In this article we calculate the one-loop supersymmetric QCD (SQCD) corrections to the decay $\tilde{u}_1 \to c\tilde{\chi}_1^0$ in the minimal supersymmetric standard model with generic flavor structure. This decay mode is phenomenologically important if the mass difference between the lightest squark $\tilde{u}_1$ (which is assumed to be mainly stoplike) and the neutralino lightest supersymmetric particle $\tilde{\chi}_1^0$ is smaller than the top mass. In such a scenario $\tilde{u}_1 \to t\tilde{\chi}_1^0$ is kinematically not allowed and searches for $\tilde{u}_1 \to Wb\tilde{\chi}_1^0$ and $\tilde{u}_1 \to c\tilde{\chi}_1^0$ are performed. A large decay rate for $\tilde{u}_1 \to c\tilde{\chi}_1^0$ can weaken the LHC bounds from $\tilde{u}_1 \to Wb\tilde{\chi}_1^0$ which are usually obtained under the assumption $Br(\tilde{u}_1 \to Wb\tilde{\chi}_1^0) = 100\%$. We find the SQCD corrections enhance $\Gamma(\tilde{u}_1 \to c\tilde{\chi}_1^0)$ by approximately 10\% if the flavor violation originates from bilinear terms. If flavor violation originates from trilinear terms, the effect can be ±50\% or more, depending on the sign of $A'$. We note that connecting a theory of supersymmetry breaking to LHC observables, the shift from the DR to the on-shell mass is numerically very important for light stop decays.

I. INTRODUCTION

Natural supersymmetry requires light stops in order to cancel the quadratic divergences of the Higgs self-energies involving a top quark while the other supersymmetric partner can be much heavier [1,2]. Theoretical motivation for light stops also comes from the fact that when starting at a high scale with universal squark masses, the renormalization group evolution (RGE) (known at the two-loop level [3–5]) generically drives the masses of the third generation squarks to lower values as for example in gravity mediated supersymmetry (SUSY)-breaking scenarios (see for example [6]). In addition, light stops are also welcome in order to accommodate for the observed relic density within the minimal supersymmetric standard model (MSSM) [7–12] and to realize baryogenesis [13–21].

On the experimental side, the bounds on the stop mass are much weaker than the ones on the other strongly interacting SUSY particles, i.e. squarks of the first two generations [22,23] and the gluino (see for example [24] for a recent overview of ATLAS and CMS results). Light stops might even be welcome in the light of recent LHC data for $W$-pair production where the observed cross section [25,26] is slightly above the standard model (SM) predictions [27].

This can be interpreted as a hint for light sleptons, light charginos and/or light stops [28,29]. However, in order to accommodate the measured Higgs mass of around 125 GeV [30,31] rather heavy stops are required. This tension can be solved if the stop-mixing angle is large (or even maximal [32]), by promoting the MSSM to the next-to-minimal supersymmetric standard model (NMSSM) or $\alpha$SUSY [33,34] or by adding D-term contributions [35].

Concerning the exclusion limits on stop masses from the LHC, there are still regions in parameter space in which light stops are allowed. If the mass splitting between the stop and the neutralino is bigger than the top mass, the main search channel is $\tilde{u}_1 \to t\tilde{\chi}_1^0$ and the constraints are stringent [36,37]. However, if the mass difference is smaller than $m_t$ the limits on the stop mass come from searches for $\tilde{u}_1 \to Wb\tilde{\chi}_1^0$ and the limits are much weaker [38–41]. If the mass difference between the stop and the neutralino is even smaller than $m_W + m_b$ the limits are obtained from searches for the flavor-changing decay $\tilde{u}_1 \to c\tilde{\chi}_1^0$ [42,43].

The decay $\tilde{u}_1 \to c\tilde{\chi}_1^0$ has important experimental implications, both for scenarios with minimal and nonminimal flavor violation.

In the case of minimal flavor violation [44–48] the decay rate is suppressed leading to a sizable stop decay length, which can be used to determine the flavor structure [49,50] and is in principle measurable at the LHC [51–54]. The most plausible scenario with a suppressed stop decay rate $\tilde{u}_1 \to c\tilde{\chi}_1^0$ is to assume a flavor-blind SUSY-breaking

If the decay rate for $\tilde{u}_1 \to c\tilde{\chi}_1^0$ is small, the four-body decay $\tilde{u}_1 \to b\tilde{\chi}_1^+f^+f^-$ [55] (also searched for at the LHC [40]) can have a significant impact on the branching ratio for $\tilde{u}_1 \to c\tilde{\chi}_1^0$ [56].
mechanism at some high scale $\Lambda$, for example the grand unified theory (GUT) scale. In this case, flavor off-diagonal elements in the squark mass matrices are induced by the renormalization group for which the decay width has been calculated in Ref. [57] and the finite part of the one-loop electroweak corrections has been computed in Ref. [56].

In the case of nonminimal flavor violation the decay width for $\tilde{u}_1 \rightarrow c\tilde{\chi}^0_1$ can be significantly enhanced since the flavor-changing elements in the up sector are rather poorly constrained from flavor changing neutral current processes. It has been noticed in Ref. [62] (see also [63,64] for later analysis) that an enhanced branching ratio for $\tilde{u}_1 \rightarrow c\tilde{\chi}^0_1$ can weaken the bounds from $\tilde{u}_1 \rightarrow t\tilde{\chi}^0_1$, for which a branching ratio of 100% is commonly assumed in the experimental analysis, allowing for lighter stop masses. We point out that a similar effect occurs concerning the limits extracted from $\tilde{u}_1 \rightarrow Wb\tilde{\chi}^0_1$ searches. Since $\tilde{u}_1 \rightarrow Wb\tilde{\chi}^0_1$ is a three-body decay, it is kinematically suppressed compared to the two-body decay $\tilde{u}_1 \rightarrow t\tilde{\chi}^0_1$. Therefore, already a much smaller amount of flavor violation, as the one necessary to affect the limits from $\tilde{u}_1 \rightarrow t\tilde{\chi}^0_1$, would be sufficient to significantly weaken the limits extracted from $\tilde{u}_1 \rightarrow Wb\tilde{\chi}^0_1$. This observation is especially interesting taking into account that the bounds on the stop mass from $\tilde{u}_1 \rightarrow Wb\tilde{\chi}^0_1$ are currently anyway the weakest ones. Therefore, very light stop masses for $m_w < m_{\tilde{u}_1} < m_{\tilde{\chi}^0_1}$ are allowed, especially in the case of nonminimal flavor violation.

In this article we investigate the one-loop supersymmetric QCD (SQCD) corrections to $\tilde{u}_1 \rightarrow c\tilde{\chi}^0_1$ in the MSSM with generic flavor structure. These $\alpha_s$ corrections are the leading ones in the case of nonminimal flavor violation. Furthermore, assuming a flavor-blind SUSY-breaking mechanism at a high scale $\Lambda$ the counting of the loop effects is as follows: The leading order effect is the one-loop electroweak running from $\Lambda$ to $m_{SUSY}$. To this leading effect the next-to-leading order (NLO) corrections are the two-loop $\overline{\text{MS}}$ effects originating from $\alpha_s$ and the one-loop QCD corrections to the decay width at the SUSY scale which we calculate here.

