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Determination of Precise Satellite Orbits and Geodetic Parameters using Satellite Laser Ranging

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Three Pillars of Satellite Geodesy

Geometry
Determination of geometrical three-dimensional positions and velocities (in global, regional, and local reference frames),

Gravity
Determination of the Earth's gravity field and its temporal variations,

Rotation
Modeling and observing of geodynamical phenomena (tectonic plates, loading crustal deformations) including the rotation and orientation of the Earth (polar motion, Length-of-day, precession and nutation).
Observation Techniques in Satellite Geodesy

Satellite and space observation techniques contributing to the Global Geodetic Observing System (GGOS):

- **Techniques used for the definition of the International Terrestrial Reference Frame (ITRF):**
  - Global Navigation Satellite Systems (GNSS, e.g., GPS, GLONASS, Galileo, Beidou),
  - Very Long Baseline Interferometry (VLBI),
  - Doppler Orbitography and Radiopositioning Integrated by Satellite (Détermination d’Orbite et Radiopositionnement Intégré par Satellite, DORIS),
  - Satellite Laser Ranging (SLR) and Lunar Laser Ranging (LLR).

- **Other techniques of satellite geodesy:**
  - Satellite altimetry (e.g., TOPEX-Poseidon, Jason–1/2),
  - Interferometric Synthetic Aperture Radar (InSAR), e.g., TerraSar–X, TanDEM–X),
  - Satellite gravimetry (e.g., GRACE, GRAIL, GOCE),
  - Satellite optical imagery,
  - Other remote sensing techniques (e.g., geomagnetic field mapping, e.g., SWARM).
Satellite Laser Ranging (SLR)

SLR is one of the space–geodetic techniques used for precise positioning, for determination of Earth's gravity field, and for measurement of geodynamical phenomena.

In SLR the basic observable is the **two-way travel time of a laser pulse** between a ground station and a satellite. The time of flight can be transformed into a direct distance by multiplying the time of flight of a laser pulse by the velocity of light.

\[
\Delta t^s_r = \frac{2}{c} (d^s_r + \delta_{tro} + \delta_{rel}) + \frac{1}{c} \delta_{sys} + \epsilon^s_{tr},
\]
The International Laser Ranging Service (ILRS) coordinates all operational and scientific activities of the institutions involved in scientific lunar and satellite laser ranging since 1998. The ILRS provides official products primarily based on SLR observations to the spherical geodetic satellites: LAGEOS–1/2 and Etalon–1/2. The official ILRS products are based on 7-day solutions and consist of station coordinates, polar motion, and length–of–day parameters.
Geodetic SLR Satellites

Current ILRS products:

- Based on LAGEOS–1/2 & Etalon–1/2 solutions only,
- On average ~3000 normal points to LAGEOS–1/2 and ~300 normal points to Etalon–1/2 per week,
- The impact of Etalon–1/2 on the solution is virtually negligible

<table>
<thead>
<tr>
<th></th>
<th>AJISAI</th>
<th>Starlette/Stella</th>
<th>LAGEOS-1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter [m]</td>
<td>2.15</td>
<td>0.24</td>
<td>0.60</td>
</tr>
<tr>
<td>Mass [kg]</td>
<td>685</td>
<td>47/48</td>
<td>407/405</td>
</tr>
<tr>
<td>Area-to-mass [m²kg⁻¹]</td>
<td>58.0e-4</td>
<td>9.6e-4/9.4e-4</td>
<td>6.9e-4/7.0e-4</td>
</tr>
<tr>
<td>Radiation coeff. ( C_R )</td>
<td>1.03</td>
<td>1.134/1.131</td>
<td>1.13</td>
</tr>
<tr>
<td>Semi-major axis [km]</td>
<td>7.866</td>
<td>7.335/7.176</td>
<td>12.274/12.158</td>
</tr>
<tr>
<td>Orbit altitude [km]</td>
<td>1.500</td>
<td>800-1.100/830</td>
<td>5.860/5.620</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.0016</td>
<td>0.0205/0.0010</td>
<td>0.0039/0.0137</td>
</tr>
<tr>
<td>Inclination [deg]</td>
<td>50.04</td>
<td>49.84/98.57</td>
<td>109.90/52.67</td>
</tr>
<tr>
<td>Drift of node [days]</td>
<td>116.77</td>
<td>90.97/364.7</td>
<td>1050.1/569.5</td>
</tr>
<tr>
<td>Drift of perigee [days]</td>
<td>141.1</td>
<td>108.7/122</td>
<td>1680.3/822.7</td>
</tr>
<tr>
<td>Draconitic year [days]</td>
<td>89</td>
<td>72.8/182</td>
<td>560/222</td>
</tr>
<tr>
<td>( S_2 ) alias period [days]</td>
<td>44.5</td>
<td>36.5/91</td>
<td>280/111</td>
</tr>
<tr>
<td>A priori CoM corr.</td>
<td>1010 mm</td>
<td>78 mm</td>
<td>CoM¹</td>
</tr>
</tbody>
</table>

