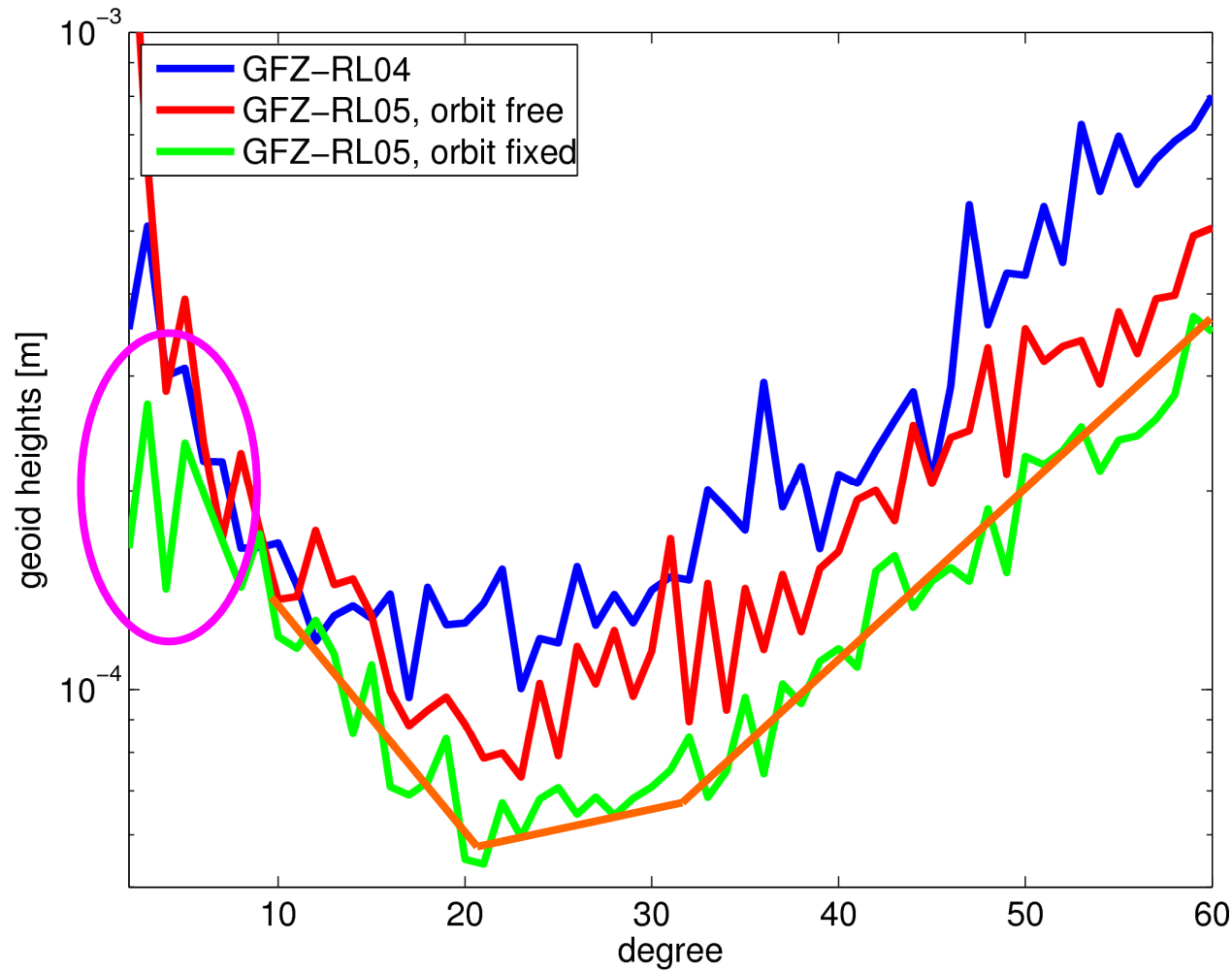
A priori model:  
**EIGEN-6C**  
(incl. time-var. d/o 50)

