Status of GRAIL Gravity Field Determination Using the Celestial Mechanics Approach

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Outline

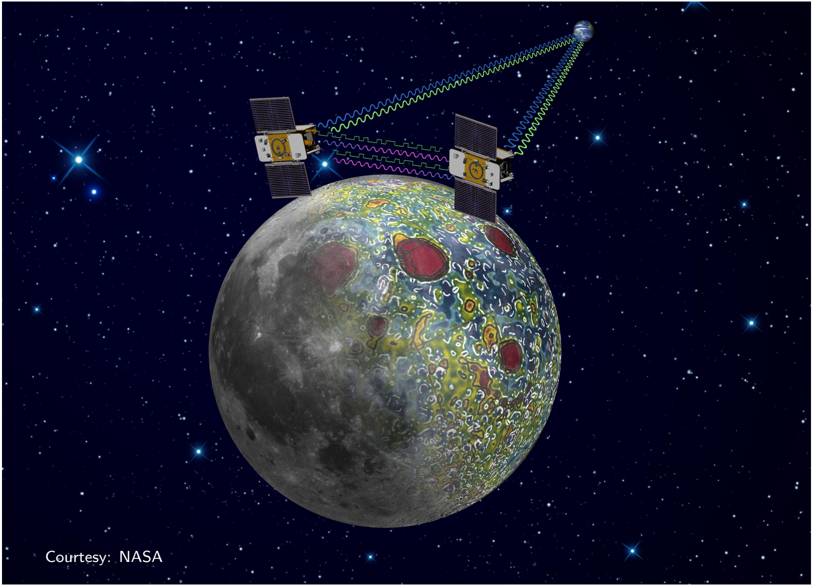
The GRAIL mission

The Celestial Mechanics Approach

Results

Conclusion & Outlook

The GRAIL mission


Courtesy: NASA

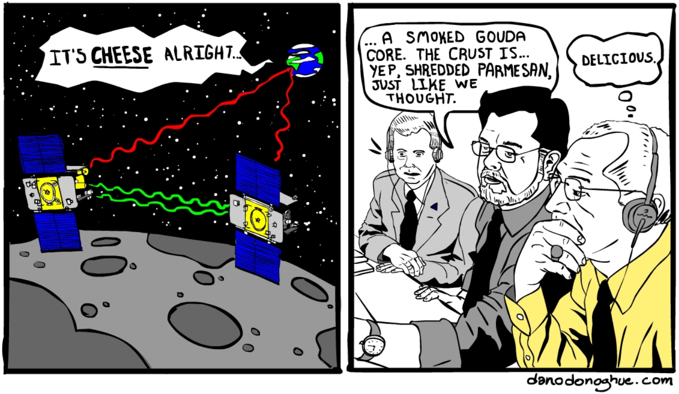
The GRAIL mission


Courtesy: NASA

- Gravity Recovery and Interior Laboratory
- Satellites “Ebb” (GRAIL-A) and “Flow” (GRAIL-B)
- Launched on 10 Sep 2011, entered lunar orbits ($i = 89.2°$) on 1 Jan 2012
- 2 science phases ($\beta > 49°$):
  - Primary mission phase: 1 Mar - 29 May, $h \sim 55\,\mathrm{km}$
  - Extended mission phase: 30 Aug - 14 Dec, $h \sim 23\,\mathrm{km}$
- Controlled crash on 17 Dec

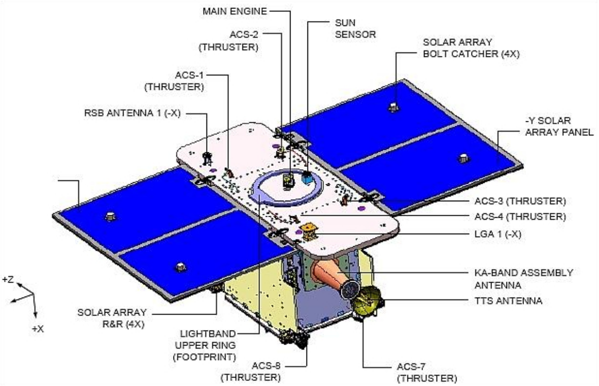
The GRAIL mission

Science objectives



- Determine structure of lunar interior, from crust to core
  → Subsurface structure of impact basins, mascons, ...
- Understand (asymmetric) thermal evolution of Moon

