

Statistical Analysis of Data from Sensitive Question Techniques

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Questions: Theory and Data Collection Methods“

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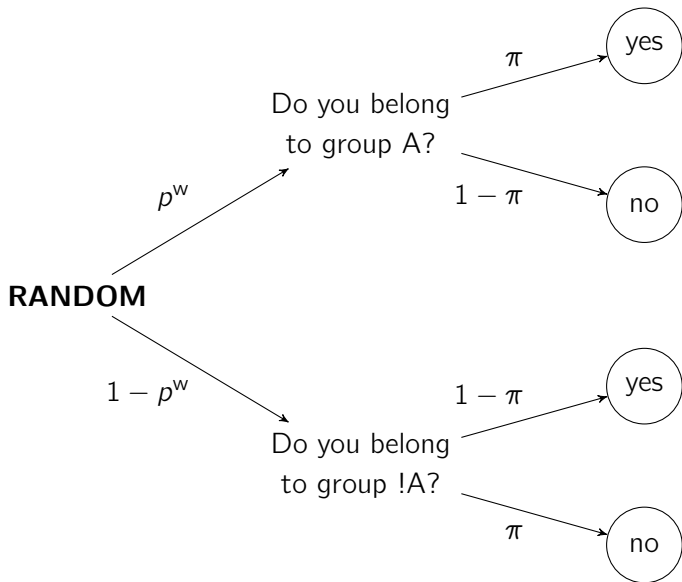
Outline

- Introduction
- Prevalence estimators for various RRT schemes
- Generalized regression estimators for RRT
- Cheating correction in RRT
- Analysis of Item Count Data

Introduction

- Various special techniques have been proposed to collect data for sensitive questions.
- The basic idea of these techniques is to anonymize answers by adding noise to the data (e.g. replacing some of the answers by random answers, aggregating answers from several questions)
- As long as the properties of the misclassification procedure are known, the statistical distribution of the sensitive question can be recovered.
- Some of these techniques are
 - ▶ Randomized Response Technique (RRT) in various variants
 - ★ Warner, unrelated question, forced-response, Mangat, Kuk, Crosswise Model, ...
 - ▶ Item Count Technique (ICT) a.k.a. List Experiment

Warner's RRT (Warner 1965)



Warner's RRT (Warner 1965)

- “Group A” is the sensitive group, i.e. belonging to group A is equivalent to answering “yes” to the sensitive question ($SQ = 1$).
- Point estimate for $\pi = \Pr(\text{“belongs to group A”}) = \Pr(SQ = 1)$?

$$\Pr(\text{“yes”}) = \lambda = p^w \pi + (1 - p^w)(1 - \pi)$$

$$\pi = \frac{\lambda + p^w - 1}{2p^w - 1}, \quad p^w \neq 0.5$$

$$\hat{\lambda} = \frac{1}{n} \sum_{i=1}^n y_i \quad \text{where} \quad y_i = \begin{cases} 1 & \text{if “yes”} \\ 0 & \text{if “no”} \end{cases}$$

$$\hat{\pi} = \frac{\hat{\lambda} + p^w - 1}{2p^w - 1}$$

Warner's RRT (Warner 1965)

Sampling variance of $\hat{\pi}$?

Delta method:

$$\text{Var}\{f(x)\} = \left(\frac{df(x)}{dx}\right)^2 \text{Var}(x)$$

if $f(x)$ is a linear transformation.

$$f(\hat{\lambda}) = \frac{\hat{\lambda} + p^w - 1}{2p^w - 1} \quad \Rightarrow \quad f' = \frac{1}{2p^w - 1}$$

$$\widehat{\text{Var}}(\hat{\lambda}) = \frac{\hat{\lambda}(1 - \hat{\lambda})}{n}$$

$$\widehat{\text{Var}}(\hat{\pi}) = \frac{\hat{\lambda}(1 - \hat{\lambda})}{n(2p^w - 1)^2} = \frac{\hat{\pi}(1 - \hat{\pi})}{n} + \frac{p^w(1 - p^w)}{n(2p^w - 1)^2}$$