The article is structured as follows: In the next section we establish our conventions and recall the tree-level expression for the decay rate for $\tilde{u}_1 \rightarrow c\tilde{\chi}^0_1$. Section III describes the calculation as well as the renormalization followed by a numerical analysis in Sec. IV. Finally we conclude in Sec. V.

II. CONVENTIONS AND TREE-LEVEL DECAY

In this section we define our conventions and discuss the tree-level decay width. First, we denote the term in the Lagrangian for the coupling of an up quark $u_i$ to an up squark $\tilde{u}_i$ and a neutralino $\tilde{\chi}^0_1$ as

$$\bar{\tilde{u}}_i \gamma_\mu \tilde{\chi}^0_1 \left[ \Gamma_{\tilde{u},u_i}^{pL} P_L + \Gamma_{\tilde{u},u_i}^{pR} P_R \right] u_i + \text{H.c.},$$

where $P_L$ and $P_R$ are chiral projectors. For the coupling of quarks to squarks and gluinos we introduce a similar notation:

$$\bar{\tilde{u}}_i \tilde{g} \left[ \Gamma_{\tilde{u},u_i}^{gL} P_L + \Gamma_{\tilde{u},u_i}^{gR} P_R \right] u_i + \text{H.c.}$$

In the following, we will order the mass eigenstates for the neutralino $p = 1–4$ and of the up squarks $s = 1–6$ in increasing order and $u_3, u_2$ and $u_1$ correspond to the $t, c$ and $u$ quark, respectively. For the neutralino mass matrix we use the convention

$$\mathcal{M}_\tilde{\chi} = \begin{pmatrix}
M_1 & 0 & -\frac{v_1}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} \\
0 & M_2 & \frac{v_1}{\sqrt{2}} & -\frac{v_2}{\sqrt{2}} \\
-\frac{v_1}{\sqrt{2}} & \frac{v_2}{\sqrt{2}} & 0 & -\mu \\
d\frac{v_1}{\sqrt{2}} & -\frac{v_2}{\sqrt{2}} & -\mu & 0
\end{pmatrix},$$

with $v = \sqrt{2m_w/g_2} \approx 174$ GeV and $v_u/v_d = \tan \beta$. $M_1$ and $M_2$ denote the gaugino masses and $\mu$ is the higgsino mass. The up-squark mass term in the Lagrangian is given by

$$-(\tilde{u}_L \cdot \tilde{u}_R) \mathcal{M}_\tilde{u}^2 \begin{pmatrix}
\tilde{u}_L \\
\tilde{u}_R
\end{pmatrix},$$

where both $\tilde{u}_L$ and $\tilde{u}_R$ are three-vectors in flavor space. The squark mass(-squared) matrix is given by

$$\mathcal{M}_\tilde{u}^2 = \begin{pmatrix}
V_{uL}^2 & v_y Y_u \mu \cot \beta \\
V_{uR}^2 + v^2 Y_u^2
\end{pmatrix},$$

with

$$\Delta^{uLR} = \Delta^{uRL\dagger} = -v_u (A_u + Y_u \mu \cot \beta),$$

$$\mathbf{m}_{uL}^{L2} = V^\dagger \mathbf{m}_{Q}^2 V.$$
Yukawa couplings and used LR conventions for them and the A terms [67]. Note that in Eq. (5) the Yukawa couplings and not the quark masses enter which is a relevant difference since we are computing one-loop SQCD corrections in this article.\(^5\)

Equation (5) is given in the super-CKM basis which we define to be the basis in which the Yukawa couplings of the MSSM superpotential are diagonal, both for quarks and squarks, so that supersymmetry is manifest:

\[
Y_u = \begin{pmatrix} Y_{u_1} & 0 & 0 \\ 0 & Y_{u_2} & 0 \\ 0 & 0 & Y_{u_3} \end{pmatrix}.
\]

Note that in the literature the super-CKM basis is often defined to be the basis with diagonal quark mass matrices. However, this definition has the disadvantage that the basis changes with every loop order.

We diagonalize the full Hermitian 6 \times 6 squark mass-squared matrix \(\mathcal{M}_u^2\) and the symmetric 4 \times 4 neutralino mass matrix \(\mathcal{M}_\chi^2\) as

\[
\begin{align*}
W_{s,s'}^{\tilde{u}_s} (\mathcal{M}_u^2)_{s,s'} W_{t,t'}^{\tilde{u}_t} &= m_{\tilde{u}_s}^2 \delta_{st}, \\
\mathcal{Z}_N^{ij} \mathcal{M}_\chi^2 \mathcal{Z}_N^{ji} &= m_{\tilde{\chi}_j}^2 \delta_{pq},
\end{align*}
\]

where \(Z_N\) and \(W_{\tilde{u}}\) are unitary matrices. With these conventions we get for the squark-quark-neutralino couplings in Eq. (1)

\[
\Gamma_{\tilde{u}_1 \to u_1 \tilde{\chi}_i^0} = \frac{m_{\tilde{u}_1}}{16\pi} \left( 1 - \frac{m_{\tilde{\chi}_i^0}^2}{m_{\tilde{u}_1}^2} \right)^2 \left( |\mathcal{W}_{21}^u|^2 + |\mathcal{W}_{51}^u|^2 \right).
\]

If the LSP is mostly binlike, we can further simplify the expression neglecting very small neutralino mixing and small charm Yukawa couplings:

\[
\Gamma_{\tilde{u}_1 \to u_1 \tilde{\chi}_i^0} = \frac{m_{\tilde{u}_1}}{16\pi} \frac{g_1^2}{18} \left( 1 - \frac{m_{\tilde{\chi}_i^0}^2}{m_{\tilde{u}_1}^2} \right)^2 \left( |\mathcal{W}_{21}^u|^2 + 16|\mathcal{W}_{51}^u|^2 \right).
\]

Note that the decay to a right-handed charm quark is enhanced by a factor 16 which can be traced back to hyperchargets.

### III. Calculation of the SQCD Corrections

In this section we discuss in detail the calculation of the one-loop SQCD corrections including our renormalization scheme. Our calculation involves the following steps:

1. Renormalization of the quark sector.
2. Renormalization of the squark sector.
3. Calculation of the gluon contributions to the decay width including real emission corrections, i.e. the decay \(\tilde{u}_1 \to c \tilde{\chi}_1^0 q\).
4. Calculation of the gluino contributions [including the cancellation of ultraviolet (UV) divergences].

We renormalize the fundamental parameters entering the decay width of the stop decay at tree level, which receive SQCD corrections at the one-loop level, in the dimensional reduction (DR) scheme. These quantities are

- (i) the Yukawa couplings \(Y_{ui}\) of the MSSM superpotential,
- (ii) the trilinear \(A_{ui}\) terms, and
- (iii) the bilinear squark mass terms \(m_{ij}^u\) and \(m_{ij}^\tilde{q}\).