The GRAIL mission



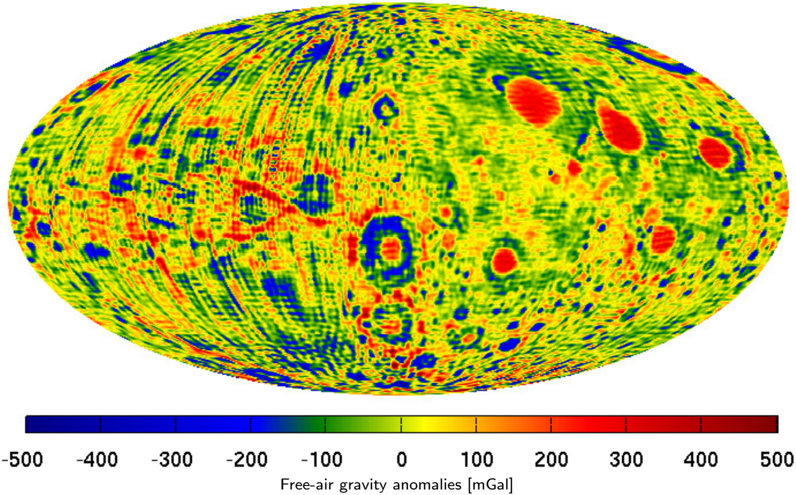
Courtesy: NASA

Main scientific instrument: Lunar Gravity Ranging System (LGRS):

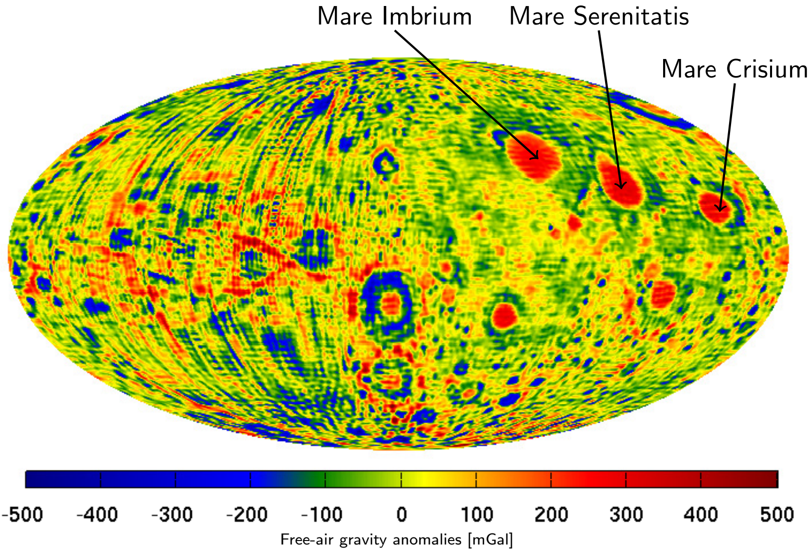
- Same principle as for GRACE
- Ultra-precise inter-satellite Ka-band range data available
- For the first time near- and far-side with same quality

