Algebra of Theoretical Term Reductions in the Sciences

Dale Jacquette

Abstract: An elementary algebra identifies conceptual and corresponding applicational limitations in John Kemeny and Paul Oppenheim's (K-O) 1956 model of *theoretical reduction* in the sciences. The K-O model was once widely accepted, at least in spirit, but seems afterward to have been discredited, or in any event superceeded. Today, the K-O reduction model is seldom mentioned, except to clarify when a reduction in the Kemeny-Oppenheim sense is not intended. The present essay takes a fresh look at the basic mathematics of K-O comparative vocabulary theoretical term reductions, from historical and philosophical standpoints, as a contribution to the history of the philosophy of science. The K-O theoretical reduction model qualifies a theory replacement as a successful reduction when preconditions of explanatory adequacy and comparable systematicization are met, and there occur fewer numbers of theoretical terms identified as replicable syntax types in the most economical statement of a theory's putative propositional truths, as compared with the theoretical term count for the theory it replaces. The challenge to the historical model developed here, to help explain its scope and limitations, involves the potential for equivocal theoretical meanings of multiple theoretical term tokens of the same syntactical type.

Keywords: John Kemeny, Kemeny-Oppenheim (K-O) model of theoretical reduction, Paul Oppenheim, theoretical reduction, science, scientific theory

1. Kemeny-Oppenheim (K-O) Model

The reduction of secondary to primary sciences encounters difficulties where reduction procedures are described as involving comparisons of unspecified 'terms' that leave their individuation and denumeration undetermined. The 1956 Kemeny-Oppenheim (K-O) model of scientific and more generally theoretical reduction prescribes a reduction procedure that involves a method for the array and elimination of theoretical 'terms,' but does not explain what is to count as a *term*.

The omission turns out to have important implications in applying the K-O model. The difficulties entailed by this lack of clarity about the nature of terms apply to Kemeny and Oppenheim's treatment of theoretical reduction, but can also be raised in a general way against any attempt to set forth procedures of reduction that involve enumerations of theoretical terms and vocabularies before and after the replacement of one set of equivalently explanatorily capable competing theories for another at the propositional level. Kemeny and Oppenheim do not suggest that a reduction is achieved when the number of theoretical terms is reduced from alternative equally explanatorily powerful and systematic theory.

When K-O preconditions are satisfied, when the reducing theory explains all the same relevant observational data that the reduced theory explains, and the reducing theory is at least as well systematized as the reduced theory, however the concept of being systematized is more exactly interpreted and applied, then there results a K-O theoretical reduction marked criteriologically by a reduction in the number of theoretical term types from the reduced to the reducing theoretical vocabulary. The further moral in the fate of the K-O model of theoretical reduction in the sciences has to do with the limits of considering only syntax tokens and types, and the need also to go beyond Kemeny and Oppenheim by including the meanings and full-blooded semantic interpretations of terms and expressions in an adequate metatheory of the term token economy in comparative theoretical explanation. There is, in other words, more to reduction, even when K-O preliminary conditions are satisfied, than counting up the number of minimally needed term tokens on both sides of a theoretical reduction undertaken at the propositional level, when a reduced theory is replaced by a reducing theory.

The informal discussion Kemeny and Oppenheim present in their influential co-authored essay "On Reduction" explains theoretical reduction in the sciences in terms of several factors. When satisfied, they are supposed to produce as a consequence a numerical reduction in the number of theoretical terms needed to express the truths of reduced and reducing scientific theory for purposes of comparing their respective cardinalities. The comparative vocabulary K-O theoretical reduction model, as the authors acknowledge and intend, is easily and equally attractively extended to all systematic branches of knowledge possessing an identifiable terminology in which explanations are expressed. It is accordingly not just our understanding and ability intelligently to pursue theoretical reductions within the natural sciences in a narrow sense that is at stake, but all propositional knowledge involving explanatory propositions. Kemeny and Oppenheim believe that theoretical reduction contributes to progress in scientific understanding, because it brings science closer to more basic principles of explanation, which can in turn make a scientific theoretical explanation more practically applicable, easier to grasp in its most fundamental principles, and potentially establishing insightful conceptual connections between the special sciences.