We write the bare quantities of the Lagrangian [labeled with a superscript \(0\)] as

\[
Y_{ui}^{(0)} = Y_{ui} + \delta Y_{ui}, \quad A_{ij}^{(0)} = A_{ij} + \delta A_{ij},
\]

\[
m_{ij}^{u, \tilde{q}} = m_{ij}^{u, \tilde{q}}, \quad m_{ij}^{u, \tilde{q}} + \delta m_{ij}^{u, \tilde{q}}.
\]

Since we renormalize all quantities in a minimal renormalization scheme, i.e. the DR scheme, \(A_u, m_{ij}^{u, \tilde{q}}\) and \(Y_u\) are understood to be the renormalized ones in the DR scheme. However, in the decay width of the stop, the on-shell squark mass also enters. Therefore, a conversion from the on-shell squark mass to the DR is necessary. In addition, the Yukawa couplings have to be related to the measured quark masses of the SM by running and threshold corrections.

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\(^5\)The threshold corrections connecting the Yukawa couplings and the quark masses are known to be very large in the down sector [68–77] and have been computed at the two-loop level [78–83].

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A. Renormalization of the quark sector

SQCD corrections to quark masses and Yukawa couplings can be calculated from the quark self-energies (see Fig. 1). The UV renormalization of the Yukawa couplings (in the \( \text{DR-bar} \) scheme) is given by

\[
\delta Y_{ii}^u = -\frac{\alpha_s}{2\pi} C_F Y_{ii}^u, \tag{16}
\]

where \( C_F = 4/3 \) is a color factor. In our approach we compute only Lehmann-Symanzik-Zimmermann (LSZ) factors corresponding to flavor-diagonal self-energies, which are also the only UV divergent ones. All other contributions from self-energies can be calculated as one-particle irreducible diagrams \([84-86]\).

Therefore, the LSZ factor for left- and right-handed quarks is

\[
\delta Z_{ii}^L = \delta Z_{ii}^a + \delta Z_{ii}^b, \tag{17}
\]
\[
\delta Z_{ii}^R = \delta Z_{ii}^a + \delta Z_{ii}^b. \tag{18}
\]

Here, the superscript \( a \) denotes the flavor-independent gluon piece, while the index \( b \) refers to the gluino piece whose finite part is in general flavor dependent:

\[
\delta Z_{ii}^a = \frac{\alpha_s}{4\pi} C_F (1 - (1 - \xi)) \left[ \frac{1}{\epsilon_{\text{IR}}} - \frac{1}{\epsilon} \right], \tag{19}
\]
\[
\delta Z_{ii}^{L,b} = \Sigma_{ii}^{\xi}, \tag{20}
\]
\[
\delta Z_{ii}^{R,b} = \Sigma_{ii}^{\xi}. \tag{21}
\]

\( \xi \) denotes the gauge parameter which is involved in the gluon propagator (e.g. \( \xi = 1 \) would correspond to the Feynman gauge). Furthermore, \( \epsilon_{\text{IR}} \) denotes the dimensionally regularized infrared (IR) divergence and \( \epsilon \) the UV one while \( \Sigma_{ii}^{LL} \) and \( \Sigma_{ii}^{RR} \) are defined in Eqs. (22) and (23).

1. Threshold corrections

In order to determine the actual values of the Yukawa couplings we have to make the connection to the quark masses determined within the SM.\(^6\) The self-energies with heavy virtual particles, in our case the one with squarks and gluinos, lead to threshold corrections modifying the tree-level relation \( v_u Y_{ii}^u = m_{ii} \). In order to write down these corrections we decompose the quark self-energies originating from squark-gluino loops as

\[
\Sigma_{ii}^u(p^2) = \Sigma_{ii}^{\nu \nu} (p^2) + \Sigma_{ii}^{\nu \bar{\nu}} (p^2) + \Sigma_{ii}^{\bar{\nu} \bar{\nu}} (p^2) \tag{22}
\]

Since in the decay \( \tilde{u}_i \to c_i \tilde{b}_i \) we are dealing with external charm quarks on the mass shell it is sufficient to evaluate Eq. (22) at vanishing external momenta, i.e. neglecting finite terms of the order \( m_c^2/m_{\text{SUSY}}^2 \):

\[
\Sigma_{ii}^{\nu \nu} (0) = \Sigma_{ii}^{\nu \bar{\nu}} (0), \quad \Sigma_{ii}^{\bar{\nu} \bar{\nu}} (0) = \Sigma_{ii}^{\bar{\nu} \bar{\nu}} (0). \tag{23}
\]

With these notations the relation between the Yukawa couplings of the MSSM superpotential and the running quark masses of the SM (evaluated at the scale \( m_{\text{SUSY}} \)) is given by

\[
\left[ m_{ii} \left( 1 - \frac{1}{2} \Sigma_{ii}^{LL} + \Sigma_{ii}^{RR} \right) \right]_{\text{finite}} = v_u Y_{ii}^u. \tag{24}
\]

B. Renormalization of the squark sector

As for the quarks, we compute in our approach only LSZ factors corresponding to flavor-diagonal squark self-energies (i.e. \( \tilde{u}_i \to \tilde{u}_i \) transitions), while all other contributions from squark self-energies are calculated as one-particle irreducible diagrams. The LSZ factors for the squarks then read

\[
\delta \tilde{Z}_{ii}^a = \frac{\alpha_s}{4\pi} C_F (2 + (1 - \xi)) \left[ \frac{1}{\epsilon} - \frac{1}{\epsilon_{\text{IR}}} \right], \tag{25}
\]
\[
\delta \tilde{Z}_{ii}^{b,p^2} = \left. \frac{\partial \Sigma_{ii}^{\tilde{b} \tilde{b}} (p^2)}{\partial p^2} \right|_{p^2 = m_{ii}^2} = -\frac{\alpha_s}{2\pi} C_F \frac{1}{\epsilon} + \text{finite}. \tag{26}
\]

Like in the quark case \( a \) refers to the gluon part and \( b \) to the gluino and squark-tadpole part and \( \Sigma_{ii}^{\tilde{b} \tilde{b}} (p^2) \) denotes the sum of Eqs. (A19) and (A24). From Eqs. (A19)-(A24) in the Appendix we find that the sum of those UV divergent parts of the squark self-energies which are independent of the external momentum (i.e. the masslike contribution) is given by.
\[
\Sigma_{u,\bar{u}}^{1\text{UV}}(0) = \frac{\alpha_s}{2\pi} C_F \left( \frac{1}{\epsilon} \left[ \left( \frac{\xi - 1}{2} m_{\bar{u}}^2 + m_{\bar{u}}^2 \right) \delta_{\alpha\beta} + 2 \sum_{j=1}^{3} \left( W_{j+3,a}^{\bar{u}\bar{u}} m_{\bar{u}}^2 W_{j+3,a}^{\bar{u}\bar{u}} + W_{j+3,a}^{\bar{u}\bar{u}} \Delta_{ij}^{LL} W_{j+3,a}^{\bar{u}\bar{u}} \right) \\
+ 3 \sum_{i,j=1}^{3} \left( W_{i,j+3}^{\bar{u}\bar{u}} \Delta_{ij}^{LR} W_{j+3,a}^{\bar{u}\bar{u}} - 2 m_{\bar{u}} \sum_{j=1}^{3} \left( W_{j+3,a}^{\bar{u}\bar{u}} m_{\bar{u}}^2 W_{j+3,a}^{\bar{u}\bar{u}} + W_{j+3,a}^{\bar{u}\bar{u}} \Delta_{ij}^{LR} W_{j+3,a}^{\bar{u}\bar{u}} \right) \right) \right] \right). \tag{27}
\]