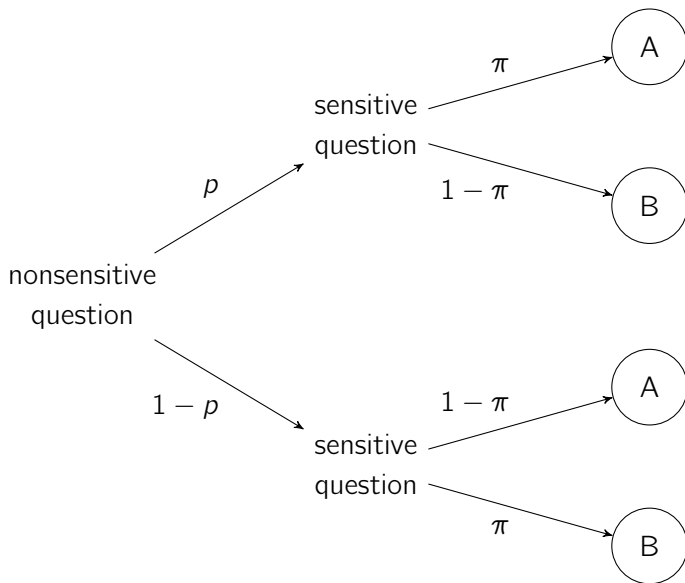
Crosswise Model (Yu et al. 2008)

- Ask a sensitive question and a nonsensitive question and let the respondent indicate whether ...
 - A** the answers to the questions are the same (both “yes” or both “no”)
 - B** the answers are different (one “yes”, the other “no”)

		nonsensitive question	
		no	yes
sensitive question	no	A	B
	yes	B	A

- ▶ Assumption: The two questions are uncorrelated.
- ▶ $p = \Pr(\text{“yes”})$ of the nonsensitive question must not be 0.5.

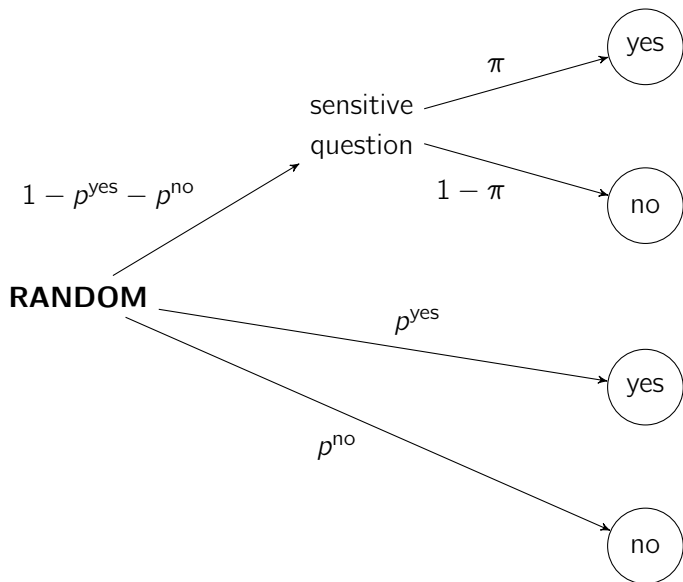
Crosswise Model (Yu et al. 2008)



Crosswise Model (Yu et al. 2008)

- The Crosswise Model is formally equivalent to Warner's RRT with $p^w = p$.

Forced Response RRT (Boruch 1971)



Forced Response RRT (Boruch 1971)

$$\Pr(\text{"yes"}) = \lambda = (1 - p^{\text{yes}} - p^{\text{no}})\pi + p^{\text{yes}}$$