¹ station-specific CoM (Appleby et al. 2012)
## List of Parameters in 7-day SLR Solutions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LAGEOS</th>
<th>LEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station Coordinates</td>
<td>Weekly</td>
<td>Weekly</td>
</tr>
<tr>
<td>Earth Rotation Parameters</td>
<td>PWL daily</td>
<td>PWL daily</td>
</tr>
<tr>
<td>Geocenter Coordinates</td>
<td>Weekly</td>
<td>Weekly</td>
</tr>
<tr>
<td>Gravity field</td>
<td>Up to d/o 4</td>
<td>Up to d/o 4</td>
</tr>
<tr>
<td>Range Biases</td>
<td>Selected stations</td>
<td>All stations</td>
</tr>
<tr>
<td>Osculating Elements</td>
<td>Weekly</td>
<td>Weekly</td>
</tr>
<tr>
<td>Constant along-track S0</td>
<td>Weekly</td>
<td>-</td>
</tr>
<tr>
<td>Air Drag Scaling Factor</td>
<td>Weekly</td>
<td>Daily</td>
</tr>
<tr>
<td>Once-per-rev SS, SC</td>
<td>Weekly</td>
<td>Daily</td>
</tr>
<tr>
<td>Once-per-rev WS, WC</td>
<td>Weekly</td>
<td>Daily</td>
</tr>
<tr>
<td></td>
<td>(when not estimating gravity field)</td>
<td></td>
</tr>
<tr>
<td>Pseudo-Stochastic Pulses</td>
<td>-</td>
<td>Once-per-rev in along-track</td>
</tr>
</tbody>
</table>

Bernese GNSS Software, v.5.3
Precise Orbit Determination of SLR Satellites
### Satellite Perturbing Forces