Here, \(m_{\tilde{g}}\) denotes the gluino mass. To Eq. (27), the divergent squark mass terms induced by the LSZ factors in Eqs. (25) and (26)

\[
\frac{\alpha_s}{4\pi} C_F (1 - \xi) \frac{1}{\epsilon} m_{\tilde{u}}^2 \delta_{\alpha\beta} \tag{28}
\]

have to be added, canceling the divergence involving \(m_{\tilde{g}}^2\). In order to see the cancellation of the remaining UV divergences in Eq. (27), we consider the bare mass matrix which is given in the super-CKM basis

\[
\mathcal{M}_{\bar{u}}^{(0)}(0) = \left( \begin{array}{cc}
\mu & \Delta_{ij}^{LL} \\
\Delta_{ij}^{LL} & \mu
\end{array} \right)
\]

Since the squark-mixing matrix \(W^{\bar{u}}\) diagonalizes the renormalized mass matrix, the bare mass matrix is not diagonal in this basis but rather has the form

\[
W^{\bar{u}^\prime}(\mathcal{M}_{\bar{u}}^{(0)}(0)).
\]

Comparing Eqs. (27) and (28) to Eq. (30), we observe that the counterterms

\[
v_{u} \delta Y^{u} = -\frac{\alpha_s}{2\pi} C_F m_{u}, \tag{31}
\]

and

\[
\delta A_{ij}^{u} = -\frac{\alpha_s}{2\pi} C_F (A_{ij}^{u} + 2m_{\bar{u}} Y^{u} \delta_{ij}), \tag{32}
\]

\[
(\delta m_{ij}^{LL})_{ij} = (\delta m_{ij}^{LR})_{ij} = -\frac{\alpha_s}{2\pi} C_F m_{\bar{u}}^2 \delta_{ij} \tag{33}
\]

cancel the divergences. As required by supersymmetry, Eq. (31) equals Eq. (16). Therefore, no renormalization of the squark-mixing matrices \(W\) is necessary in this formalism.\footnote{Furthermore, note that since the renormalization of the Yukawa couplings is fixed from the quark sector to be in a minimal renormalization scheme, it would not be consistent to absorb the finite pieces of the loop corrections into a redefinition of the squark-mixing matrices.}

In the numerical analysis, we will use the connection between the on-shell and the \(\overline{\text{DR}}\) mass. This relation is given by

\[
m_{u}^{2\overline{\text{OS}}} = m_{u}^{2\overline{\text{DR}}} + \Sigma_{u,\bar{u}}^{\text{finite}}(p^2 = m_{\tilde{u}}^2). \tag{34}
\]

### C. Gluon contributions

Here, we combine the virtual gluon contributions with the real radiation (see Fig. 2) and show the cancellation of the infrared and collinear divergences. In our calculations all singularities are regularized dimensionally; more precisely, we use dimensional reduction and introduce the renormalization scale in the form \(\mu^2 e^\gamma/(4\pi)\), where \(\gamma = 0.577...\) is the Euler constant.

For the vertex correction diagram due to gluon exchange (left diagram in Fig. 3) we get

\[
V^{g} = A_0 \frac{\alpha_s}{4\pi} C_F \left[ \frac{(1 - (1 - \xi))}{\epsilon} - \frac{1}{\epsilon_{\text{IR}}} \right] + \frac{-2 + (1 - \xi) - 2L_{\mu} + 2 \ln(1 - x_1)}{\epsilon_{\text{IR}}} - \frac{2 - \pi^2/12}{2}
\]

\[\begin{align*}
-2L_{\mu}^2 - 2L_{\mu} + 4L_{\mu} \ln(1 - x_1) - 2\ln^2(1 - x_1) + 2 \ln(1 - x_1) - 2L_{\mu_2}(x_1) \].
\tag{35}
\]
using the abbreviations $x_1 = m_{\tilde{g}}^2/m_{\tilde{u}}^2$ and $L_\mu = \ln(\mu/m_{\tilde{g}})$. $\xi$ denotes the gauge parameter which is involved in the gluon propagator. As before, poles of the form $1/e$ correspond to ultraviolet singularities, while poles of the form $1/e^2_\text{IR}$ and $1/e_\text{IR}$ are due to infrared and collinear singularities. Finally, $A_0$ is the tree-level amplitude [originating from Eq. (1)], reading

$$A_0 = i\bar{u}(p_{u_1}) \left( \Gamma^{L*}_{\bar{u}u_2} P_R + \Gamma^{R*}_{\bar{u}u_2} P_L \right) v(p_{\tilde{u}_1}).$$

(36)

To get the renormalized result $A^\vartheta$ for the amplitude, we need to add the contributions induced by the gluon part of the LSZ factors of the (massless) charm quark and the stop squark [see Eqs. (19) and (25), respectively], as well as the effects induced by the renormalization constants for the coupling constants $e$ and $Y_{\tilde{u}}$, appearing in the tree-level squark-quark-neutralino vertex. These renormalization constants are written as $Z_e = 1 + \delta Z_e^g$, $\delta Z_e^g$ for the gauge coupling $e$ and $Z_{Y_{\tilde{u}}} = 1 + \delta Z_{Y_{\tilde{u}}}^g + \delta Z_{Y_{\tilde{u}}}^\vartheta$ for the Yukawa coupling $Y_{\tilde{u}}$. The parts due to gluon corrections, which are relevant in this subsection, read

$$\delta Z_e^g = -\frac{\alpha_s}{4\pi} C_F \frac{3\, \ln \frac{\mu}{\Lambda}}{2\, \epsilon}.$$  

(37)

As expected, these expressions are independent of the gauge parameter $\xi$. Adding up the mentioned contributions, we get the renormalized amplitude

$$A^\vartheta = A_0 \frac{\alpha_s}{4\pi} C_F \left[ -\frac{1}{e_\text{IR}} + \frac{-5/2 - 2L_\mu + 2\ln(1 - x_1)}{e_\text{IR}} - 2 - \frac{\pi^2}{12} 

- 2L_\mu^2 - 2L_\mu + 4L_\mu \ln(1 - x_1) - 2\ln^2(1 - x_1) + 2\ln(1 - x_1) - 2Li_2(x_1) \right].$$

(38)

This result is, as required by consistency, again independent of the gauge parameter $\xi$. After squaring the renormalized amplitude and performing the phase-space integrals (working consistently in $d = 4 - 2\epsilon$ dimensions), we get the decay width

$$\Gamma^{\text{virt}} = \Gamma_0 \frac{\alpha_s}{4\pi} C_F \left[ -\frac{2}{e_\text{IR}} + \frac{-9 - 8L_\mu + 8\ln(1 - x_1)}{e_\text{IR}} - 22 + \frac{\pi^2}{3} - 16L_\mu^2 

- 30L_\mu + 32L_\mu \ln(1 - x_1) - 16\ln^2(1 - x_1) + 30\ln(1 - x_1) - 4Li_2(x_1) \right].$$

(39)

where $\Gamma_0$ is the corresponding decay width at order $\alpha_s^0$ given in Eq. (13).