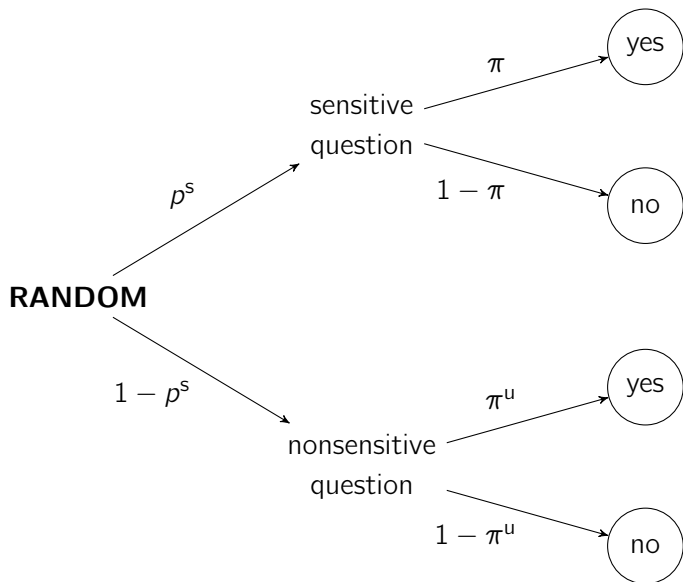
Hence

$$\hat{\pi} = \frac{\hat{\lambda} - p^{\text{yes}}}{1 - p^{\text{yes}} - p^{\text{no}}}$$

and

$$\widehat{\text{Var}}(\hat{\pi}) = \frac{\hat{\lambda}(1 - \hat{\lambda})}{n(1 - p^{\text{yes}} - p^{\text{no}})^2}$$

Unrelated Question RRT (Horvitz et al. 1967)



Unrelated Question RRT (Horvitz et al. 1967)

$$\Pr(\text{"yes"}) = \lambda = p^s \pi + (1 - p^s) \pi^u$$

Let

$$p^s = 1 - p^{\text{yes}} - p^{\text{no}}, \quad \pi^u = \frac{p^{\text{yes}}}{p^{\text{yes}} + p^{\text{no}}}$$

then

$$\begin{aligned} \lambda &= (1 - p^{\text{yes}} - p^{\text{no}}) \pi + (1 - (1 - p^{\text{yes}} - p^{\text{no}})) \frac{p^{\text{yes}}}{p^{\text{yes}} + p^{\text{no}}} \\ &= (1 - p^{\text{yes}} - p^{\text{no}}) \pi + p^{\text{yes}} \end{aligned}$$

Unrelated Question RRT (Horvitz et al. 1967)

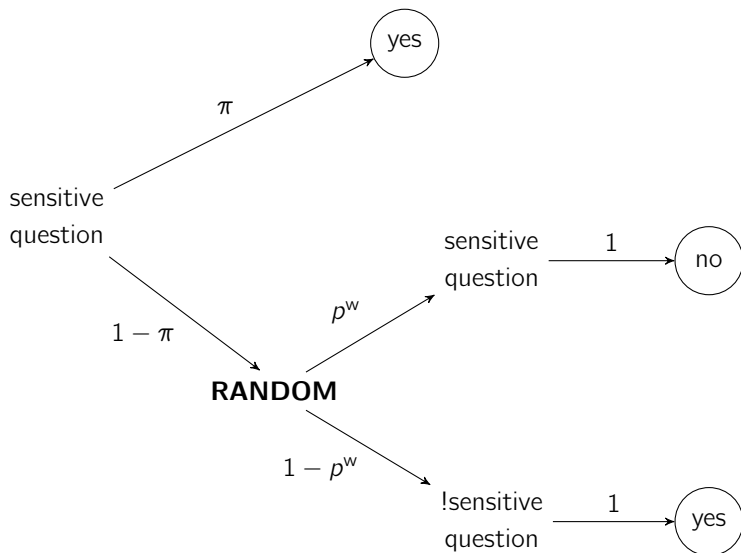
- Hence, if π^u is known, the Unrelated Question RRT is formally equivalent to the Forced Response RRT with

$$p^{\text{yes}} = (1 - p^s)\pi^u, \quad p^{\text{no}} = (1 - p^s)(1 - \pi^u)$$

- If π^u is unknown, it has to be estimated from a control sample. This does not change the formula for the point estimate, but it has consequences for the sampling variance (increase). Use bootstrap for variance estimation in this case.
- Alternatively, here's the variance formula (assuming that π^u is estimated using an independent sample):

$$\hat{\pi} = \frac{1}{p^s} \hat{\lambda} - \frac{1 - p^s}{p^s} \hat{\pi}^u \Rightarrow \widehat{\text{Var}}(\hat{\pi}) = \left(\frac{1}{p^s}\right)^2 \widehat{\text{Var}}(\hat{\lambda}) + \left(\frac{1 - p^s}{p^s}\right)^2 \widehat{\text{Var}}(\hat{\pi}^u)$$