Kemeny and Oppenheim formulate what has come to be known as a vocabulary count model of theoretical reduction. They adopt Thomas Nagel's terminology to formulate the basic principle of reduction in the sciences:

> In a reduction we are presented with two theories T_1 and T_2, and with the observational knowledge of today represented by the complex sentence

> t O...The theoretical vocabulary of T_2, Voc(T_2), contains terms which are not in Voc(T_1)...But it turns out that T_1 can explain all that T_2 can, and it is no more complex. Hence we drop T_2 from our body of theories, and strike out all the terms in Voc(T_2) which are not in T_1. Then we say that T_2 has been reduced to T_1.[1]

The authors consider four definitions to bring precision and clarity to the concept of theoretial reduction in the form of a rational reconstruction of the general requirements for an adequate reduction.[2] They outline three conditions that are supposed to be sufficient to effect a reduction in the sciences from a secondary scientific theory T_2 to a primary scientific theory T_1. According to their interpretation, T_2 has been reduced to T_1 when:

(i) T_1 can explain all that T_2 can.

(ii) T_1 is no more complex than T_2.

(iii) Hence: Drop T_2 from our body of scientific theories, and strike out all the terms in Voc(T_2) that do not occur in Voc(T_1).[3]

The account has the form of an enthymematic practical syllogism, or a sequential procedure to follow in effecting a theoretial reduction. It considers the theoretical terms of any pair of theories under consideration, with reference to potential differences in the cardinalities of the sets of theoretical terms contained in competing reduction candidates. The theory is judged ontically most economical among those with equivalent explanatory competence and systematization, whose complete set of theoretical terms has the least cardinality. We assume whatever systematization Kemeny and Oppenheim expect in their requirement (3) for a reducing theoretical explanation relative to any theory it reduces. When conditions (i) and (ii) are satisfied by theories T_1 and T_2, then we are instructed to implement directive (iii), by which the theoretical reduction of T_2 to T_1 is supposed to be achieved.[4]

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[1] Kemeny and Oppenheim 1956, 9.

[2] Kemeny and Oppenheim 1956, 7: "As this process has been the subject of much philosophical controversy, it is the task of the philosopher of science to give a rational reconstruction of the essential features of reduction." See Swanson 1962, Schaffner 1967, and the papers collected in Agazzi 1991 and in Milkov and Peckhaus 2013.

[3] Kemeny and Oppenheim 1956, 8-9.

[4] Nagel (1951; 1961) is often acknowledged as the chief exponent of classical reduction in the sciences. In 1951, 299, Nagel distinguishes between reduction principles of definability and derivability. Feyerabend (1962) attacks the principle of derivability in Nagel's analysis of scientific reduction on the grounds that the meaning-invariance it presupposes does not obtain in possible instances of reduction. See also Coffa 1967, 500. Kemeny and Oppenheim's

2. Resilience of the K-O Model

The continuing intrigue of the K-O model depends largely on the fact that it features what is arguably the only objective measure of anything belonging to reduced theories having been literally *reduced* conceptually or in the cardinality of a new science's referential domain of existent objects. The K-O model proposes to count the theoretical term syntax tokens in the complete statements of a theory's putative propositional truths, and in particular to tally up the theoretical term tokens in the propositions that each theory advances as true explanations. When the relevant propositions are written out, assuming we know which terms are theoretical, we can simply highlight every occurrence of a theoretical term syntax token as though the words were presented in a two-dimensional matrix.

The > -1 algebra for K-O reductions applied to theoretical term tokens in a random theory's inscribed explanations nevertheless reveals the limitations of a purely syntactical albeit the only objective criterion of theoretical reduction. The implication is that the K-O model, whatever its fate at the hands of previous lines of criticism, and regardless of its current reputation and range of philosophical acceptance and acknowledged application, or the reverse, should either be: (a) rejected and replaced by a metatheory that interprets theoretical reductions in terms of the *meanings* of theoretical terms in reduction candidate theories, and not just the syntax of the theoretical term tokens scattered among a theory's propositions; or else (b) a major overhaul of the K-O model would be needed to accommodate semantic as well as purely syntactical dimensions of theoretical reductions from one choice of theoretical explanatory propositions to another. If meaning in the relevant theoretical expressions cannot be understood as purely objective, then a further apparently inescapable implication is that theoretical reduction in the sciences is also not a purely objective relation, phenomenon or occurrence.

However tempting it may be to turn away from the K-O model as old-fashioned or unsuited to a significant number of recognized theoretical reductions, to follow a trend of disregard for its usefulness in contemporary philosophy of science, to the point where few have studied its details, the K-O account of theoretical reduction cannot be so easily discounted, even as it braves indifference. The model succeeds in its most general form despite criticism and neglect because in the end it interprets theoretical reduction as involving a literal comparative numerical reduction in the theoretical vocabularies of competing scientific theories as the only objective measure of their comparative conceptual and consequently respective explanatory economies. To know to what concepts and entities a theory makes explanatory ontological commitments, the K-O model says that we must count the words that appear as

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discussion ignores the principle of derivability for the most part, dealing with the principle of definability as bypassing the problem of meaning-invariance.

specific syntax items in a typically inscribed statement of the theory's explanations.