We now turn to the bremsstrahlung corrections (see Fig. 2). Using the information given in Section A 1 of the Appendix on the three-particle phase space and making use of the mathematica package HypExp 2.0 [87], it is straightforward to derive the decay width for $\bar{u}_1 \to \vartheta^0_{1}\tilde{g}$. We obtain

8The parts $\delta Z_e^g$ and $\delta Z_{Y_{\tilde{u}}}^g$, which are due to gluino corrections, will be taken into account in the following subsection.

FIG. 2. Feynman diagrams showing the real emission of a gluon, i.e. the process $\bar{u}_1 \to c + g + \chi_{1}^0$.

FIG. 3. Genuine vertex corrections involving gluons (left diagram) and gluinos (right diagram).
As expected, the collinear and infrared singularities canceled and the result is finite. Adding the virtual corrections (39) and the gluon bremsstrahlung corrections (40), we get

$$
\Gamma^\text{brem} = \frac{\alpha_s}{4\pi} C_F \left[ \frac{9 + 8L_\mu - 8\ln(1-x_1)}{\varepsilon_{IR}} - \frac{5\pi^2}{3} + \frac{69 - 71x_1}{2(1-x_1)} + 16L_\mu^2 + 36L_\mu - 32L_\mu \ln(1-x_1) + 16\ln^2(1-x_1) - 4(9 + \ln(x_1))\ln(1-x_1) - x_1(4 - 3x_1) \frac{1}{(1-x_1)^2} \ln(x_1) - 4L_{i2}(x_1) \right].
$$

Adding the virtual corrections (39) and the gluon bremsstrahlung corrections (40), we get

$$
\Gamma^y = \frac{\alpha_s}{4\pi} C_F \left[ - \frac{4\pi^2}{12} + \frac{25 - 27x_1}{2(1-x_1)} + 6L_\mu - 2(3 + 2 \ln(x_1))\ln(1-x_1) - x_1(4 - 3x_1) \frac{1}{(1-x_1)^2} \ln(x_1) - 8L_{i2}(x_1) \right].
$$

As expected, the collinear and infrared singularities canceled and the result is finite.  

### D. Gluino and squark-tadpole contributions

We write the amplitude containing the tree level and the contribution of loop diagrams involving gluinos and the squark tadpole (right diagram in Fig. 4) as

$$
A^{\tilde{u}} = i\bar{u}(p_{\tilde{u}}) \left[ \left( \Gamma_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Ls} + \Gamma_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Rs} + \sum_{f=1}^{3} X^{\tilde{u}_{\tilde{u}}} \Gamma_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Ls} + \sum_{f=1}^{6} \Gamma_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Ls} \tilde{X}^{\tilde{u}_{\tilde{u}}} \bar{u} \right) P_R + (R\leftrightarrow L) \right] v(p_{\tilde{u}}). 
$$

Here, \( \Gamma_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Ls} \) encodes the tree-level contribution and \( \Gamma_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Rs} \), given in Eq. (A9) of the Appendix, denotes the genuine vertex correction involving the gluino. Furthermore, \( X^{\tilde{u}_{\tilde{u}}} \) and \( \tilde{X}^{\tilde{u}_{\tilde{u}}} \) originate from quark and squark self-energy diagrams, respectively. The explicit expressions read

$$
\tilde{X}^{\tilde{u}_{\tilde{u}}} = \left\{ \begin{array}{ll}
\frac{\Sigma_{s,s'}(p^2=m_{s'}^2)}{m_{s'}^2-m_{s}^2} & \text{for } s \neq t,
\frac{1}{2} \delta Z^{b}_{i} & \text{for } s = t,
\end{array} \right.
$$

Let us briefly discuss the ultraviolet singularities in Eq. (42) and how they get canceled. All divergences in the off-diagonal elements of \( \tilde{X}^{\tilde{u}_{\tilde{u}}} \) are canceled by the counterterms induced through the renormalization of \( Y^{u}_{ij}, A^{u}_{ij}, (m_{UL}^{Ls})_{ij} \), and \( (m_{UR}^{Rs})_{ij} \) in the squark mass matrix, while the off-diagonal elements of \( X^{\tilde{u}_{\tilde{u}}} \) are finite \textit{ab initio}. Therefore, we are effectively left in Eq. (42) with the singularities in the flavor conserving parts of \( X^{\tilde{u}_{\tilde{u}}} \) and \( \tilde{X}^{\tilde{u}_{\tilde{u}}} \), which originate from LSZ factors, and with the singularities present in the vertex correction \( \Gamma_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Rs} \). Using the unitarity of the squark-mixing matrices in Eq. (A9), the latter singularities read

$$
\begin{align*}
\Lambda_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Ls} & = \frac{-\alpha_s}{2\pi} C_F \left[ \frac{1}{1+\varepsilon} \right] W_{1,3}^{u} Y_{\tilde{u}_{\tilde{u}}}^{41L_{s}}., \\
\Lambda_{\tilde{u}_{\tilde{u}}}^{\tilde{u},Rs} & = \frac{-\alpha_s}{2\pi} C_F \left[ \frac{1}{1+\varepsilon} \right] W_{1,3}^{u} Y_{\tilde{u}_{\tilde{u}}}^{41R_{s}}.,
\end{align*}
$$

As the renormalization scheme in Ref. [66] is quite different from ours, a full comparison is difficult. It was, however, possible to compare the gluino vertex correction, the virtual gluon corrections and the gluon bremsstrahlung corrections individually. Taking into account that in Ref. [66] the two-particle phase space (and a corresponding part of the three-particle phase space) is in \( d \) dimensions and that the renormalization scale is of the form \( \mu^{2\varepsilon}(1-\varepsilon)/(4\pi)^{4\varepsilon} \), we found that the results are in agreement. In our calculation we used a \( d \)-dimensional phase space [and introduce the renormalization scale in the form \( \mu^{2\varepsilon} e^{-\varepsilon}/(4\pi)^{4\varepsilon} \)].
It is straightforward to see that the remaining singularities get canceled against those which are induced by the gluino parts \(\delta Z_u^b \) and \(\delta Z_{\gamma_s}^b\) of the renormalization constants of the gauge coupling \(e\) and \(Y^{u_i}\) present in the tree-level squark-quark-neutralino vertex. These renormalization constants read\(^{10}\)

\[
\delta Z_u^b = -\delta Z_{\gamma_s}^b, \quad (46)
\]

\[
\delta Z_{\gamma_s}^b = -\frac{\alpha_s}{8\pi} C_F \frac{1}{\epsilon} , \quad (47)
\]

where \(Z_u^b\) is given in Eq. (37).