Stochastic accelerations:  
60 min, in R,S,W

## Signal



# Signal and Noise in monthly models (GRACE)



Reference:  
EIGEN-6C

Expected

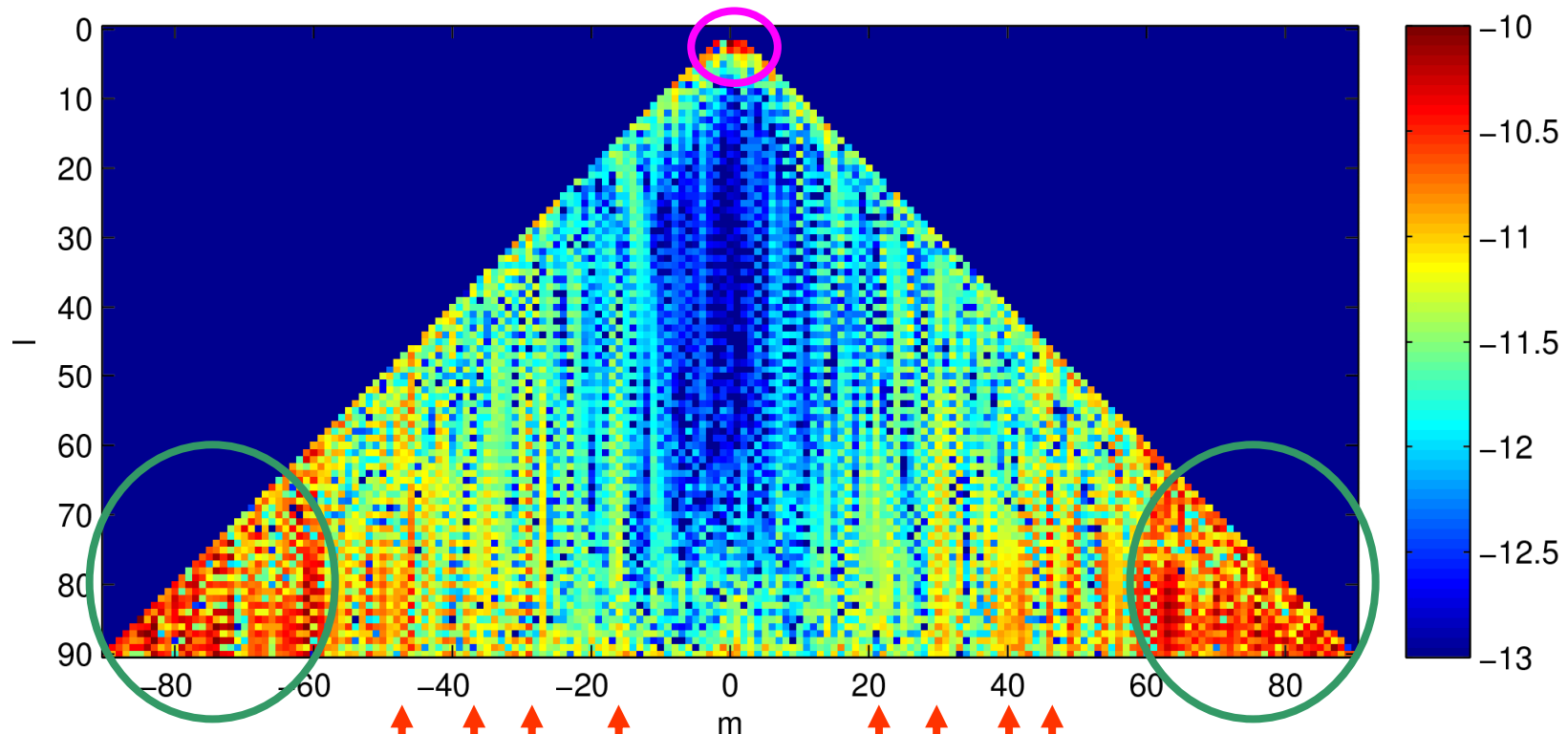
Surprising!