What else are we supposed to be able to do, if we are proceeding objectively, scientifically, in arriving at these metatheorical comparisons in support of the conclusion that one theory is reducible or has in fact been reduced to another? Even in the case of still living theorists who can further explicate their explanations by offering forth still more words to digest, and certainly with respect to the documented written heritage of theory development in a culture, there seems to be no available method except to read or otherwise process and evaluate the syntax in which a theory's explanations are expressed. Such considerations provide strong if not finally decisive justification for some form of the K-O model in the metatheory of theoretical reduction and its expected scientific methodology.

Elementary algebraic relations of > -1 govern the relative numbers of theoretical term tokens that belong to a theory than to the theory to which it is K-O reduced. As always, in the original K-O model, explanatory adequacy on both sides of theoretical reduction is presupposed, along with other condition to be met, so that theoretical reduction, as Kemeny and Oppenheim insist, can contribute to scientific progress. Differences in syntax token numbers in different choices of theoretical explanations can be understood as signifying both comparative economic differences in the numbers of concepts and entities to which a theory is ontologically committed, and, secondly, also, the comparative simplicity or complexity of such explanations, as reflected in the number of times a theoretical term must be employed within a theory's explanations in order to express its explanations.

3. Critique of the Comparative Vocabulary Reduction Model

An objection to the K-O model is that all three of the conditions in (i)-(iii) can be fulfilled in circumstances in which a theoretical reduction of scientific or other explanatory theory T_1 from T_2 in Kemeny and Oppenheim's sense is not effected.

What the underlying algebraic structure of the K-O model seems to reveal, demonstrated in a highly simplified application that nevertheless meets the K-O conditions, is that the K-O model is woefully inadequate in its inability to support correct evaluations of reduction relations in the overwhelming number of possible reduction candidates among choices of theoretical terms in the vocabularies of competing explanatory theories. The K-O model fails in particular for the vast number of random combinatorially available *syntactically token replicative* cases. The argument suggests that the K-O model, on these specific grounds, must either be rejected as an inappropriate interpretation of the comparative vocabulary concept of theoretical reduction, or, if the interpretation is judged correctly to capture the comparative vocabulary concept, then the idea of theoretical reduction itself must be rethought as a descriptive model of or prescriptive guideline for ideal scientific practice.

Theoretical reduction on the K-O model, as previously mentioned, is supposed to be progressive, resulting in theoretical explanatory improvements.[5] A genuine theoretical reduction must entail no loss in ability to explain phenomena when one theory is reduced to another, and the theory to which another is reduced must constitute a simpler or more economical way of explaining the same phenomena as the theory from which it is reduced. For Kemeny and Oppenheim, the simplification that is expected to result from a scientific theoretical reduction produces a greater economy in the number of terms in the scientific vocabulary. They begin by asking:

> What are the special features of reduction? Since it is to be progress in science, we must certainly require that the new theory should fulfill the role of the old one, i.e., that it can explain (or predict) all those facts that the old theory could handle. Secondly, we do not recognize the replacement of one theory by another as progress recognize the replacement of one theory by another as progress unless the new theory compares favorably with the old one in a feature that we can very roughly describe as its simplicity...And the special feature of reduction is that it accomplishes all this and at the same time allows us to effect an economy in the theoretical vocabulary of science.[6]

The objection to this reasonable proposal is that the K-O conditions (i)-(iii) do not necessarily guarantee reduction in the sciences in the relevant sense of 'simplicity', by effecting a theoretical economy in the scientific vocabulary. The argument to demonstrate the limitations of the K-O model of reduction begins with an elementary secondary science T_2 in which the following conditions obtain between scientific principles (A,B,C,D) and theoretical observations (O_1,O_2,O_3,O_4):

(1) A explains O_1

(2) B explains O_2

(3) C explains O_3

(4) D explains O_4

Here there are four explanatory scientific laws in one-one correspondence with four observations to be explained. This is already an unrealistic simplification, because scientific laws are ordinarily assigned the task of explaining many observations, and several laws are often needed to explain a single observation. Needless to say, besides, most scientific theories additionally include more than four scientific laws. Although the theory is simplified in at least these ways, it should nevertheless serve the purpose of illustrating a

[5] Kemeny and Oppenheim 1956, 6: "The label 'reduction' has been applied to a certain type of progress in science."

[6] *Ibid*.

general point about the limitations of the K-O model of theoretical reduction. There can, after all, be theories as basic as this interpretation of T_2, and if the K-O reduction model does not work even in this simple case, then it should probably not be expected to provide correct results when extended to increasingly more complex and to that extent potentially more realistic applications.