Therefore, the renormalized version of the amplitude is obtained by just taking the finite part of Eq. (42). The corresponding contribution to the decay width is then obtained by inserting the renormalized amplitude into Eq. (13) and working out the interference term, i.e. the term proportional to \(\alpha_s\).

\[
M_u^2 = \begin{pmatrix}
(2\text{ TeV})^2 & 0 & 0 & 0 & 0 & 0 \\
0 & (2\text{ TeV})^2 & \Delta_{LL}^L & 0 & 0 & \Delta_{LR}^L \\
0 & \Delta_{LL}^L & (m_{L}^{L})^2 & 0 & -\epsilon \Delta_{LL}^R & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \Delta_{RR}^L & 0 & (2\text{ TeV})^2 & 0 \\
0 & \Delta_{RR}^L & -\epsilon \Delta_{LR}^R & 0 & (m_{R}^{R})^2 & 0 \\
\end{pmatrix} . \quad (48)
\]

Here, \(\Delta_{ij} = \delta_{ij} \sqrt{M_{a,ii}^2 M_{a,jj}^2} \) parametrizes the flavor change (and is assumed to be small compared to the diagonal elements) and we choose \(A' = \pm 1\text{ TeV}\). In the following, we will consider the case of \(m_{33}^{RR} = m_{33}^{LL}\) (i.e. maximal mixing). For the neutralino, which we assume to be binolike, we choose a mass of 250 GeV and use \(\alpha_s(m_{\text{SUSY}}) = 0.087\) as an input.

At tree level, the scheme for the stop mass is not defined. At the one-loop level the quantities of the MSSM supersymmetry must be renormalized in a process-independent way in order to respect supersymmetry; e.g. the Yukawa couplings have to be renormalized in the DR scheme. For consistency, also all other elements of the squark mass matrix should be renormalized in this scheme as well and should be given at the same renormalization scale. After diagonalization of the squark mass matrix, the eigenvalues correspond to DR masses which can be translated to on-shell masses if necessary or desired. This is the case for \(\tilde{u}_1 \rightarrow c\tilde{\chi}_1^0\) where the masses entering the decay width in Eq. (13) are on-shell masses.

The shift between the DR and the on-shell mass [see Eq. (34)] turns out to be numerically especially important for our scenario with a light stop because it scales like\(^{11}\)

\[
m_u^{LL} = m_u^{RR} \frac{m_{L}^{LL}}{m_{R}^{RR}} . \quad (49)
\]

In Fig. 5 we show the ratio \(m_u^{DR}/m_u^{OS}\) as a function of the gluino mass at the one-loop level for \(m_{a_{u_1}}^{OS} = 275\text{ GeV}\). For this we set all flavor off-diagonal elements \(\Delta_{ij}\) in Eq. (48) to zero.

Note that for large gluino masses the on-shell stop mass is smaller than the DR mass. This has interesting consequences for model building with light stops: Assuming that there is already a splitting between the DR squark masses of the first two generations and the stop squark (for example due to the running from the GUT scale to the SUSY scale) at the SUSY scale, then this splitting is significantly increased for heavy gluinos, making the stop even lighter. Therefore, light stop

\(^{11}\)Even though the correction is very large, perturbation theory still works, because the parametric enhancement \(m_{u_{L}}^{2}/m_{u_{R}}^{2}\) can only appear once at any loop level. Therefore, higher loop corrections will have the size of ordinary SQCD effects compared to the one-loop result. The relation is known at the two-loop level [88].
scenarios, which are interesting for the decay $\tilde{u}_1 \to c\chi^0_1$, can be even generated via finite loop effects.

For the numerical analysis of the SQCD corrections to $\tilde{u}_1 \to c\chi^0_1$, we choose $m_{33}^L = m_{33}^R$ in Eq. (48) in such a way, that a given on-shell mass for $\tilde{u}_1$ (275 GeV in our example) results after diagonalizing Eq. (48) and shifting the so-obtained DR squark masses to the corresponding on-shell masses. This procedure we do for both the tree-level decay width $\Gamma^{\text{tree}}$ and for the SQCD corrected version $\Gamma^{\text{one-loop}}$ calculated in this paper.

In Fig. 6 (Fig. 7) we illustrate the effect of the one-loop contributions for positive (negative) $A'$ for the four different sources of flavor violation: $\delta_{21}^{RR}$, $\delta_{23}^{LL}$, $\delta_{23}^{RL}$ and $\delta_{23}^{LR}$. Here, we defined the ratio $R = \Gamma^{\text{one-loop}}/\Gamma^{\text{tree}}$ of the partial widths. In each of the four curves in Figs. 6 and 7 the indicated $\delta_{ij}$ is put to 0.01, while the other $\delta$'s are switched off. Note that the actual numerical values of the mentioned $\delta$'s drop out in this ratio to a very good approximation. We find that if bilinear terms are the only sources of flavor violation, the SQCD effects are around 10%, while if flavor violations originate from trilinear terms the corrections can reach ±50% or even more. The large corrections in the case of $\delta_{23}^{RL}$ and $\delta_{23}^{LR}$ can be traced back to the suppressed decay width for left-handed charm quarks.

As stated above, we renormalize all quantities in a minimal renormalization scheme, i.e. in the DR scheme, and relate the Yukawa couplings to the ones obtained within the SM by running and threshold corrections. Reference [66] used an on-shell renormalization scheme in which the counterterms to the quark- and squark-mixing matrices are fixed in such a way that they exactly cancel the anti-Hermitian parts of the LSZ factors of the quarks and the squarks. While to all orders in perturbation theory the final result must of course be independent of the renormalization scheme, at a finite loop order, different schemes have (dis)advantages compared to each other. A minimal renormalization scheme is process independent and allows combining different quantities (like for example those appearing in the squark mass matrix) if they are given at the same renormalization scale. Furthermore, if one aims at studying the consequences of a model with high-scale SUSY breaking, a minimal renormalization scheme is preferred since running quantities are involved so that the different scales can be connected. Furthermore, when using a mixed on-shell DR scheme, one must be careful not to renormalize dependent quantities (connected by SUSY relations) entering at various places in a different way. For example, renormalizing the squark-mixing matrices on shell might be in conflict with the DR renormalization of the Yukawa couplings within the squark mass matrices, entering the determination of $W^u$.

V. CONCLUSIONS

In this article we computed the one-loop SQCD corrections to the decay $\tilde{u}_1 \to c\chi^0_1$ in the MSSM with generic sources of flavor violation. This decay is phenomenologically very important if the mass splitting between the neutralino and the lightest stop is smaller than the top mass. In particular, we pointed out that a sizable partial width for $\tilde{u}_1 \to c\chi^0_1$, which is possible in the presence of nonminimal sources of flavor violation, can significantly weaken the LHC exclusion bounds obtained from $\tilde{u}_1 \to Wb\chi^0_1$ where usually a branching ratio of 100% is assumed.