# S,C-Coefficients

Difference: common estimation – orbit fixed

Example: March 2008

Expected



Surprising

Dominated by noise

Hotine-Marussi, 17<sup>th</sup>-21<sup>st</sup> June 2013, Roma

# Direct – Spacewise – Timewise Analysis

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- Direct approach:  
generalized orbit determination problem
  - arc specific parameters
  - model parameters
- Space wise approach:  
grid values are interpolated from observations  
=> S,C–Analysis (integral formulas)
- Time wise approach:  
observations as timeseries along orbit  
Fourier–Analysis => Lumped Coefficients  
=> Spherical Harmonic Coefficients



# Timewise approach

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- Potential along orbit
- Gravitational observations in satellite fixed frame
- Orbit perturbations relative to reference orbit (in satellite fixed frame)
- Inter-satellite observations
- Time derivatives

# Gravity potential along orbit

$$\begin{aligned}
 V(r, I, u, \Lambda) = & \frac{GM}{r} \sum_{m=0}^L \sum_{k=-L}^L \sum_{l=\max(m, |k|)}^L \left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I) \\
 & \left\{ \begin{array}{l} \bar{C}_{lm} \cos \psi_{km} + \bar{S}_{lm} \sin \psi_{km} \\ -\bar{S}_{lm} \cos \psi_{km} + \bar{C}_{lm} \sin \psi_{km} \end{array} \right\} \begin{array}{l} l-m \text{ even} \\ l-m \text{ odd} \end{array} \\
 = & \sum_{m=0}^L \sum_{k=-L}^L A_{mk}^V \cos \psi_{km} + B_{mk}^V \sin \psi_{km}
 \end{aligned}$$

Inclination Functions

Lumped Coefficients

$$\psi_{km} = ku + m\Lambda$$

# Lumped Coefficients: potential

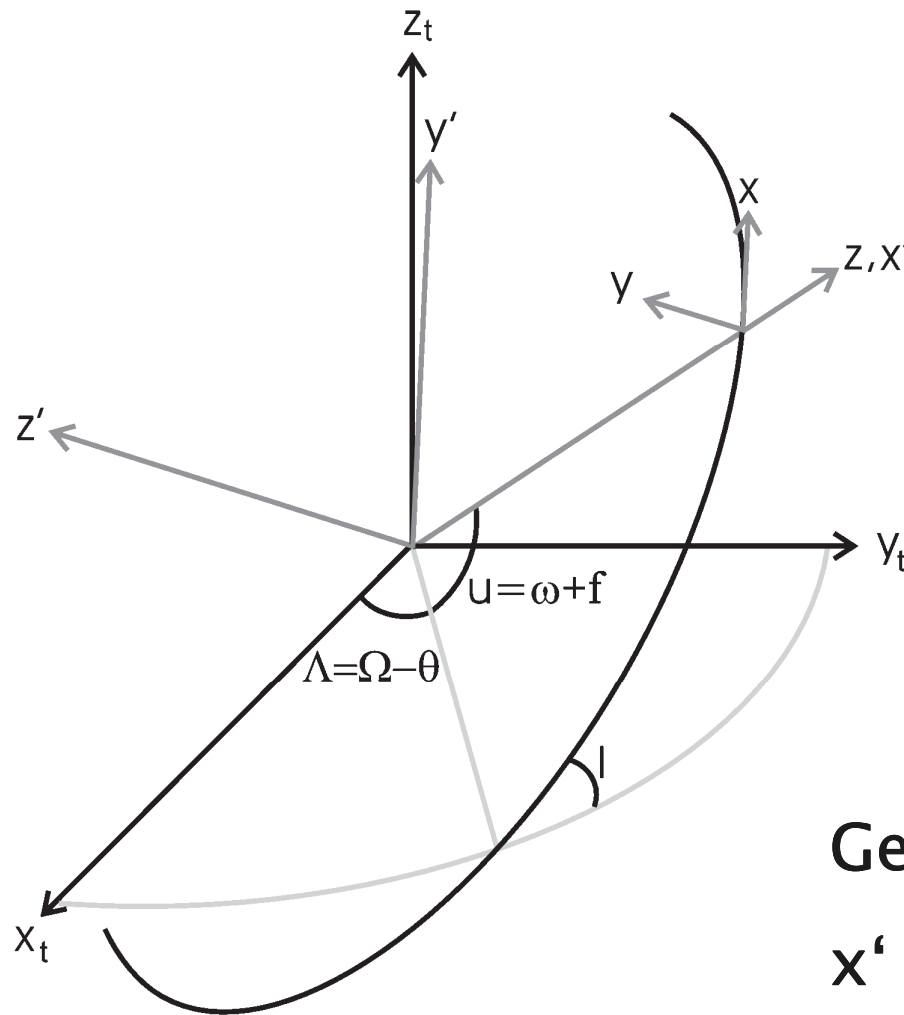
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$$A_{mk}^V = \sum_{l=\max(m,|k|)}^L \bar{H}_{lmk}^V \begin{cases} \bar{C}_{lm} & l-m \text{ even} \\ -\bar{S}_{lm} & l-m \text{ odd} \end{cases}$$