<table>
<thead>
<tr>
<th>Perturbing accel.</th>
<th>Accel. on LAGEOS</th>
<th>Accel. on AJISAI</th>
<th>Accel. on LARES</th>
<th>Accel. on Stella</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gravitational perturbations:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Earth’s monopole</td>
<td>2.7</td>
<td>6.4</td>
<td>6.5</td>
<td>7.7</td>
</tr>
<tr>
<td>• Earth’s oblateness $C_{20}$</td>
<td>$1.0 \times 10^{-3}$</td>
<td>$6.2 \times 10^{-3}$</td>
<td>$6.3 \times 10^{-3}$</td>
<td>$8.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>• Low-order grav. $C_{22}$</td>
<td>$6.0 \times 10^{-6}$</td>
<td>$3.6 \times 10^{-5}$</td>
<td>$3.7 \times 10^{-5}$</td>
<td>$5.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>• Low-order grav. $C_{56}$</td>
<td>$8.6 \times 10^{-8}$</td>
<td>$3.1 \times 10^{-6}$</td>
<td>$3.2 \times 10^{-6}$</td>
<td>$6.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>• Mid-order grav. $C_{20,20}$</td>
<td>$8.1 \times 10^{-13}$</td>
<td>$1.5 \times 10^{-8}$</td>
<td>$1.6 \times 10^{-8}$</td>
<td>$1.1 \times 10^{-7}$</td>
</tr>
<tr>
<td>• Grav. attr. of Moon</td>
<td>$2.1 \times 10^{-6}$</td>
<td>$1.4 \times 10^{-6}$</td>
<td>$1.4 \times 10^{-6}$</td>
<td>$1.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>• Grav. attr. of Sun</td>
<td>$9.6 \times 10^{-7}$</td>
<td>$6.4 \times 10^{-7}$</td>
<td>$6.5 \times 10^{-7}$</td>
<td>$5.7 \times 10^{-7}$</td>
</tr>
<tr>
<td>• Grav. attr. of Venus</td>
<td>$1.3 \times 10^{-10}$</td>
<td>$8.5 \times 10^{-11}$</td>
<td>$8.5 \times 10^{-11}$</td>
<td>$7.8 \times 10^{-11}$</td>
</tr>
<tr>
<td>• Solid Earth tides</td>
<td>$3.7 \times 10^{-6}$</td>
<td>$2.0 \times 10^{-5}$</td>
<td>$2.0 \times 10^{-5}$</td>
<td>$2.9 \times 10^{-5}$</td>
</tr>
<tr>
<td>• Ocean tides</td>
<td>$3.7 \times 10^{-7}$</td>
<td>$1.9 \times 10^{-6}$</td>
<td>$2.0 \times 10^{-6}$</td>
<td>$3.0 \times 10^{-6}$</td>
</tr>
<tr>
<td><strong>General relativity:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Schwarzschild effect</td>
<td>$2.8 \times 10^{-9}$</td>
<td>$1.1 \times 10^{-8}$</td>
<td>$1.1 \times 10^{-8}$</td>
<td>$1.4 \times 10^{-8}$</td>
</tr>
<tr>
<td>• Lense-Thirring effect</td>
<td>$2.7 \times 10^{-11}$</td>
<td>$1.3 \times 10^{-10}$</td>
<td>$1.4 \times 10^{-10}$</td>
<td>$1.8 \times 10^{-10}$</td>
</tr>
<tr>
<td>• Geodetic precession</td>
<td>$3.4 \times 10^{-11}$</td>
<td>$4.2 \times 10^{-11}$</td>
<td>$4.2 \times 10^{-11}$</td>
<td>$4.3 \times 10^{-11}$</td>
</tr>
</tbody>
</table>

**Non-gravitational perturbations:**

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>Accel. on LAGEOS</th>
<th>Accel. on AJISAI</th>
<th>Accel. on LARES</th>
<th>Accel. on Stella</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Solar radiation pressure</td>
<td>$3.5 \times 10^{-9}$</td>
<td>$2.5 \times 10^{-8}$</td>
<td>$1.1 \times 10^{-9}$</td>
<td>$4.4 \times 10^{-9}$</td>
</tr>
<tr>
<td>• Earth radiation pressure</td>
<td>$4.4 \times 10^{-10}$</td>
<td>$8.6 \times 10^{-9}$</td>
<td>$3.9 \times 10^{-10}$</td>
<td>$1.8 \times 10^{-9}$</td>
</tr>
<tr>
<td>• Thermal re-radiation</td>
<td>$5.0 \times 10^{-11}$</td>
<td>$4.1 \times 10^{-10}$</td>
<td>$1.9 \times 10^{-11}$</td>
<td>$6.9 \times 10^{-11}$</td>
</tr>
<tr>
<td>• Light aberration</td>
<td>$1.1 \times 10^{-13}$</td>
<td>$1.1 \times 10^{-12}$</td>
<td>$5.1 \times 10^{-14}$</td>
<td>$2.0 \times 10^{-13}$</td>
</tr>
<tr>
<td>• Atmospheric drag (~ min)</td>
<td>$0.8 \times 10^{-14}$</td>
<td>$3.0 \times 10^{-11}$</td>
<td>$2.6 \times 10^{-12}$</td>
<td>$5.0 \times 10^{-11}$</td>
</tr>
<tr>
<td>• Atmospheric drag (~ max)</td>
<td>$2.0 \times 10^{-13}$</td>
<td>$5.9 \times 10^{-10}$</td>
<td>$4.8 \times 10^{-11}$</td>
<td>$5.0 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