The GRAIL mission

Pre-GRAIL lunar gravity missions: Lunar Prospector (NASA, 1998-99)
JGL165P1

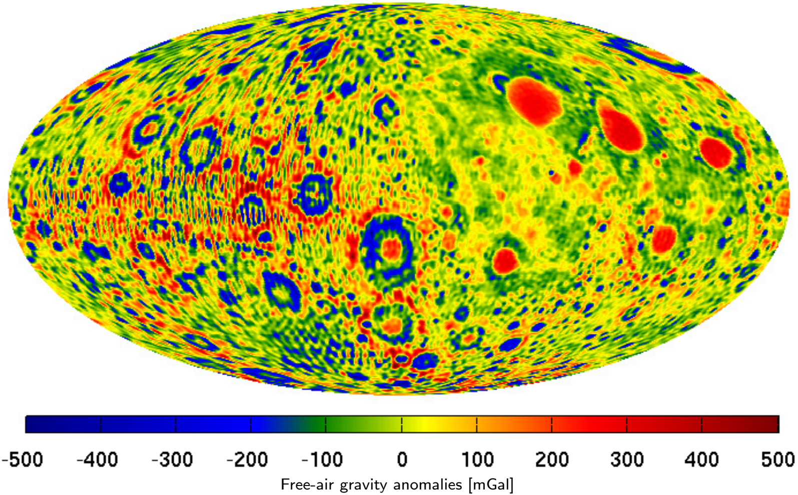


The GRAIL mission



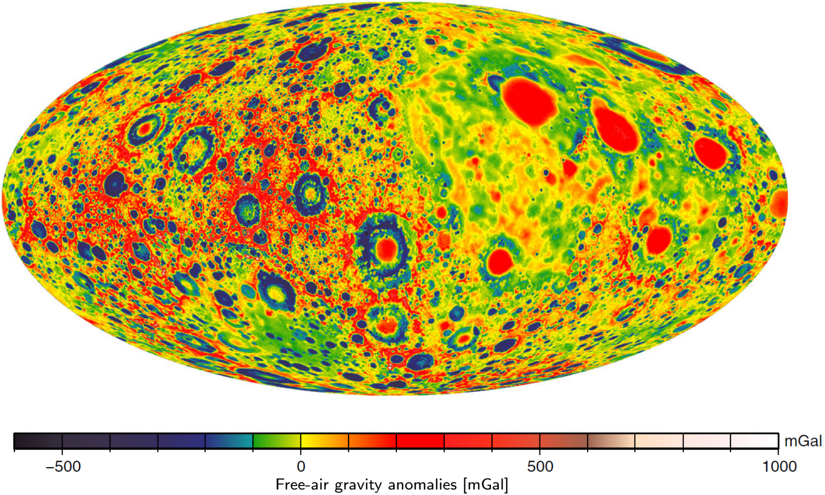
The GRAIL mission

Pre-GRAIL lunar gravity missions: SELENE (JAXA, 2007-09)
SGM150J

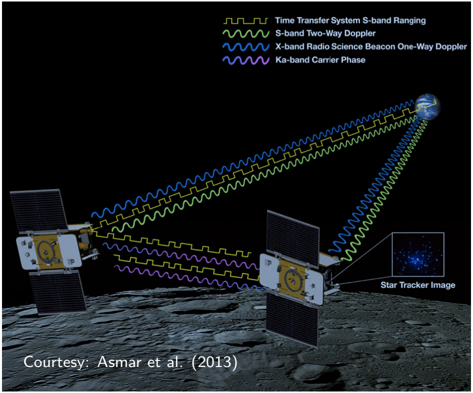


The GRAIL mission

GRAIL: First official $l_{\mathrm{max}} = 660$ (half wavelength $8.3\,\mathrm{km}$) gravity field models: GRGM660PRIM (Lemoine et al., 2013), GL0660B (Konopliv et al., 2013)



The GRAIL mission: Satellite signals

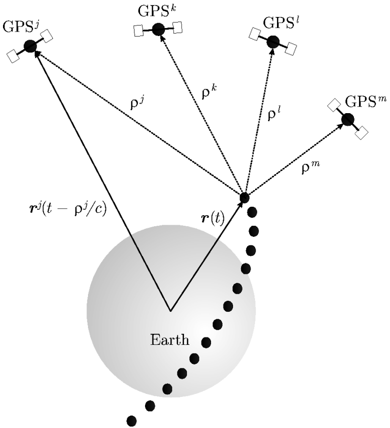


Courtesy: Asmar et al. (2013)