Mangat's RRT (Mangat 1994)



Mangat's RRT (Mangat 1994)

$$\Pr(\text{"red"}) = \lambda = \pi p_1 + (1 - \pi)p_2$$

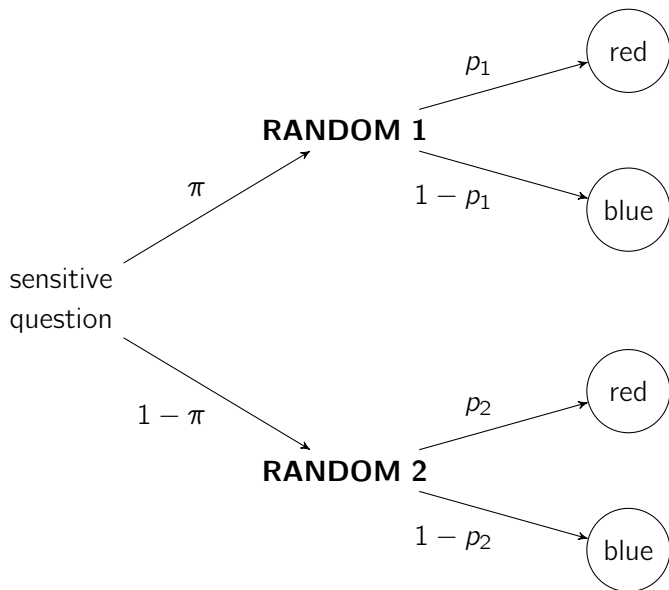
Hence

$$\hat{\pi} = \frac{\hat{\lambda} - p_2}{p_1 - p_2}, \quad p_1 \neq p_2$$

and

$$\widehat{\text{Var}}(\hat{\pi}) = \frac{\hat{\lambda}(1 - \hat{\lambda})}{n(p_1 - p_2)^2}$$

Kuk's RRT (Kuk 1990)



Kuk's RRT (Kuk 1990)

$$\Pr(\text{"yes"}) = \lambda = \pi + (1 - \pi)(1 - p^w)$$

Hence

$$\hat{\pi} = \frac{\hat{\lambda} + p^w - 1}{p^w}$$

and

$$\widehat{\text{Var}}(\hat{\pi}) = \frac{\hat{\lambda}(1 - \hat{\lambda})}{n(p^w)^2} = \frac{\hat{\pi}(1 - \hat{\pi})}{n} + \frac{(1 - \hat{\pi})(1 - p^w)}{np^w}$$

Generalized regression estimator for RRT

- Let

Y_i response ($Y_i = 1$ if “yes” in RRT or “A” in CM, else $Y_i = 0$)

λ_i probability of $Y_i = 1$

π_i (unknown) prevalence of sensitive item

p_i^w probability of the non-negated question in Warner's RRT (prevalence of nonsensitive item in CM)

p_i^{yes} probability of a forced “yes”

p_i^{no} probability of a forced “no”

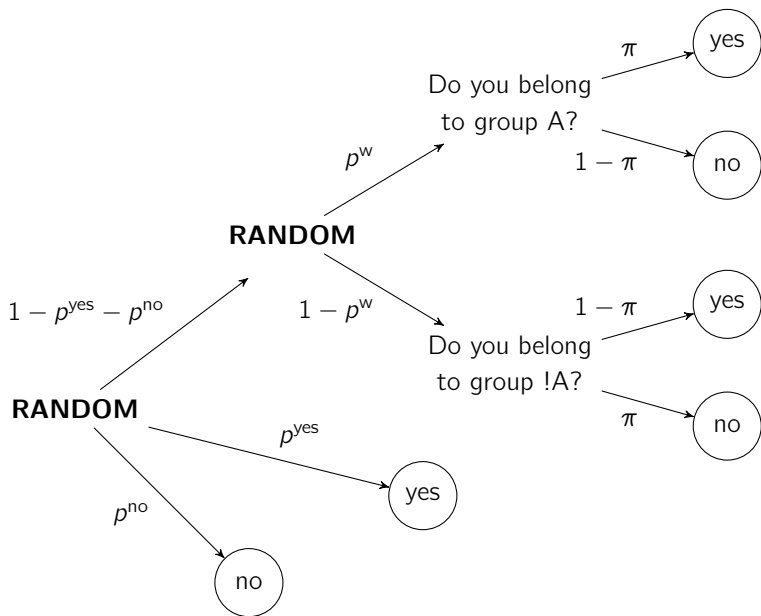
- Then

$$\lambda_i = (1 - p_i^{\text{yes}} - p_i^{\text{no}})p_i^w\pi_i + (1 - p_i^{\text{yes}} - p_i^{\text{no}})(1 - p_i^w)(1 - \pi_i) + p_i^{\text{yes}}$$