$$B_{mk}^V = \sum_{l=\max(m,|k|)}^L \bar{H}_{lmk}^V \begin{cases} \bar{S}_{lm} & l-m \text{ even} \\ \bar{C}_{lm} & l-m \text{ odd} \end{cases}$$

**Transfer:**  $\bar{H}_{lmk}^V = \frac{GM}{r} \left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I)$

# Co-Rotating frame



Earth fixed frame:

$x_t, y_t, z_t$

Satellite fixed frame:

$x$  = along-track

$y$  = cross-track

$z$  = radial

Geocentric, rotating frame:

$x' \parallel z, y' \parallel x, z' \parallel y$

# Potential => gravitational acceleration

Gradient in  
satellite  
fixed frame:

$$\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{1}{r \cos \phi'} \frac{\partial}{\partial \lambda'} \\ \frac{1}{r} \frac{\partial}{\partial \phi'} \\ \frac{\partial}{\partial r} \end{pmatrix}$$

$$\underline{\ddot{x}} = \nabla V$$



Transfer:

$$\bar{H}_{lmk}^x = \frac{1GM}{r} \left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I) k$$

$$\bar{H}_{lmk}^y = \frac{1GM}{r} \left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}^\times(I)$$

$$\bar{H}_{lmk}^z = -\frac{l+1}{r} \frac{GM}{r} \left(\frac{a_E}{r}\right)^l \bar{F}_{lmk}(I)$$

# Gravitational accelerations => orbit perturbations

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Hill (1878)

Equations  
of motion:

$$\begin{aligned} \ddot{x} + 2n\dot{z} &= \frac{\partial \mathcal{T}}{\partial x} \\ \ddot{y} + n^2 y &= \frac{\partial \mathcal{T}}{\partial y} \\ \ddot{z} - 2n\dot{x} - 3n^2 z &= \frac{\partial \mathcal{T}}{\partial z} \end{aligned}$$

Perturbing potential

x, y, z relative to circular reference orbit (n const.)

Solvable analytically (exact)!

But only valid for circular orbits (approx.)

# Transfer: orbit perturbations

$$\bar{H}_{lmk}^{dx} = \frac{(3n^2 + \dot{\psi}_{mk}^2)k - 2n\dot{\psi}_{mk}(l+1)}{\dot{\psi}_{mk}^2(n^2 - \dot{\psi}_{mk}^2)} \cdot \frac{GM}{a_E^2} \left(\frac{a_E}{r}\right)^{l+2} \bar{F}_{lmk}(I)$$

Transfer:  $\bar{H}_{lmk}^{dy} = \frac{1}{n^2 - \dot{\psi}_{mk}^2} \frac{GM}{a_E^2} \left(\frac{a_E}{r}\right)^{l+2} \bar{F}_{lmk}^\times(I)$