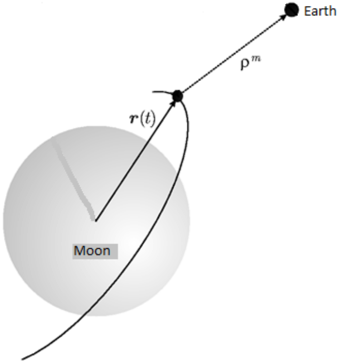
- S-band ($\sim 2\,\mathrm{GHz}$) for 2-way Doppler tracking by NASA Deep Space Network (DSN)
- X-band ($\sim 8\,\mathrm{GHz}$) for 1-way Doppler tracking
- Ka-band ($\sim 32\,\mathrm{GHz}$) inter-satellite link

Our motivation: Why not adapt our procedures for the processing of GRACE data (K-band etc.) to GRAIL, get experienced in this new environment and eventually provide an independent lunar gravity field solution?

GRAIL vs. GRACE



GRACE: Kinematic positions using GPS observations



GRAIL: DSN Doppler tracking (near-side only) yields positions

The GRAIL mission: Available data

Data releases:

- Release 2: June 12, 2013 → GRGM660PRIM, GL0660B
- Release 4: April 1, 2014 → GRGM900C, GL0900D

Selection of available data for our activities:

- 1- and 2-way Doppler data
- Ka-band range data: Ka-band range rate (KBRR)
  → 5 s-sampling in primary, 2 s-sampling in extended mission phase
- Reduced-dynamic positions (GNI1B) of GRAIL-A and GRAIL-B (by-product of gravity field estimation)
  → 5 s-sampling in primary and extended mission phase

Using the GNI1B positions as pseudo-observations allows us to gain first experience in GRAIL orbit and gravity field determination without the necessity to process DSN data!

However: Not independent!

The Celestial Mechanics Approach

The Celestial Mechanics Approach (CMA)

Selenocentric equation of motion for satellit i

$$\ddot{\mathbf{r}}_i = -GM_M \frac{\mathbf{r}}{r^3} + \mathbf{f}(t, \mathbf{r}, \dot{\mathbf{r}}, q_1, ..., q_d)$$

$$\mathbf{f} = \nabla V + \mathbf{a}_b + \mathbf{a}_t + \mathbf{a}_r + \mathbf{a}_e + \mathbf{a}_n$$

V Lunar gravity potential:

$$V(r, \lambda, \phi) = \frac{GM_M}{r} \sum_{l=1}^{l_{\text{max}}} \left(\frac{R_M}{r} \right)^l \sum_{m=0}^{l} \bar{P}_{lm}(\sin\phi) \left(\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda \right)$$

$\mathbf{a}_b$ 3rd body perturbations (Earth, Sun, Jupiter, Venus, Mars, according to JPL ephemerides DE421)

The Celestial Mechanics Approach (CMA)

Selenocentric equation of motion for satellit i

$$\ddot{\mathbf{r}}_i = -GM_M \frac{\mathbf{r}}{r^3} + \mathbf{f}(t, \mathbf{r}, \dot{\mathbf{r}}, q_1, ..., q_d)$$

$$\mathbf{f} = \nabla V + \mathbf{a}_b + \mathbf{a}_t + \mathbf{a}_r + \mathbf{a}_e + \mathbf{a}_n$$

$\mathbf{a}_t$ Tidal deformation of Moon due to Earth and Sun. IERS2010 conventions:

$$\Delta\bar{C}_{lm} - i\Delta\bar{S}_{lm} = \frac{k_{lm}}{2l+1} \sum_{j=2}^{3} \frac{GM_j}{GM_M} \left(\frac{R_M}{r_j}\right)^{l+1} \bar{P}_{lm}(\sin\Phi_j) e^{-im\lambda_j}$$

Use Love numbers k_{20}, k_{21}, k_{22} and k_{30} from Lemoine et al. (2013), neglect change of deg. 4 coefficients due to deg. 2 tides.