and hence

$$\pi_i = \frac{\lambda_i - (1 - p_i^{\text{yes}} - p_i^{\text{no}})(1 - p_i^w) - p_i^{\text{yes}}}{(2p_i^w - 1)(1 - p_i^{\text{yes}} - p_i^{\text{no}})}$$

Generalized regression estimator for RRT



Generalized regression estimator for RRT

- By parametrizing π_i we can formulate regression models.
- For example, assuming $\pi_i = X_i'\beta$, we can estimate β by applying least squares regression to a transformed response variable

$$\tilde{Y}_i = \frac{Y_i - (1 - p_i^{\text{yes}} - p_i^{\text{no}})(1 - p_i^{\text{w}}) - p_i^{\text{yes}}}{(2p_i^{\text{w}} - 1)(1 - p_i^{\text{yes}} - p_i^{\text{no}})}$$

- This is because

$$E(\text{SQ} = 1|X_i) = \frac{E(Y_i|X_i) - (1 - p_i^{\text{yes}} - p_i^{\text{no}})(1 - p_i^{\text{w}}) - p_i^{\text{yes}}}{(2p_i^{\text{w}} - 1)(1 - p_i^{\text{yes}} - p_i^{\text{no}})}$$

Generalized regression estimator for RRT

- More reasonable might be to assume a functional form such as $\ln(\pi_i/(1 - \pi_i)) = X_i'\beta$ (logit), i.e. $\pi_i = e^{X_i'\beta}/(1 + e^{X_i'\beta})$.
- In this case, we can derive the log likelihood as

$$\begin{aligned}\ln L &= \sum_{i=1}^n [Y_i \ln(\lambda_i) + (1 - Y_i) \ln(1 - \lambda_i)] \\ &= \sum_{i=1}^n [Y_i \ln(R_i) + (1 - Y_i) \ln(S_i) - \ln(1 + e^{X_i'\beta})]\end{aligned}$$

with

$$\begin{aligned}R_i &= c_i + q_i e^{X_i'\beta} & c_i &= (1 - p_i^{\text{yes}} - p_i^{\text{no}})(1 - p_i^{\text{w}}) + p_i^{\text{yes}} \\ S_i &= (1 - c_i) + (1 - q_i) e^{X_i'\beta} & q_i &= (1 - p_i^{\text{yes}} - p_i^{\text{no}}) p_i^{\text{w}} + p_i^{\text{yes}}\end{aligned}$$

and estimate β using maximum likelihood methods.

Two Stata commands

- Least-squares estimation with $\pi_i = X_i'\beta$ (Jann 2008):

```
rrreg depvar [indepvars] [if] [in] [weight] [, regress_options  
      pwarner(#|varname) pyes(#|varname) pno(#|varname) ]
```

- Maximum likelihood estimation with $\pi_i = e^{X_i'\beta} / (1 + e^{X_i'\beta})$ (Jann 2005):

```
rrlogit depvar [indepvars] [if] [in] [weight] [, logit_options  
      pwarner(#|varname) pyes(#|varname) pno(#|varname) ]
```

- `rrlogit` may make more sense in terms of functional form. However, `rrreg` is more robust, especially if there is noncompliance with the RRT procedure.