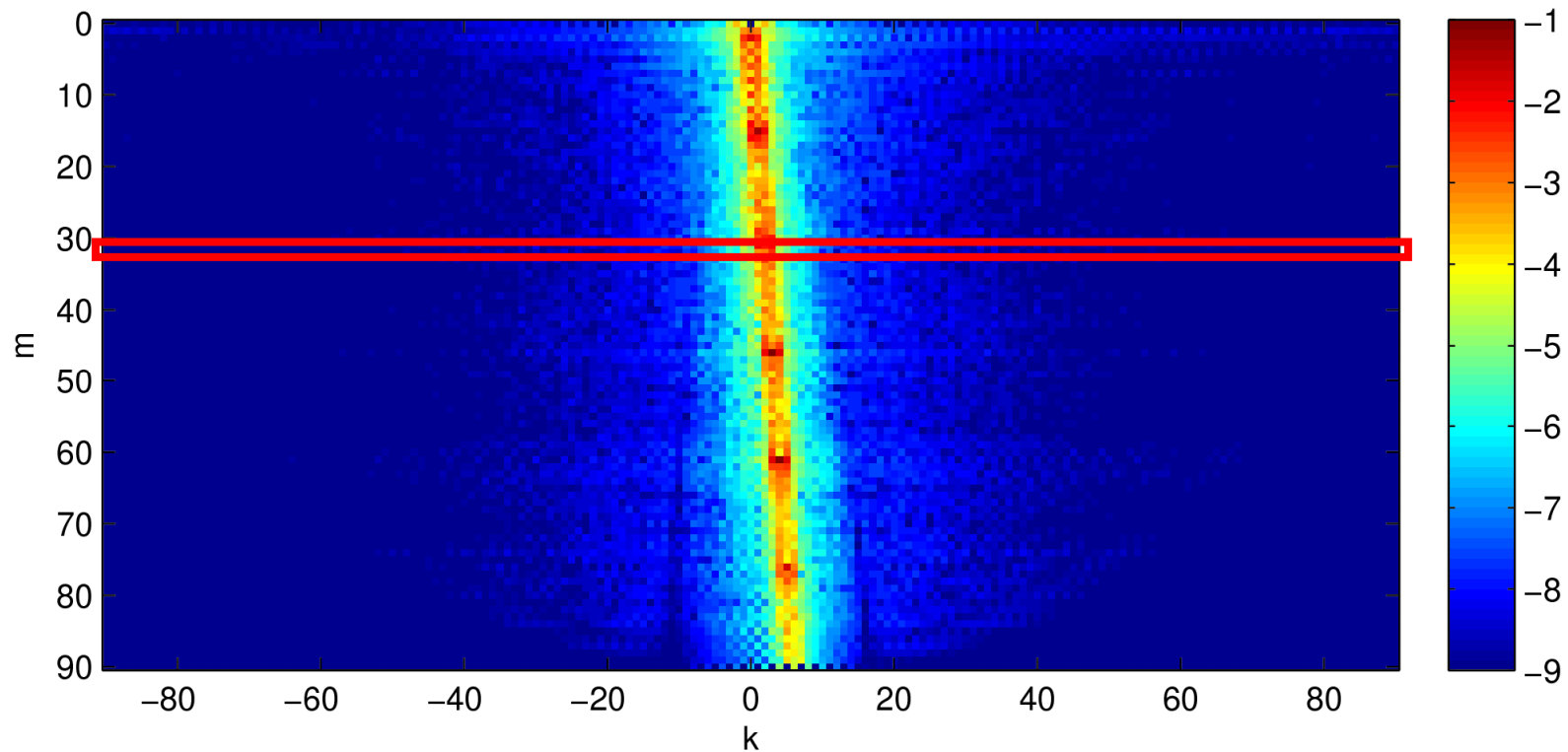
$$\bar{H}_{lmk}^{dz} = \frac{2nk - (l+1)\dot{\psi}_{mk}}{\dot{\psi}_{mk}(n^2 - \dot{\psi}_{mk}^2)} \cdot \frac{GM}{a_E^2} \left(\frac{a_E}{r}\right)^{l+2} \bar{F}_{lmk}(I)$$