Consider what the K-O model would call the theoretical 'terms' contained within or by means of which the four scientific laws (A,B,C,D) of T_2 are expressed.[7] Again, somewhat artificially for the sake of argument, suppose that the vocabulary of theory T_2, Voc(T_2), consists of the follwing *vocabulary matrix* of 'terms':

Voc(T_2) Secondary Theory (Nonreplicative Case)

A = {a,a',a'',a'''} explains O_1

B = {b,b',b'',b'''} explains O_2

C = {c,c',c'',c'''} explains O_3

D = {d,d',d'',d'''} explains O_4

Grammatically and in other ways formally well-formed combinations of these theoretical terms associated with each law make it possible to explain each correlated observation. The terms in A, for example, {a,a',a'',a'''}, are used to explain O_1, and so on for O_2, O_3, and O_4. Collectively, the terms belonging to the four laws are the theoretical vocabulary of T_2, Voc(T_2), and presented above in a matrix array.

4. Theoretical Terms Nonreplicative Cases

The question is how a reduction of a secondary theory T_2 to a primary theory T_1 can be effected according to the K-O comparative vocabulary model. Two patterns of reduction are distinguished, designated as 'replicative' and 'nonreplicative.' As a paradigm of the nonreplicative case, to begin explaining the difference, suppose that T_2 above is reduced to T_1, where T_1 consists of the scientific laws (E,B,C,D), and where law E does a more economical job of explaining observation O_1, by virtue of containing only three terms (e,e',e''). The theoretical vocabulary of T_1 is thus:

Voc(T_1) Primary Theory (Nonreplicative Case)

E = {e,e',e''} explains O_1

B = {b,b',b'',b'''} explains O_2

[7] Observational or theoretical or both or neither; 'terms' simpliciter. See Jacquette 2004.

$C = \{c,c',c'',c'''\}$ explains O_3

$D = \{d,d',d'',d'''\}$ explains O_4

It appears that:

(i) T_1 by hypothesis can explain all that T_2 can in explaining O_1-O_4.

(ii) T_1 is no more complex than T_2, for it contains the same number of laws, and it contains fewer scientific terms ($Voc(T_1) = x = 16$, $Voc(T_2) = x - 1 = 15$).

(iii) Hence, we can drop T_2 from our body of theories, replace it with T_1, and strike out all the terms in the relevant vocabulary that occur in $Voc(T_2)$ but that do not occur in $Voc(T_1)$, viz.: the terms (a,a',a'',a'''). In this way we eliminate all of law A which has become superfluous in explaining O_1 after the discovery, verification, or acceptance of the more economical law E, by eliminating its theoretical terms.[8]

The fulfillment of these three conditions of the comparative vocabulary K-O model qualifies this first example as a genuine instance of theoretical reduction on the authors' terms. For we have eliminated four terms from the theoretical vocabulary (a,a',a'',a'''), and added only three (e,e',e''). Conditions (i)-(iii) are satisfied, and the replacement of T_2 by T_1 represents a simplification and greater economy of theoretical vocabulary. The example is unproblematic in the sense that it effects what Kemeny and Oppenheim would regard as scientific progress in theoretical reduction. It is a nonreplicative reduction, by virtue of the fact that it does not involve the replication of distinct tokens of any single syntactical term type, distributed over the theory's explanatory propositions. We assume throughout in what follows that the comparative vocabulary K-O model preconditions of explanatorily covering all the relevant observational data and being at least as well systematized (whatever this is finally understood to mean) when a reducing theory replaced a reduced theory.

5. Theoretical Term Tokens in Vertically Replicative Cases

The nonreplicative case is well-behaved but statistically atypical of scientific reductions. The percentage of possible nonreplicative reductions, supporting a matrix of four laws consisting of four scientific terms each, is swamped by the percentage of possible replicative cases in which conditions (i)-(iii) are satisfied,

[8] Kemeny and Oppenheim are hesitant about utterly eliminating superfluous terms from Voc(T2). See 1956, 17, note 3. Assume that the terms are definitely proven superfluous, thereby avoiding extralogical questions of convenience in holding on to technically unnecessary theoretical terms. We might consider, however, that such terms not be entirely eliminated from broader vocabularies of terms useful in strictly nonscientific explanations (e.g., in lay or historical explanations of scientific theories).

but in which nothing that Kemeny and Oppenheim would allow intuitively to count as a genuine reduction results. The problem is illustrated by the following replicative application of the K-O theoretical reduction model.