The Celestial Mechanics Approach (CMA)

Selenocentric equation of motion for satellit i

$$\ddot{\mathbf{r}}_i = -GM_M \frac{\mathbf{r}}{r^3} + \mathbf{f}(t, \mathbf{r}, \dot{\mathbf{r}}, q_1, ..., q_d)$$

$$\mathbf{f} = \nabla V + \mathbf{a}_b + \mathbf{a}_t + \mathbf{a}_r + \mathbf{a}_e + \mathbf{a}_n$$

$\mathbf{a}_r$ Relativistic corrections

$\mathbf{a}_e$ Empirical forces

$\mathbf{a}_n$ Non-gravitational accelerations, especially solar radiation pressure → **not yet explicitly modeled**

Orbit parametrization: $\mathbf{r}_i(t; a, e, i, \Omega, \omega, u_0; Q_1, ..., Q_d, P_1, ..., P_s)$

Q_i: Dynamic parameters (general and arc-specific)
P_i: Pseudo-stochastic parameters (pulses)

The Celestial Mechanics Approach (CMA)

- Development version of Bernese GNSS software.
- Linearization of orbit around a priori orbit.
- Numerical integration (with a priori parameters) of equations of motion and variational equations.
- Set up of position and Ka-band normal equations (NEQs) on a daily basis.
- Combination of position and Ka-band NEQs with apropriate weighting.
- NEQ manipulation: Preelimination of parameters and accumulation to weekly, monthly and three-monthly NEQs, which are then inverted without applying any regularization.

All parameters estimated simultaneously

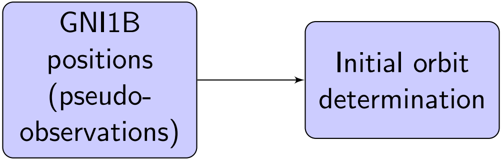
$\longrightarrow$ Gravity field estimation $=$ extended orbit determination problem

Results

(Based on release-2 and release-4 data of primary mission phase)

Orbit determination: Positions only

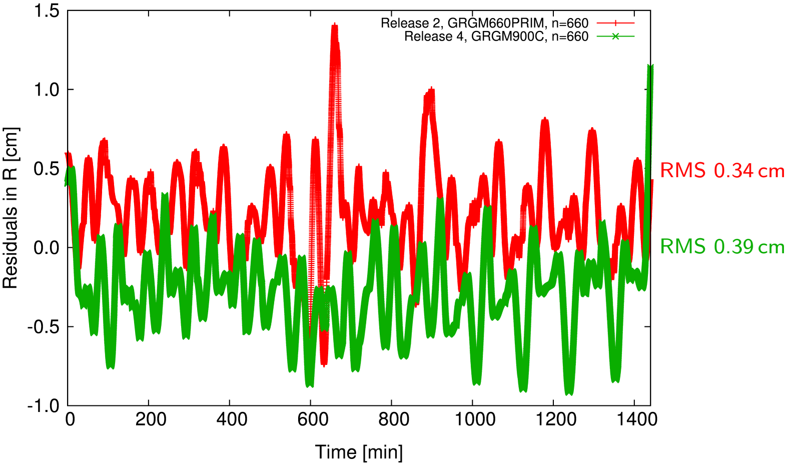
Use the GNI1B positions as pseudo-observations for an initial orbit determination for GRAIL-A and GRAIL-B.