**AJISAI:**
- Diameter: 2.15 m
- Mass: 685 kg
- Area-to-mass: $A/m: 58 \times 10^{-4} \ m^2 \ kg^{-1}$

**LAGEOS:**
- Diameter: 0.60 m
- Mass: 407 kg
- Area-to-mass: $A/m: 6.9 \times 10^{-4} \ m^2 \ kg^{-1}$

**LARES:**
- Diameter: 0.36 m
- Mass: 387 kg
- Area-to-mass: $A/m: 2.7 \times 10^{-4} \ m^2 \ kg^{-1}$
Solar Radiation Pressure

Standard value of the solar radiation pressure coeff. used by most ILRS ACs:

- \( C_R = 1.13 \) for LAGEOS-1 & LAGEOS-2

From this study:

- \( C_R = 1.125 \pm 0.015 \) for LAGEOS-1
- \( C_R = 1.094 \pm 0.012 \) for LAGEOS-2
Albedo

Earth Radiation Pressure:
- Visible Earth’s reflectivity
- Infrared Earth’s emissivity

The Earth emissivity renders about 60% of total albedo effect and introduces a rather constant force in the radial direction.

Source of data: monthly global maps from the CERES mission, grids 2.5x2.5 (Rodriguez-Solano et al., 2011)
Albedo – impact on LAGEOS orbits

A difference in the estimated semi-major axis $a$ when introducing an additional constant force in the radial direction $R_0$ can be expressed as:

$$\Delta a = -\frac{4 a^3 R_0}{3 GM}.$$ 

Scale of the reference frame is affected by 0.07 ppb = 0.5 mm

- $\Delta a = -0.5$ mm due to refl.
- $\Delta a = -1.0$ mm due to emiss.
- $\Delta a = -1.5$ mm due to refl+emiss.
Atmospheric Drag

Assuming the laminar air currents, and that the atmosphere is co-rotating with the Earth, and neglecting thermal motion of molecules, the acceleration due to the atmospheric drag can be expressed as (Beutler, 2005):

\[ a_D = -\frac{C_D}{2} \rho(h,T,\lambda,\phi,F_{10.7},A_p) \frac{A}{m} \frac{r^2}{|r'|} \]

Graph showing atmospheric drag effects on satellites AJISAI, Starlette, and Stella over 24 hours of solar time.
Atmospheric Drag

\[ \Delta a = -12 \text{ m/year for AJISAI}, \]
\[ \Delta a = -14 \text{ m/year for Starlette}, \]
\[ \Delta a = -30 \text{ m/year for Stella}. \]

For LAGEOS–1 and LAGEOS–2 the secular decays of semi-major axes amount to 0.20 m/year and 0.23 m/year, respectively, and are caused by the Yarkovsky and the Yarkovsky–Schach effects.
Geodetic parameters:
Station coordinates
Atmospheric Pressure Loading

The atmospheric Blue-Sky Effect
Atmospheric Pressure Loading— the Blue–Sky Effect

Scale of the reference frame is affected by ~1 mm

<table>
<thead>
<tr>
<th>SLR station</th>
<th>Number of normal points (1999–2010)</th>
<th>Mean impact of Atmospheric Pressure Loading</th>
<th>Blue–Sky effect [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Golosiv, Ukraine</td>
<td>330</td>
<td>6.6</td>
<td>4.4</td>
</tr>
<tr>
<td>Wuhan, China</td>
<td>1052</td>
<td>4.9</td>
<td>3.2</td>
</tr>
<tr>
<td>Beijing–A, China</td>
<td>189</td>
<td>2.7</td>
<td>2.5</td>
</tr>
<tr>
<td>Helwan, Egypt</td>
<td>223</td>
<td>3.2</td>
<td>2.4</td>
</tr>
<tr>
<td>Altay, Russia</td>
<td>1776</td>
<td>6.7</td>
<td>2.3</td>
</tr>
</tbody>
</table>
Atmospheric Pressure Loading

Atmospheric pressure loading:
- Atmospheric tidal loading (ATL)
- Atmospheric non-tidal loading (ANTL)