Suppose for simplicity sake that primary science T_1 consists of the scientific laws (E,B,C,D), and that, as above, the scientific terms of law E = (e,e',e''). Then, where $Voc(T_2) = x$, $Voc(T_1) = x - 1$ theoretical terms belonging to the two vocabularies. Now suppose also that the matrix of theoretical terms for both T_2 and T_1 for laws (B,C,D) in the nonreplicative case is not identical to the matrix of theoretical terms in the following replicative case. We stipulate again that E explains O_1, B explains O_2, C explains O_3, and D explains O_4, when T_2 is reduced to T_1. We permit restricted replication of scientific terms vertically in the matrix, but do not consider horizontal replication. To be precise, we specify the scientific terms of the two modified theories in the replicative case in this way:

$Voc(T_2^r)$ Secondary Theory (Replicative Case)

A = {a,a',a'',a'''} explains O_1

B = {b,b',b'',b'''} explains O_2

C = {c,a',c'',c'''} explains O_3

D = {d,d',d'',d'''} explains O_4

The theory is replicative in an obvious sense, because laws A and C share a single term a', rather than each containing completely different distinct scientific terms. The secondary theory is now K-O theoretically reduced to:

$Voc(T_1^r)$ Secondary Theory (Replicative Case)

E = {e,e',e''} explains O_1

B = {b,b',b'',b'''} explains O_2

C = {c,a',c'',c'''} explains O_3

D = {d,d',d'',d'''} explains O_4

There are difficulties for the K-O model that the replicative case immediately brings to light. Condition (i) is satisfied because both T_2 and T_1 adequately explain O_1-O_4. Condition (ii) is also satisfied because $Voc(T_2) = x = 16$, $Voc(T_1) = x-1 = 15$ scientific terms. When we attempt to fulfill condition (iii), however, as the K-O model requires whenever conditions (i) and (ii) are met, an interesting problem arises.

Satisfying condition (iii), we drop T_2 from our body of theories, and strike out the now superfluous theoretical terms in Voc(T_2) that do not appear in Voc(T_1). In the nonreplicative case we eliminate four terms (a,a',a'',a''') and add only three terms (e,e',e''), so that fulfillment of conditions (i)-(iii) effect a genuine reduction in such a way as to represent scientific theoretical progress. By contrast, in the replicative case, if we strike out the scientific terms that occur in Voc(T_2) that do not occur in Voc(T_1), we can strike out only the terms (a,a'',a'''), but not (a'), because (a') also occurs as a restricted vertical replication instance in Voc(T_1). After all, condition (iii) instructs us only to strike those terms from Voc(T_2) that do *not* occur in Voc(T_1). Law C in T_2 and in T_1 here consists of the terms (e,a',e'',e'''). Thus, we can only eliminate three terms (a,a'',a''') from the scientific vocabulary of T_1 in Voc(T_1).

However, since we have also added three terms to Voc(T_1) (e,e',e''), then, despite satisfying K-O model conditions (i)-(iii), no real *reduction* has been effected in the relevant simplification (comparative economy) sense of the K-O model, because the theoretical vocabulary of T_1 has not been simplified or made more economical than that of T_2. The net economy of a K-O comparative theoretical vocabulary reduction is necessary for the kind of progress that is supposed to characterize a genuine reduction through the replacement of one scientific theory by another. Kemeny and Oppenheim are quoted above as insisting:

> ...we do not recognize the replacement of one theory by another as progress unless the new theory compares favorably with the old one in a feature that we can very roughly describe as its simplicity... And the special feature of reduction is that it accomplishes all this and at the same time allows us to effect an economy in the theoretical vocabulary of science.[9]

Thus, there are instances in which all three conditions of the K-O model of theoretical reduction are satisfied, but where the theory that follows upon fulfilment of the conditions does not constitute a genuine theoretical reduction, given all that Kemeny and Oppenheim have informally to say about the requirements. The reason is that no economy in the scientific vocabulary and therefore no progress in science results when the conditions are satisfied in some term replicative applications. Is the replicative case significant? Can we ignore the problems it poses in light of the usefulness of the nonreplicative cases and the limited possibilites of the replicative case, the unlikelihood that it will appear among the reductions of otherwise methodologically scrupulous systematized theoretical explanations? It is easy to see that the replicative case is not a degenerate construction, because the percentage of its occurrences in a body of scientific theories projecting a matrix of theoretical terms as they appear

[9] *Ibid.*, 6.

in two dimensions, containing theoretical terms in the relevant propositions both horizontally and vertically in a list of the theory's putative truths, is enormous compared to the alternative. The replicative case vastly outnumbers the nonreplicative cases of theoretical reduction on the K-O model. Moreover, important actual T_2 secondary theories in the history of science and in contemporary theoretical explanation almost always (98-99% of the logically possible cases) embed a disqualifying vertical syntactical replication of theoretical term tokens.