GNI1B positions (pseudo-observations)

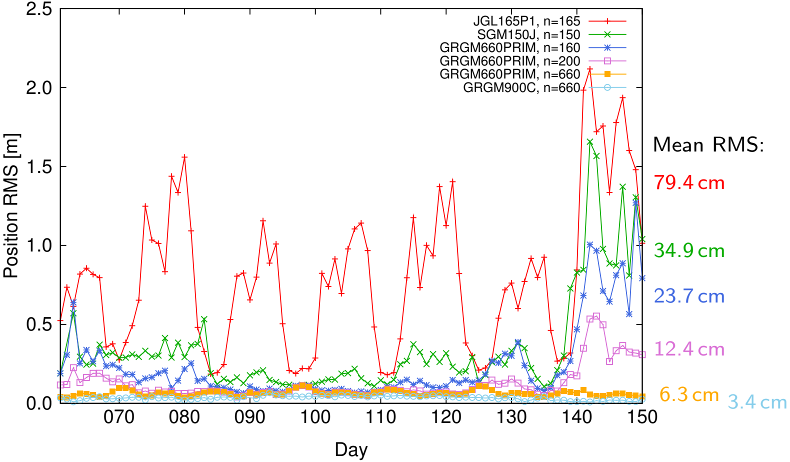
Initial orbit determination

Orbit determination: Positions only



Position residuals for GRAIL-A and day 080 when using a GRAIL gravity field as background field. Pulses every 40'.

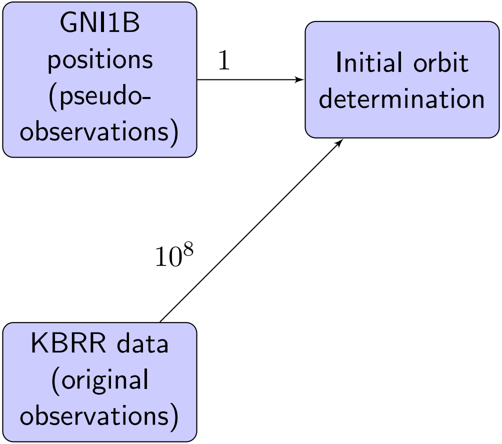
Orbit determination: Positions only



Daily RMS values of GNI1B position fit over the whole primary mission phase, using different gravity field models. Slightly worse fits for beginning and end of primary mission phase when using GRGM660PRIM to $n = 200$.

Orbit determination: Combined

Add the Ka-band range rate data to improve orbit determination.



GNI1B positions (pseudo-observations)

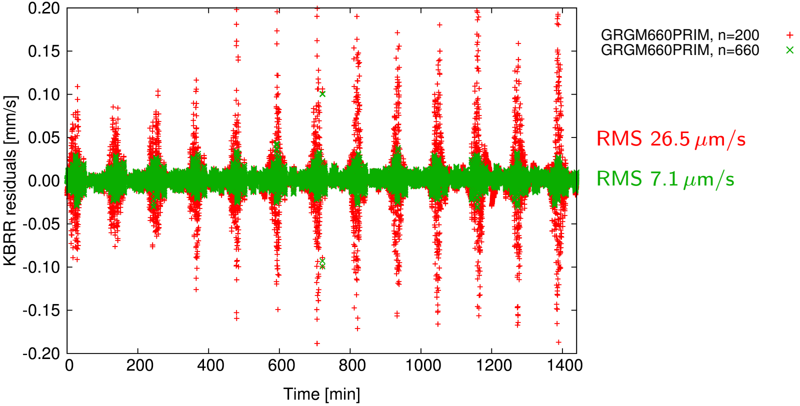
1

Initial orbit determination

10^8

KBRR data (original observations)