Solution 1
- 

Solution 2
- ATL

Solution 3
- ATL+ANTL
Multi-SLR Satellite Solutions

LAGEOS-1 vs. LAGEOS-2 vs. LAGEOS-1, LAGEOS-2, Starlette, Stella, AJISAI

Station coordinate repeatability in LAGEOS-1/2-only and in the combined solutions

Better repeatability in combined solutions

Better repeatability in LAGEOS solutions

Stations ordered by increasing number of weekly solutions
Multi-SLR Satellite Solutions – Correlations

Correlation matrix

LAGEOS-1/2 solution

LAGEOS-1/2 + AJISAI solution
TRF scale estimated from the Helmert 7-parameter transformation of weekly SLR solutions w.r.t. ITRF2008 (SLRF2008).

Orbit modeling deficiencies related to non-gravitational forces appear as the periods of the draconitic year.

Draconitic years of geodetic satellites:
- 222 days: LAGEOS-2,
- 560 days: LAGEOS-1,
- 89 days: AJISAI,
- 73 days: Starlette,
- 182 days: Stella.
Geodetic parameters:
Geocenter coordinates
Geocenter coordinates from SLR and GNSS
Geocenter coordinates from SLR and GNSS

The impact of APL is different on the GNSS and SLR solutions, despite applying the same a priori APL model. The differences are due to:

- the correlations in GNSS solutions between the vertical component and other estimated parameters, e.g. station clock corrections or troposphere delays. None of these parameters have to be estimated in the SLR solutions.
- in GNSS the solution is based on double-difference phase observations, in SLR it is based on direct undifferenced ranges,
- in GNSS the orbit modeling deficiencies are typically reflected in draconitic year periods, and thus, accumulated in the annual signal of geocenter coordinates,
- seasonal pressure variations are more visible in GNSS time series, whereas SLR sites are affected by the Blue-Sky effect.
Geocenter coordinates from SLR and GNSS

The SLR-derived geocenter coordinates are affected by the inhomogeneous distribution of SLR stations.

A priori model – Vienna APL v.2.0

a priori model on the inhomogeneous SLR network
From the analysis of correlations coefficients between the Z geocenter coordinate and the SC orbit parameter, the correlation coefficients are $-0.83$, and $0.58$ for LAGEOS–1, and LAGEOS–2, respectively in LAGEOS–only solutions. These correlations are reduced to $-0.23$ and $0.15$ in the multi–SLR solutions.
Geodetic parameters: geometry & rotation & gravity
Three pillars of satellite geodesy in one SLR solution

Is it possible to derive simultaneously all three pillars from the combined SLR solution?
For LoD, the simultaneous estimation of the gravity field parameters:

- 1. reduces the offset of LoD estimates,
- 2. substantially reduces the a posteriori error of estimated LoD. The mean a posteriori error of LoD is 1.3, 16.9, 7.1, and 44.6 μs/day in the multi-SLR solution with gravity, multi-SLR solution without gravity, LAGEOS-1/2 solution without gravity, and SLR–LEO solution without gravity field parameters, respectively.
- 2. reduces peaks in the spectrum analysis, which correspond, e.g., to orbit modeling deficiencies (peaks of 222 days, i.e., a draconitic year of LAGEOS-2, 280 days, i.e., an eclipsing period of LAGEOS-1),
Conclusions

- Appropriate modeling of satellite perturbing forces is crucial for deriving high-accuracy geodetic parameters.
- Solar radiation pressure coefficient for LAGEOS-2 is significantly different from the standard value.
- Appropriate modeling of Atmospheric Pressure Loading reduces the Blue-Sky effect and improves the station coordinate estimates.
- Low orbiting SLR satellites remarkably improve the SLR solutions, e.g., by reducing the correlations between estimated parameters.
- Simultaneous estimation of Earth's Rotation & Geometry & Gravity is feasible and is beneficial in particular for LoD estimates.
Thank you for your attention
References:

- Sośnica K, Thaller D, Dach R, Jäggi A, Beutler G (2013c) Time variable Earth’s gravity field from SLR and the comparison with polar motion, CHAMP, and GRACE results. To be submitted to J Geod