**0-RESONANCE**

**N-RESONANCE**

# Lumped Coef.: Along-track orbit perturbations

Difference: common estimation – orbit fixed

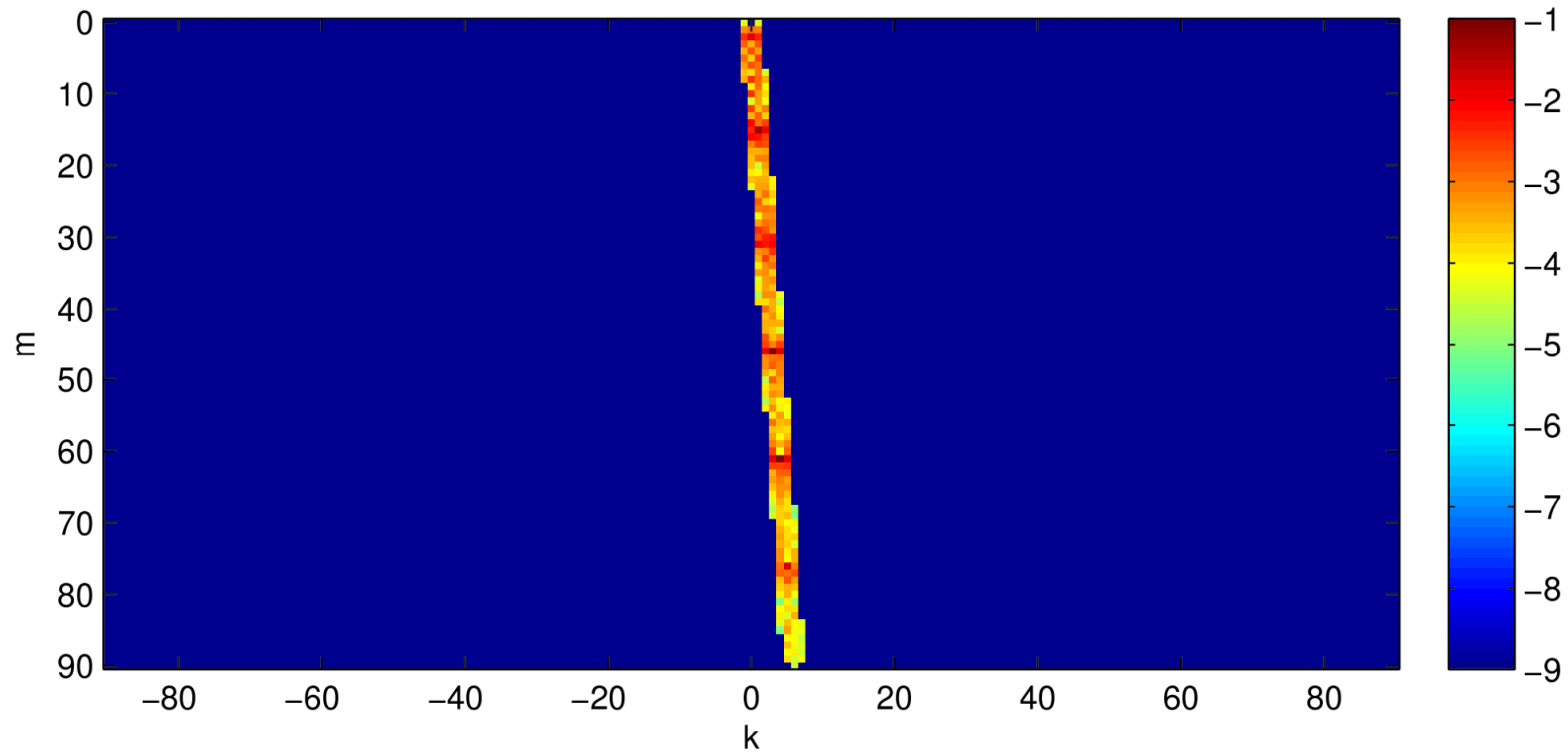


$S_{lm}$ ,  $C_{lm}$  depend on  $A_{mk}$ ,  $B_{mk}$  of same order  $m$



# Lumped Coef.: Along-track orbit perturbations

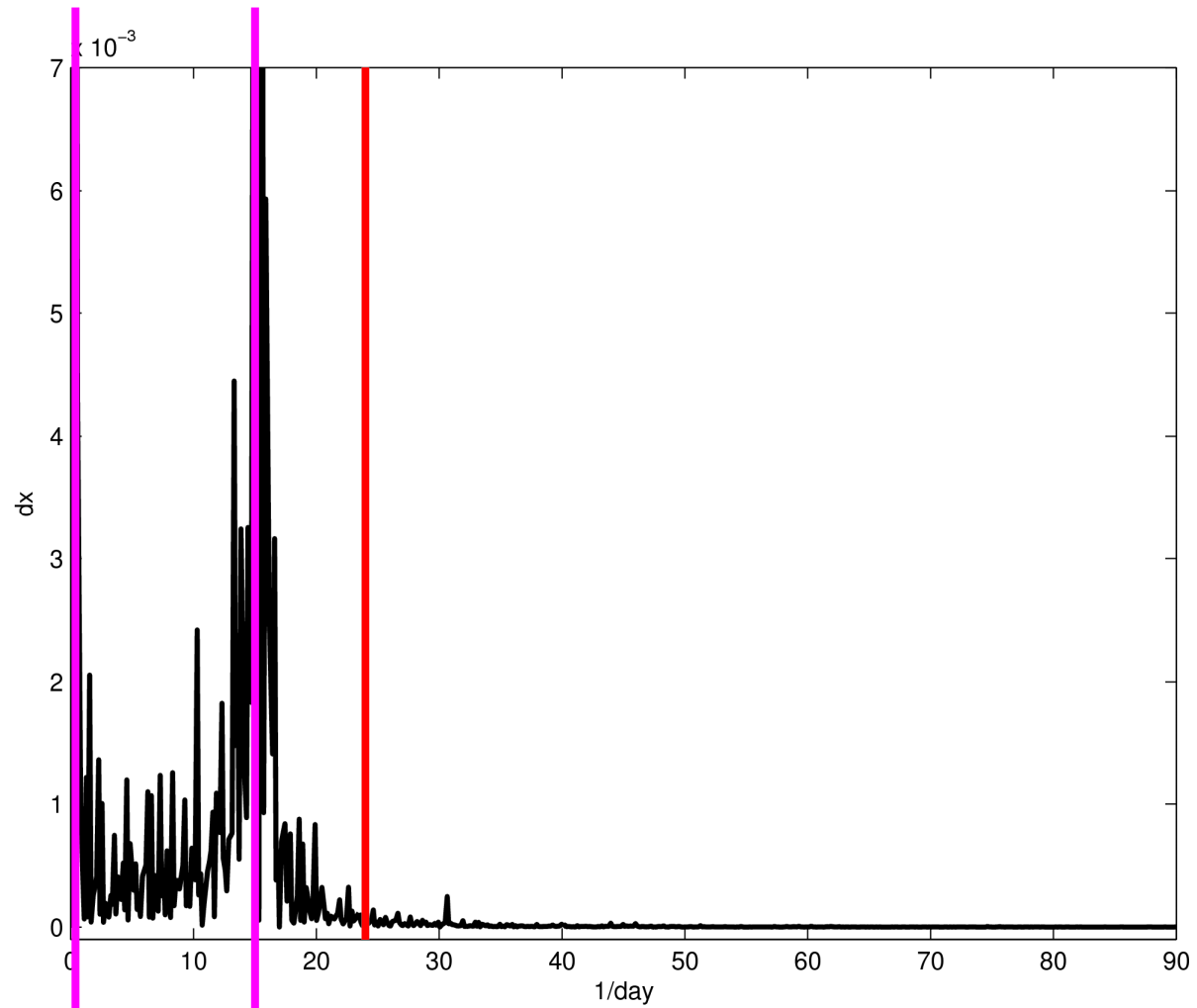
Frequency < 24 rev/day



$$\dot{\psi}_{km} = k\dot{u} + m\dot{\Lambda}$$

# Amplitude Spectrum (Lumped Coef. dx)

Frequency of  
stoch.  
accelerations



Resonance  
of Transfer

# Discussion

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- Stochastic orbit parameters increase consistency between a priori and estimated gravity field.
- Aggravated when correlations are broken.
- Whole S,C-spectrum is affected by only few low frequent stochastic accelerations.
- Can be explained via lumped coefficients by timewise analysis.
- Could probably be useful to regularize lumped coefficients (instead of S, C).
- Is complicated by resonance effects in case of orbit perturbations and derivatives.