Orbit determination: Combined

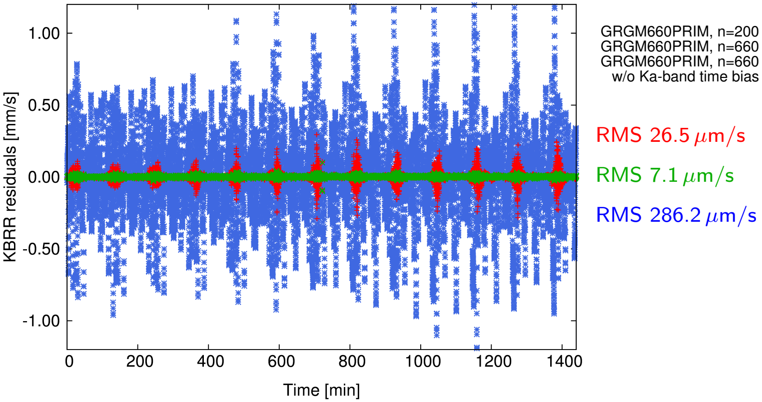


RMS 26.5 μm/s

RMS 7.1 μm/s

KBRR residuals (day 062) of a combined orbit determination when using release 2 data and GRGM660PRIM. Using the field to maximal degree reduces KBRR residuals, but they still show large systematic signals. Two orders of magnitude away from the expected noise level ($\sim 0.05\,\mu\mathrm{m/s}$).

Orbit determination: Combined



RMS 26.5 μm/s

RMS 7.1 μm/s

RMS 286.2 μm/s

KBRR residuals (day 062) of a combined orbit determination when using release 2 data and GRGM660PRIM. Using the field to maximal degree reduces KBRR residuals, but they still show large systematic signals. Two orders of magnitude away from the expected noise level ($\sim 0.05\,\mu\mathrm{m/s}$).

Estimation of Ka-band time bias is essential!

Ka-band time bias

GNI1B positions:

```
383918401 1 I 322593.033237163 -1172698.31858014 1296575.86035312 0 0 0 734.52359
383918406 1 I 326262.118730114 -1177664.29294976 1290972.99205881 0 0 0 733.10786
383918411 1 I 329924.084663656 -1182604.57682264 1285341.95561643 0 0 0 731.67577
383918416 1 I 333578.849304919 -1187519.05706519 1279682.86812418 0 0 0 730.22735
383918421 1 I 337226.331052137 -1192407.62111924 1273995.84736423 0 0 0 728.76262
383918426 1 I 340866.448444499 -1197270.15700893 1268281.01179872 0 0 0 727.28161
```

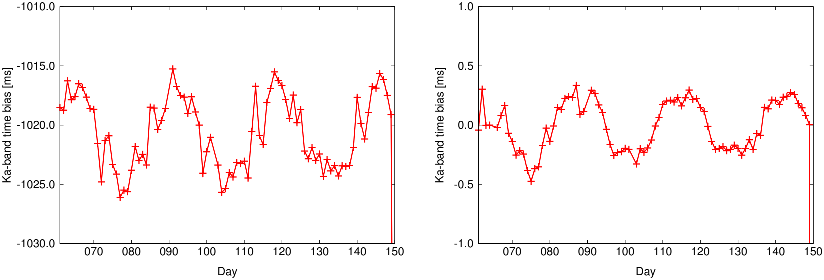
Ka-band data:

```
383918400 379799.5088783654 1.084236923065176 -0.0006847935141590895 0 -0.0021508
383918405 379804.9215565224 1.080845158969999 -0.00067181257892134‍4 0 -0.00215083
383918410 379810.3174404077 1.077519621516795 -0.0006583427723272724 0 -0.0021508
383918415 379815.6968669317 1.074262729372726 -0.0006442424652639405 0 -0.0021508
383918420 379821.0601893955 1.071078743875103 -0.0006292736830879165 0 -0.0021507
383918425 379826.4077803511 1.067970271752883 -0.0006141255745387579 0 -0.0021506
```

All time stamps should refer to Barycentric Dynamical Time (TDB). However, the Ka-band observation epochs are shifted: Ka-band time bias.