Example

```
. use gr/rrt07
(Sensitive Questions Online Survey 2007)
. fre grp
grp — Experimental group
```

		Freq.	Percent	Valid	Cum.
Valid	1 direct	609	38.42	38.42	38.42
	2 manual coin toss	169	10.66	10.66	49.09
	3 electronic coin toss	188	11.86	11.86	60.95
	4 banknote with phone	227	14.32	14.32	75.27
	5 banknote without phone	190	11.99	11.99	87.26
	6 phone number	202	12.74	12.74	100.00
	Total	1585	100.00	100.00	

```
. generate rrt = inrange(grp,2,6)
. regress keepchange if rrt==0
```

Source	SS	df	MS			
Model	0	0	.	Number of obs = 608		
Residual	149.748355	607	.246702397	F(0, 607) = 0.00		
Total	149.748355	607	.246702397	Prob > F = .		
				R-squared = 0.0000		
				Adj R-squared = 0.0000		
				Root MSE = .49669		

keepchange	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_cons	.5608553	.0201435	27.84	0.000	.5212959	.6004147

(Data fromm Coutts and Jann 2011)

Example

```
. rrreg keepchange if rrt==1, pyes(0.5)
```

Randomized response regression

```
Number of obs   =      927
F(   0,      926) =      0.00
Prob > F        =          .
R-squared       =      0.0000
Adj R-squared   =      0.0000
Root MSE       =      0.8046
```

keepchange	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_cons	.5943905	.0264269	22.49	0.000	.542527	.646254

```
Pr(non-negated question) = 1
Pr(surrogate "yes")      = 0.5
Pr(surrogate "no")       = 0
```

Example

```
. generate pyes = cond(rrt==1, 0.5, 0)
. rrreg keepchange rrt, pyes(pyes)
```

```
Randomized response regression
```

Number of obs	=	1535
F(1, 1533)	=	0.84
Prob > F	=	0.3581
R-squared	=	0.0006
Adj R-squared	=	-0.0001
Root MSE	=	0.6991

keepchange	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
rrt	.0335352	.0364839	0.92	0.358	-.0380285 .1050989
_cons	.5608553	.0283522	19.78	0.000	.505242 .6164685

```
Pr(non-negated question) = 1
Pr(surrogate "yes")      = pyes
Pr(surrogate "no")       = 0
```

Example

```
. rrreg keepchange rrt highschool, pyes(pyes)
```

Randomized response regression

```
Number of obs   =      1535  
F(   2,   1532) =       4.53  
Prob > F        =      0.0110  
R-squared       =      0.0059  
Adj R-squared   =      0.0046  
Root MSE       =      0.6975
```

keepchange	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
rrt	.0347934	.0364012	0.96	0.339	-.036608	.1061948
highschool	.1055695	.0368606	2.86	0.004	.0332669	.1778721
_cons	.4936589	.0367501	13.43	0.000	.4215731	.5657446

```
Pr(non-negated question) = 1
```

```
Pr(surrogate "yes")      = pyes
```

```
Pr(surrogate "no")       = 0
```

Example

```
. generate rrtXhs = rrt*highschool  
. rrreg keepchange rrt highschool rrtXhs, pyes(pyes)
```

```
Randomized response regression
```

Number of obs	=	1535
F(3, 1531)	=	3.32
Prob > F	=	0.0193
R-squared	=	0.0065
Adj R-squared	=	0.0045
Root MSE	=	0.6975

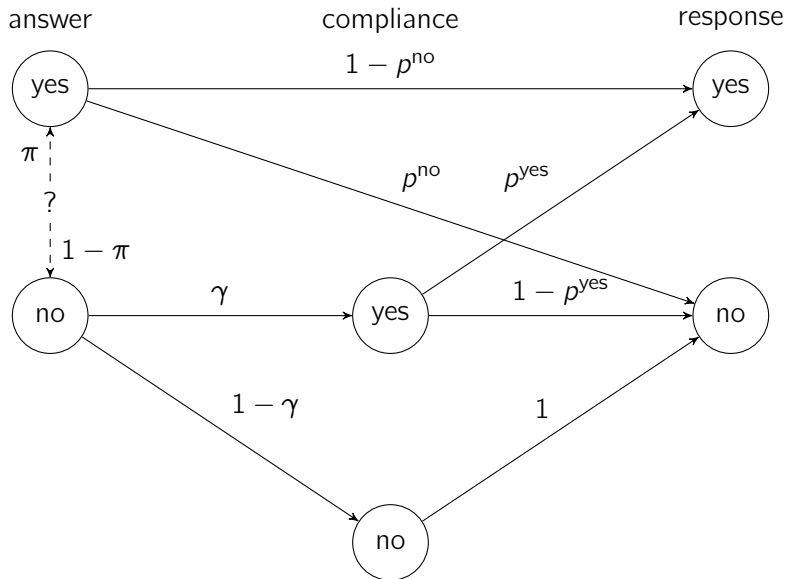
keepchange	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
rrt	.0799137	.0599938	1.33	0.183	-.0377651	.1975924
highschool	.1489237	.058808	2.53	0.011	.033571	.2642765
rrtXhs	-.071412	.0754755	-0.95	0.344	-.2194583	.0766344
_cons	.4660633	.0469181	9.93	0.000	.3740329	.5580938

```
Pr(non-negated question) = 1  
Pr(surrogate "yes") = pyes  
Pr(surrogate "no") = 0
```