Working in the super-CKM basis with diagonal Yukawa couplings and renormalizing all parameters in the DR scheme, we explicitly checked for the cancellation of UV divergences and verified that SUSY relations are satisfied. In particular, in the squark sector all divergences are eliminated by flavor-conserving counterterms to Yukawa couplings, $A$ terms and the bilinear terms, meaning

\[ m_{33}^L = m_{33}^R \]

determined in this way will depend on the gluino mass $m_{\tilde{g}}$ and on the renormalization scale $\mu$. 

\[ \delta_{ij} \]
that no renormalization of the squark-mixing matrices is necessary. Concerning the gluon corrections we regularized all divergences dimensionally and verified their cancellation in a general $R_\xi$ gauge.

Numerically, we observe a large shift between the on-shell and the $\overline{\text{DR}}$ mass of the stop. Due to the inherited quadratic divergence, the shift involves a term proportional to $m^2_{\tilde{g}}/m^2_{\tilde{q}}$. Since for large gluino masses the on-shell stop mass is driven to smaller values compared to the $\overline{\text{DR}}$ mass, it is important to take into account this shift for model building. Taking the on-shell stop mass as an input, we find a SQCD enhancement of the decay width compared to the tree level for $\tilde{u}_1 \to c\tilde{\chi}^0_1$ (assuming a binolike LSP) of approximately 10% if the flavor violation is due to bilinear terms and 50% and more if the single origin of flavor violation is the trilinear terms.

For the future, a NLO SQCD calculation of $\tilde{u}_1 \to Wb\tilde{\chi}^0_1$ would be desirable and a phenomenological study of the impact of $\tilde{u}_1 \to c\tilde{\chi}^0_1$ on the exclusion bounds from $\tilde{u}_1 \to Wb\tilde{\chi}^0_1$ is planned.

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APPENDIX

1. Relevant phase-space formulas

The fully differential decay width $d\Gamma$ for a generic process $p \to p_1 + p_2 + \cdots + p_n$ can be written as

$$d\Gamma = \frac{1}{2m} |M|^2 D\Phi(1 \to n),$$

(A1)

where $|M|^2$ is the squared matrix element, summed and averaged over spins and colors of the particles in the final and initial state, respectively, and $m$ is the mass of the decaying particle.

In Refs. [89,90] useful parametrizations for the phase-space factors $D\Phi(1 \to n)$ have been given for $n = 3, 4$, for the case where all final-state particles are massive. In our problem we only use the case $n = 3$ where only the neutralino is massive; this means that the general formula simplifies. In the following subsection we see that the three-particle phase space can be parametrized in terms of two parameters $\lambda_1$ and $\lambda_2$, which run independently in the range $[0, 1]$. Of course, all scalar products involved in $|M|^2$ can be expressed in terms of these parameters.

a. Phase-space parametrization for the three-particle final state

In our application we identify $p_1$ with the neutralino, $p_2$ with the (massless) charm quark and $p_3$ with the gluon and define $x_1 = m^2_{\tilde{u}_1}/m^2_{\tilde{u}_1}$. Starting from Eq. (2.10) of Ref. [89], one gets

$$D\Phi(1 \to 3) = \frac{m_{\tilde{u}_1} \gamma_2 d^{-d} \tilde{\chi}^0_1}{\Gamma(d-2)} [(1 - \lambda_1) \tilde{\lambda}_1 \tilde{\lambda}_2^{-d}] [(1 - \lambda_2) \tilde{\lambda}_2 d]^{-3} \times (1 - x_1)^2 \tilde{\lambda}_2 (1 - x_1) + x_1 \tilde{\lambda}_2 d \tilde{\lambda}_2 d\tilde{\lambda}_2.$$  

(A2)

The scalar products of the momenta $p_i$, encoded in the quantities $s_{ij} = (p_i + p_j)^2/m^2_{\tilde{u}_1}$, can be written in terms of the parameters $\lambda_1$ and $\lambda_2$ as

$$s_{13} = \lambda_2 (1 - x_1) + x_1,$$

$$s_{12} = \frac{\lambda_1 (\lambda_2 - 1) \lambda_2 (1 - x_1)^2 - x_1}{\lambda_2 (x_1 - 1) - x_1}.$$  

2. Loop functions

The one-loop functions which appear at various places in this Appendix are defined as

$$A_0(m^2) = \frac{16\pi^2 \mu^2 e^{\epsilon \epsilon_6}}{(4\pi)^{\epsilon}} \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\ell^2 - m^2}$$  

(A3)

$$B_0(p^2; m^2_1, m^2_2) = \frac{16\pi^2 \mu^2 e^{\epsilon \epsilon_6}}{(4\pi)^{\epsilon}} \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{(\ell^2 - m^2_1)(\ell^2 + p^2 - m^2_2)}$$

$$= \mu^2 e^{\epsilon \epsilon_6} \Gamma(\epsilon)$$

$$\times \int_0^1 [-x(m^2_1 - m^2_2 + p^2) + m^2_1 + p^2 x^2]^{-\epsilon}.$$  

(A4)

Finally, we have

$$B_1(p^2; m^2_1, m^2_2) p^\mu = \frac{16\pi^2 \mu^2 e^{\epsilon \epsilon_6}}{(4\pi)^{\epsilon}} \int \frac{d^4 \ell}{(2\pi)^4} \frac{\ell^\mu}{(\ell^2 - m^2_1)(\ell^2 + p^2 - m^2_2)}$$

$$= \frac{p^\mu A_0(m^2_1) - A_0(m^2_2) - (p^2 + m^2_1 - m^2_2) B_0(p^2; m^2_1, m^2_2)}{2p^2}.$$  

(A5)
\[ B_2(p^2; m_1^2, m_2^2) = \frac{16\pi^2 \mu^2 e^{\sigma_{rec}}}{(4\pi)^d} \int \frac{d^d \ell}{(2\pi)^d} \frac{\ell^2}{(\ell^2 - m_1^2)(\ell^2 - m_2^2)} = A_0(m_2^2) + m_1^2 B_0(p^2; m_1^2, m_2^2) \] (A6)

\[ C_0(p_1^2, (p_1 - p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) = \frac{16\pi^2 \mu^2 e^{\sigma_{rec}}}{(4\pi)^d} \int \frac{d^d \ell}{(2\pi)^d} \frac{1}{(\ell^2 - m_0^2)(\ell^2 - (p_1 + p_2)^2 - m_2^2)} = -\mu^2 e^{\sigma_{rec}} \Gamma(e + 1) \int_0^1 dx \int_0^{1-x} dy [-x(m_0^2 - m_1^2 + p_1^2) - y(m_0^2 - m_2^2 + p_2^2) + p_1^2 x^2 + 2xyp_1 \cdot p_2 + p_2^2 y^2]^{-1/2} \] (A7)

\[ C_2(p_1^2, (p_1 - p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) = \frac{16\pi^2 \mu^2 e^{\sigma_{rec}}}{(4\pi)^d} \int \frac{d^d \ell}{(2\pi)^d} \frac{\ell^2}{(\ell^2 - m_0^2)(\ell^2 - (p_1 + p_2)^2 - m_2^2)} = B_0((p_2 - p_1)^2; m_1^2, m_2^2) + m_0^2 C_0(p_1^2, (p_1 - p_2)^2, p_2^2; m_0^2, m_1^2, m_2^2). \] (A8)