6. Algebraic Parameters of K-O Theoretical Reduction

More definite mathematical significance can be offered in support of this criticism of the K-O model of comparative theoretical vocabulary reduction, by comparing the percentage of possible nonreplicative cases against the percentage of possible replicative cases, using the same simplified matrix of sixteen scientific terms assigned in sets of four each to each of four scientific laws.

Suppose that the class of nonreplicative cases and the class of replicative cases logically exhaust the total possible instances of theories potentially entering into a theoretical reduction relation. In the nonreplicative case, there are in the simplified case precisely 1,820 possible combinations of terms available for nonreplicative reductions satisfying K-O model requirements (i)-(iii). This is determined combinatorially by the equation $n!/r!(n-r)!$, relying on the same pool of scientific terms, where n = the number of terms in the matrix, and r = the groupings of those terms for each law or horizontal coordinate of the matrix. In our simplified model, $n = 16$ and $r = 4$. Some of these possible configurations of scientific terms are uninteresting, such as the difference between (a,a',a'',a''') and (a,a'',a''',a'). So the importance of this mathematical information is not found in the absolute value of the cardinality of possible configurations, but in the ratio obtained by comparison of this indicated number with the total number of possible configurations permitted by the conditions of restricted vertical syntactical replications of term tokens in the replicative case.[10]

In the replicative case, further simplified to permit vertical but not horizontal replication in the matrix of scientific terms, there are 172,900 possible configurations of scientific terms. This is determined by the general formula $L(n!/(n-r)!)-(n!/r!(n-r)!)$, where, as before, n and r equal respectively the number of terms in the matrix and the size of the groupings of the subsets of

[10] Calculation of these values is extremely oversimplified in the application, which is atypical of theories and laws in propositional explanations actually used by scientists. Here only four laws are involved, with four terms each, and those laws stand in a one-one relation or correspondence with the observations they explain. The difficulties are compounded in cases dealing with more true-to-life scientific theories.

those terms, and where L is the number of scientific laws or horizontal coordinates of the matrix. The ratio of these two values of possible configurations in the matrix of sixteen scientific terms of the secondary theory to be reduced, where the nonreplicative case is compared to the total possible configurations in the universe of discourse, including both replicative and nonreplicative cases, is 0.0104166. This is to say, that in the limits of our universe of discourse of total possible nonreplicative and replicative cases, only 1.04166% can be of the sort we have called workable nonreplicative cases. The remaining 98-99% of such possible cases are unworkable because they are replicative. These are the sort of difficulties we encounter if we attempt to apply the K-O comparative vocabulary interpretation of theoretical reduction to cases of restricted vertical replication of scientific terms in a scientific theory's vocabulary matrix. The replicative instances are transparently generated by considering more than one, in fact by considering all the permitted replications in the matrix, in contrast with our simplified model in which only a' is replicated exactly once. It seems appropriate to conclude that the restricted utility (to 1.04166% of all possible instances) of the K-O comparative vocabulary model of theoretical reduction in the sciences warrants either its total rejection or major fundamental redesign.

7. Vertical Replication of Theoretical Terms in Actual Scientific Theories

We cannot prove, but we can suggest by way of examples selected entirely at random from ancient and contemporary scientific documents, that many if not most scientific theories contain laws that exemplify vertical replication of theoretical terms.[11] For these examples, and many like them, the K-O interpretation of theoretical reduction cannot be used to describe or guide a reduction to some primary science T_1.

First, Galileo writes in *De Muto Accelerato* (c. 1590):

1) …*bodies* of the same material but of unequal *volumes* move (in natural *motion*) with the same *speed*.

2) …when solids lighter than water are completely immersed in water, they are carried upward with a force measured by the difference between the weight of a *volume* of water equal to the volume of the submerged *body* and the *weight* of the *body* itself.

3) …if we wish to know at once the relative *speeds* of a given *body* in two different media, we take an amount of each medium equal to the

[11] Horizontal replications are also possible and frequently occur. For several reasons they are not considered in calculating the possible configurations of replicated terms in a matrix that specifies the scientific vocabulary of a theory Voc(Tn). The mathematical formula for computing the number of possible unrestricted replications of n terms is simply n! (n-factorial).

volume of the *body* and subtract the *weight* (of such amounts) of each medium the *weight* of the *body*.