A little bit of magic: Cheating correction in RRT

- In many RRT designs, the “self-protective no” bias can occur.
- In these designs, some of the respondents are instructed to answer “yes” by the randomization device, even though the sensitive item does not apply to them.
- There is evidence that these respondents often deviate from the instructions and answer “no”.
- Such non-compliance introduces a large bias to RRT estimates. It is noteworthy that this bias does not come from respondents who did commit the sensitive behavior and want to conceal it. It comes from respondents who did not and don't want it to look like they did.
- In a standard design, it is not possible to account for such “cheaters”. However, if the RRT design parameters are varied, this variation can be used to identify the proportion of cheaters and correct the estimates.

A little bit of magic: Cheating correction in RRT



A little bit of magic: Cheating correction in RRT

- Assumptions:
 - ▶ There is random variation in p^{yes} and p^{no} between respondents.
 - ▶ π and γ do not depend on p^{yes} and p^{no} (which may be justified if the variation in p is small)
 - ▶ Respondents do not say “yes” if instructed to say “no” by the randomization device.
- π and γ can then be estimated using the following log likelihood:

$$\ln L = \sum_{i=1}^n Y_i \ln(\ell_i) + (1 - Y_i) \ln(1 - \ell_i)$$

with

$$\ell_i = \pi_i(1 - p_i^{\text{no}} - \gamma p_i^{\text{yes}}) + \gamma p_i^{\text{yes}}$$

A little bit of magic: Analysis

```
program define rrcheat_lf
  args lnf theta1 cheat
  local p1 $rrcheat_pyess
  local p2 $rrcheat_pno
  quietly replace `lnf' = cond($ML_y1, ///
    ln(`theta1' * (1 - `p2' - (1-`cheat')*`p1') + (1-`cheat')*`p1'), ///
    ln(1 - (`theta1' * (1 - `p2' - (1-`cheat')*`p1') + (1-`cheat')*`p1'))))
end
forv i = 1/5 {
  local depvar: word `i' of $sqvar
  global rrcheat_pyess pyessQ`i'
  global rrcheat_pno pnoQ`i'
  ml model lf rrcheat_lf (`depvar': `depvar' = ) /cheat if RRT==1
  ml maximize
  eststo `depvar'
}
esttab, nonumb nostar mti se b(1) transform(100*100) ///
  eqlab(none) coef(main:_cons "RRT adjusted" cheat:_cons "Cheaters")
```


A little bit of magic: Results

	copy	notes	drugs	partial	severe
RRT adjusted	17.9 (6.5)	12.0 (6.1)	16.7 (5.6)	14.3 (6.6)	6.7 (5.9)
Cheaters	-9.5 (36.1)	-3.6 (31.9)	88.9 (36.9)	54.3 (40.1)	36.1 (31.8)
N	2855	2855	2849	2105	2104

Standard errors in parentheses

Unadjusted results for comparison:

	copy	notes	drugs	partial	severe
DQ	17.5 (1.2)	8.8 (0.9)	3.4 (0.6)	2.5 (0.6)	1.5 (0.5)
RRT	19.6 (1.2)	12.7 (1.1)	0.6 (1.0)	4.2 (1.2)	-0.6 (1.1)
CM	27.2 (2.0)	15.0 (1.9)	9.9 (1.9)	8.2 (2.1)	3.0 (2.0)

(Data from Höglinger et al. 2012)

Analysis of Item Count Data

- Item Count Design (Droitcour et al. 1991):

group A (short list)	group B (long list)
nonsensitive item 1	nonsensitive item 1
nonsensitive item 2	nonsensitive item 2
nonsensitive item 3	nonsensitive item 3
	sensitive item

- How many items do apply to you?
- Two randomized groups, one with the short list, one with the long list (single list design).