3. Vertex correction involving the gluino

The correction of the squark-quark-neutralino vertex involving the gluino (see the right frame of Fig. 3) reads

\[ \Sigma_{\tilde{g}i}^{3/2} = \frac{-1}{16\pi^2} \sum_{j,s} \left[ \bar{\tilde{g}}_{\tilde{g},j,u_1} \left( C_0 + C_{p_{\tilde{g},j}} \Gamma_{\tilde{g},j,u_1} \Gamma_{\tilde{g},j,u_1} m_3^{-2} \right) \right. \\
+ \left. (C_0 + C_{p_{\tilde{g},j}}) \Gamma_{\tilde{g},j,u_1} \Gamma_{\tilde{g},j,u_1} m_3^{-2} \right) P_R + \left( L \leftrightarrow R \right) \] (A9)

with the abbreviations

\[ C_0 \equiv C_0(m_{a_i}^2, m_{a_f}^2, 0; m_{\tilde{g}}^2, m_{\tilde{u}_a}^2, m_{\tilde{u}_b}^2) \] (A10)

\[ C_2 \equiv C_2(m_{a_i}^2, m_{a_f}^2, 0; m_{\tilde{g}}^2, m_{\tilde{u}_a}^2, m_{\tilde{u}_b}^2) \] (A11)

\[ C_{p_{\tilde{g},j}} \equiv C_{p_{\tilde{g},j}}(m_{a_i}^2, m_{a_f}^2, 0; m_{\tilde{g}}^2, m_{\tilde{u}_a}^2, m_{\tilde{u}_b}^2). \] (A12)

\[ C_{p_{\tilde{g},j}} \] is defined through the decomposition

\[ \frac{16\pi^2 \mu^2 e^{\sigma_{rec}}}{(4\pi)^d} \int \frac{d^d \ell}{(2\pi)^d} \frac{\ell^\mu}{([\ell^2 - m_0^2][\ell^2 - p_1^2 - m_2^2])^{1/2}} = p_{\tilde{g},j}^\mu C_{p_{\tilde{g},j}} + p_{\tilde{u}}^\mu C_{p_{\tilde{u}}} \] (A13)

and is given by

\[ \Sigma_{\tilde{g},i}(p^2) = \frac{\alpha_2 C_F}{\pi} \left\{ (W_{\tilde{g},i} + W_{\tilde{g},j} + W_{\tilde{g},f}) \left( B_2(p^2; m_2^2, m_3^2) + p^2 B_1(p^2; m_2^2, m_3^2) \right) \right. \\
\left. - m_3 m_0 \left( W_{\tilde{g},i} + W_{\tilde{g},j} + W_{\tilde{g},f} \right) B_0(p^2; m_2^2, m_3^2) \right\} \] (A19)

4. Self-energies of quarks and squarks

In our approximation where we put \( m_c = 0 \), the quark self-energy contribution with an internal squark and gluino is only needed at \( p^2 = 0 \):

\[ \Sigma_{fi}^{g\tilde{g}}(p^2) = \frac{\alpha_s}{2\pi} W_{fi}^\dagger W_{fi} + \frac{\alpha_s}{2\pi} C_F m_0 B_0(0; m_0^2, m_0^2), \] (A15)

\[ \Sigma_{fi}^{q\tilde{g}} = \frac{\alpha_s}{2\pi} W_{fi}^\dagger W_{fi} + \frac{\alpha_s}{2\pi} C_F m_0 B_1(0; m_0^2, m_0^2) \] (A16)

For the contribution with an internal quark and gluino we get (for arbitrary \( p^2 \))

\[ \Sigma_{fi}^{3/2}(p^2) = \frac{\alpha_2 C_F}{\pi} \left\{ \frac{d - 2}{d} B_1(p^2; m_2^2, m_3^2) \right\}, \] (A17)

\[ \Sigma_{fi}^{3/2}(p^2) = \frac{\alpha_2 C_F}{\pi} \left\{ \frac{d - 2}{d} B_1(p^2; m_2^2, m_3^2) \right\}, \] (A18)

For the squark self-energies there are three contributions:

First, the contribution with internal gluino and quark

\[ \Sigma_{\tilde{g},i}(p^2) = \frac{\alpha_2 C_F}{\pi} \left\{ \left( W_{\tilde{g},i} + W_{\tilde{g},j} + W_{\tilde{g},f} \right) \left( B_2(p^2; m_2^2, m_3^2) + p^2 B_1(p^2; m_2^2, m_3^2) \right) \right. \\
\left. - m_3 m_0 \left( W_{\tilde{g},i} + W_{\tilde{g},j} + W_{\tilde{g},f} \right) B_0(p^2; m_2^2, m_3^2) \right\} \] (A19)
and finally the contribution with a squark tadpole

\[ \Sigma_{\tilde{u}, \tilde{u}}(p^2) = \frac{\alpha_s}{4\pi} C_F (p^2 + m_{\tilde{u}}^2) B_0(p^2, m_{\tilde{u}}^2, 0) - A_0(m_{\tilde{u}}^2) \delta_{st}, \]  

(A20)

and finally the contribution with a squark tadpole

\[ \Sigma_{\tilde{u}, \tilde{u}} = -\frac{\alpha_s}{4\pi} C_F \left( \delta_{st} A_0(m_{\tilde{u}}^2) \right) \]

(A21)

\[ -2 \sum_{i,j=1}^{3} \sum_{s'=1}^{6} (W_{i+3s'}^{u*} W_{i+3s'}^{\tilde{u}} W_{j+3s'}^{\tilde{u}} + W_{i+3s'}^{u*} W_{j+3s'}^{\tilde{u}} W_{j+3s'}^{\tilde{u}} A_0(m_{\tilde{u}, s'}^2) ) \]

(A22)

\[ = -\frac{\alpha_s}{4\pi} C_F \frac{1}{e} |\delta_{st}| m_{\tilde{u}}^2, \]

(A23)

\[ -2 \sum_{i,j=1}^{3} \sum_{s'=1}^{6} (W_{i+3s'}^{u*} W_{i+3s'}^{\tilde{u}} W_{j+3s'}^{\tilde{u}} + W_{i+3s'}^{u*} W_{j+3s'}^{\tilde{u}} W_{j+3s'}^{\tilde{u}} m_{\tilde{u}, s'}^2 ) + \text{finite.} \]

(A24)

For further useful information on self-energies and LSZ factors, see Ref. [91].


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