4) …in the (natural) downward *motion* of *bodies* the ratio of the *speeds* is not equal to the ratio of the *weights* of the *bodies*…[12]

While in a modern if not especially recent tried-and-true 1968 genetics textbook taken down from the shelf we find:

1) [*Chromosomes*] duplicate precisely and divide equally in mitosis, furnishing each cell with a full complement of *chromosomes*.
2) Their behavior in *meiosis* accords with our expectations of heredity — that it is due to contributions from both parents.
3) Their random mixing and crossing over during *meiosis* provides an important source for the observed variables between individuals.
4) In addition…*chromosome* abberations can be associated with the inheritance of specific characteristics. [13], [14]

Such instances are typical rather than exceptional among the groupings of propositions in an explanatory theory in which syntactically identical term tokens appear in several of the propositions according to the pattern we have referred to as vertical replication. There is usually a network of token syntactical linkages among the propositions advanced for purposes of theoretical explanation in a theory, reflected in the matrix of each specific theory's vocabulary of theoretical terms, targeted by the K-O theoretical reduction model for comparison in establishing theoretical reduction and reducibility relations between any two or more competing explanatory theories.

8. Countercritique of Theoretical Term Reduction Model Objections

Problems of several kinds might be raised against the matrix analysis of theoretical terms in a scientific theory. We conclude by addressing two such complaints, both of which seem dangerous, but on consideration neither of which seems to be especially compelling. The second criticism pinpoints exactly the philosophical difficulties that seem to be entailed whenever theoretical reduction procedures are described by ambiguous reference to syntactical 'terms,' as the main objection to the K-O comparative vocabulary model of theoretical reduction.

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[12] Galileo 1960 [1590], 29, 33, 35.

[13] Strickberger 1968, 48.

[14] Consult the laws of a theory in almost any ancient or contemporary scientific text. For example, Galileo 1933, 203, 209, 218. Newton 1972 [1726]. Bent 1965, 15 (citing Joule's paper "On the Mechanical Equivalent of Heat" (communicated by Michael Faraday to the Royal Society in 1849)). Bloss 1971. See also Kimbrough 1979.

Criticism 1. It might be objected that counterexample replicative cases do not appropriately fit the K-O model of theoretical reduction, because if no genuine reduction is effected, that just means that condition (ii) is not satisfied. This reasoning misses the point of the argument. For initially condition (ii) is not and is not supposed or expected to be satisfied. $Voc(T_2) = x$ (16 terms), and $Voc(T_1) = x - 1$ (15 terms). The K-O interpretation then instructs us to enact condition (iii). However, when we carry out condition (iii), as previously observed, we do not effect a reduction of theories by simplification or economy in the theoretical vocabulary. Criticism 1 accordingly overlooks the precise way in which the K-O model authors have instructed us to use their schema as a kind of decision procedure, and we have followed these instructions in constructing our counterexample to criticize it on its own own erritory, and literally in its own terms. The simple-minded example presented above contains only one replication of a single term a' in the matrix of the relevant theoretical vocabulary. If two or more such terms, beginning with a' and b'', were to be vertically replicated within the matrix, then in a still significant percentage of total possible cases, condition (ii) would turn out to be initially satisfied, but no genuine reduction would result. T_1 would be 'more complex' than T_2, even under the deliberate misconstrual of the intentions of the K-O interpretation on which Criticism 1 is based.

Criticism 2. A more serious objection holds that we have no business counting a' as it occurs in law A (in T_2) as a denumerably distinct term from a' as it occurs in law C (also in T_2). If this is true, then the problems of the replicative case considered above disappear. For then $Voc(T_2) = 15$ instead of 16, and $Voc(T_1) = 15$ also, for the same reasons as before. Thus, if the argument is to get off the ground, we must eliminate a term in $Voc(T_1)$ in order to initially satisfy condition (ii).[15] Suppose that we respecify law E to consist of (e,e') instead of (e,e',e''), implying that $Voc(T_1) = x - 1$, once again, this time = 14. If we fulfill condition (iii) under these circumstances, then we do in effect what looks to be a genuine theoretical reduction according to the K-O model. We eliminate three

[15] Theory T_1 being 'no more complex than' T_2 may be sufficiently ambiguous to dismiss this objection. If theories are constructive propositional entities, then, if the propositions of one are simpler than those of the other, then the first theory should be simpler than the second. For propositions to be simpler than one another in scientific usage syntactically speaking covering the same explanatory obligations with competence and systematization can only be a matter of the number of countable theoretical terms contained in the reducing theory's explanatorily competent and well-systematized propositional replacement for the reduced theory's explanations. If you add more terms, you increase the grammatical combinatorics and hence the greater complexity of explanations. The greater number of theoretical term tokens, the greater potential for complexity, when its explanations stacked up against those of the reducing theory with a smaller syntactical theoretical term cardinality.

terms (a,a'',a''') from Voc(T_2) that do not occur in Voc(T_1), while adding only two terms (e,e'). In this application, we simplify and economize the theoretical vocabulary in replacing T_2 with T_1, thereby ridding the referential domain of as many corresponding theoretical concepts and objects. It may therefore seem that the application is an example of the nonreplicative case after all. Why indeed should we count two theoretical term tokens, a' and a' (say, in classical mechanics or kinematics, 'force' and 'force,' 'mass' and 'mass') as denumerably distinct terms? Why should we count these as two terms instead of two replications of a single term?