Analysis of Item Count Data

- Estimate of the probability of the sensitive item $\pi = \Pr(SQ = 1)$?
- Mean difference between the two groups:

$$\hat{\pi} = \bar{y}^{LL} - \bar{y}^{SL} = \frac{1}{n^B} \sum_{i \in B} y_i - \frac{1}{n^A} \sum_{i \in A} y_i$$

- Variance of $\hat{\pi}$?

$$\text{Var}(\hat{\pi}) = \text{Var}(\bar{y}^{LL}) + \text{Var}(\bar{y}^{SL})$$

Analysis of Item Count Data

- Double list design:

- ▶ Both groups answer to two sets of items. In one group, the sensitive item is paired with the first set of nonsensitive items, in the other group the sensitive item is paired with the second set of nonsensitive items.

Set 1:	group A	group B
	nonsensitive item 1	nonsensitive item 1
	nonsensitive item 2	nonsensitive item 2
	nonsensitive item 3	nonsensitive item 3
		sensitive item

Set 2:	group A	group B
	nonsensitive item 4	nonsensitive item 4
	nonsensitive item 5	nonsensitive item 5
	nonsensitive item 6	nonsensitive item 6
	sensitive item	

Analysis of Item Count Data

$$\hat{\pi}_1 = \bar{y}^{LL1} - \bar{y}^{SL1}, \quad \hat{\pi}_2 = \bar{y}^{LL2} - \bar{y}^{SL2}$$

$$\begin{aligned}\hat{\pi} &= \frac{\hat{\pi}_1 + \hat{\pi}_2}{2} = \frac{(\bar{y}^{LL1} - \bar{y}^{SL1}) + (\bar{y}^{LL2} - \bar{y}^{SL2})}{2} \\ &= \frac{(\bar{y}^{LL1} - \bar{y}^{SL2}) + (\bar{y}^{LL2} - \bar{y}^{SL1})}{2} \\ &= \frac{\frac{1}{n^B} \sum_{i \in B} (y_{1i} - y_{2i}) + \frac{1}{n^A} \sum_{i \in A} (y_{2i} - y_{1i})}{2}\end{aligned}$$

$$\begin{aligned}\text{Var}(\hat{\pi}) &= \frac{\text{Var}(\hat{\pi}_1) + \text{Var}(\hat{\pi}_2) - 2\text{Cov}(\hat{\pi}_1, \hat{\pi}_2)}{4} \\ &= \frac{\text{Var}(\bar{y}^{LL1} - \bar{y}^{SL2}) + \text{Var}(\bar{y}^{LL2} - \bar{y}^{SL1})}{4}\end{aligned}$$

Analysis of Item Count Data

- Regression model for single list design:
 - ▶ Estimate β by applying least-squares regression (with robust standard errors) to

$$Y_i = (LL_i \cdot X_i)' \beta + X_i' \gamma + \epsilon_i$$

(For more sophisticated approaches see Glynn 2010, Imai 2010, Blair and Imai 2012.)

- Regression model for single list design:
 - ▶ Approach 1: estimate separate models (as above) for Y_1 and Y_2 , combine estimates using `suest` to obtain joint variance matrix, compute average coefficients using `lincom`
 - ▶ Approach 2: estimate a system of equations (e.g. using `sureg`) for Y_1 and Y_2 with the constraint that the coefficients are the same

Conclusions

- Suitable methods for basic analysis of data from sensitive question techniques (SQT) are easy to derive.
- Canned software exists for various RRT designs.
- Outlook
 - ▶ Add support for Mangat's RRT, Kuk's RRT, ...
 - ▶ Canned software for Item Count Data
 - ▶ The presented methods treat the SQT-data as the dependent variable. What if SQT-variables are used as predictors?
 - ▶ Correlations among SQT-variables? Analysis of multiple SQT-items?
 - ▶ More sophisticated designs/methods for cheating correction?

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