The best reason for considering these terms as distinct is that the K-O model of theoretical reduction in conditions (i)-(iii) and the authors' surrounding informal discussions, deal solely with 'terms' and not with the meanings the terms might be assigned. This means that tokens of the very same term vertically replicated with the matrix of a theoretical vocabulary can potentially mean radically different things. We ought for safety sake then to denumerate these replicated terms as discrete and distinct entities in determining the number of theoretical terms in a theory's vocabulary matrix. Naturally, it is only good theoretical practice not to use syntactically indistinguishable term tokens within a descriptive and explanatory vocabulary as having different meanings. Unfortunately, there can be no logical guarantee that this is not the case with respect to any particular theory and its theoretical terms. Moreover, the K-O model, as we have seen, makes no provision for sanitizing the terms in the vocabulary of a theory in a theoretical reduction relation on semantic grounds on the basis of the meanings of replicated theoretical term tokens within the theoretical vocabulary matrix, prior to determining whether or not conditions (i)-(ii) are satisfied, and on the strength of meeting those requirements implementing condition (iii). Nor is this the problem of meaning-invariance that Paul Feyerabend raises, in which term types are thought to change meanings holistically when extended across different theoretical frameworks.[16] We refer only to term tokens composed of the identically same letters or symbols in the same order and their grammatical variants that are deliberately or even unnoticed assigned different meanings by default within a single theoretical framework. Under ideal circumstances, such ambiguities and equivocations could not arise; although in an ideal world theoretical reductions would be unnecessary anyway, since all theories would already be maximally reduced to the minimal necessary theoretical structures and the matrix of their univocally replicating theoretical terms.

An intuitively trivial example, that the K-O reduction model nevertheless does not exclude, if one can forgive the awful puns, projects a set of laws in a biological theory containing the terms 'mole' meaning 'a burrowing mammal,' 'an

[16] Feyerabend 1962, 34, 41-43.

epidermal growth of tissue,' and 'a unit of measure, especially volume.'[17] There can obviously be more subtle differences of meaning in what seem to be identical terms vertically and even horizontally replicated in any set of explanatory propositions. The metatheoretical choices here are few. The discussion has led to recognizing the following two outstanding alternatives. We can either: (a) Recalculate by stipulation supported in argument syntactically replicated term tokens as distinct entities in the vertical coordinates (and perhaps also in the horizontal coordinates) of matrices containing the vocabularies of theoretical terms belonging to specific scientific and other kinds of explanatory theories. This option has already revealed its limitations, for it is precisely the condition of the above replicative term counterexamples to the K-O model, and as such offers no respite from its damaging conclusions for the K-O model; or (b) Conclude that the K-O model be rejected outright and in its entirety, if it cannot be amended to deal adequately semantically somehow with the meanings of theoretical term tokens in a theory's theoretical vocabulary, and not just with the syntactical forms of symbols that collectively include all the tokens of the theory's theoretical terms in any single statement of the theory's totality of putative propositional truths or at least its fundamental principles or axioms, also characterizable as the theory's propositional or thetic substance or content.

What continues to fascinate about the K-O model of theoretical reduction is its confident assumption that the possibility of an episode of theoretical reduction in the history of science can only be objectively made in supposedly purely syntactical terms of competing theoretical vocabularies, in which one theory comes to be reduced by and to another. Naturally, it is the relative cardinalities of theoretical syntax items in a larger context of all theoretical explanations as they are affected by the inclusion of the reduced or reducing theory that matter. A reducing theory in genetic biochemistry might make use of many theoretical concepts that are already part of chemistry, and use overall more theoretical terms in its explanations of a predecessor pre-DNA biological theory of genes, but still result in an integrated scientific network of explanations in which overall the number of syntactically distinct theoretical terms is diminished. The applications we have considered must all be considered accordingly as miniaturized versions of the complete scientific explanatory situation before and after a reduction, in which the total number of theoretical terms are compared when a theoretical reduction is considered. They are on each side the before and after theoretical term portraits of the reduced and reducing theories in the broadest context representing all theoretical terms in all theoretical explanations.[18]

[17] Or consider the less trivial fact that 'gram' in a chemical theory can mean either 'weight' or 'mass,' or, if the term is replicated, might mean both in different laws.

[18] I am grateful to several anonymous readers who have offered useful suggestions for improvement of previous drafts of the essay.

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