Ka-band time bias

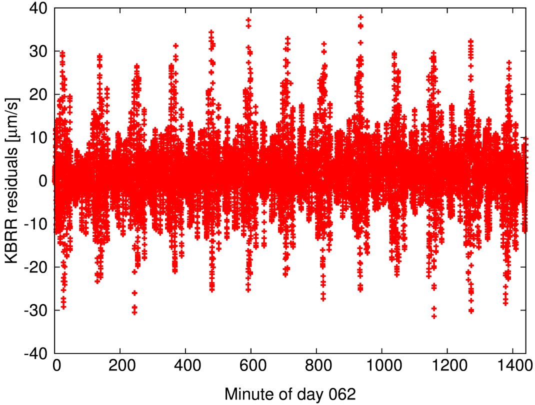
Estimated Ka-band time bias for primary mission phase:



Release 2 Release 4

- Only for release 2 data of primary mission phase ($\sim -1.02\,\mathrm{s}$)
- Reason unknown
- Fixed in release 4 of data

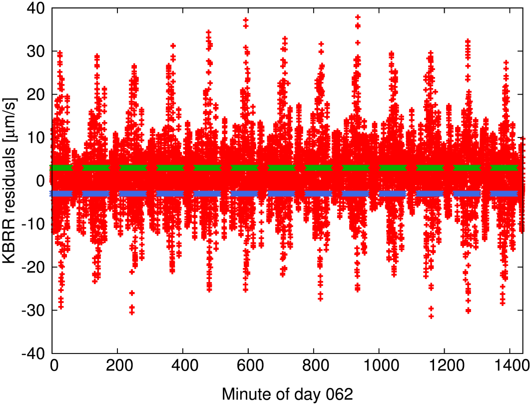
Orbit determination: Combined, release 2



Day 062

Release 2: Allowing for a Ka-band time bias strongly reduces the KBRR residuals. But they are still quite large.

Orbit determination: Combined, release 2



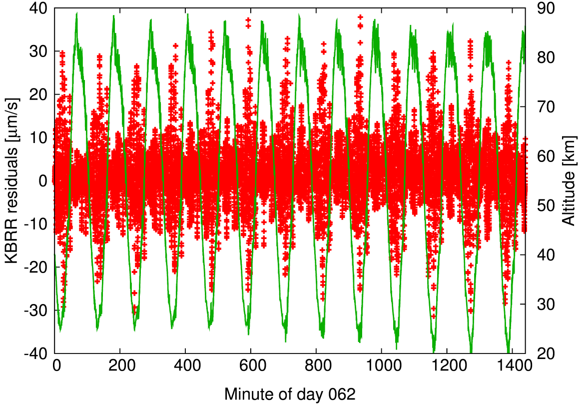
Day 062

GRAIL-A and
GRAIL-B in sunlight

Release 2: Allowing for a Ka-band time bias strongly reduces the KBRR residuals. But they are still quite large.

- Missing solar radiation pressure modelling?

Orbit determination: Combined, release 2



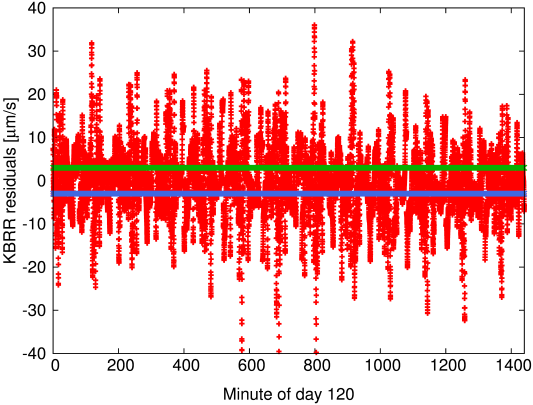
Day 062

Altitude of probes above lunar surface (using topography of LRO's Lunar Orbiter Laser Altimeter [LOLA])

Release 2: Allowing for a Ka-band time bias strongly reduces the KBRR residuals. But they are still quite large.

- Missing solar radiation pressure modelling?
- Further inconsistencies?

Orbit determination: Combined, release 2



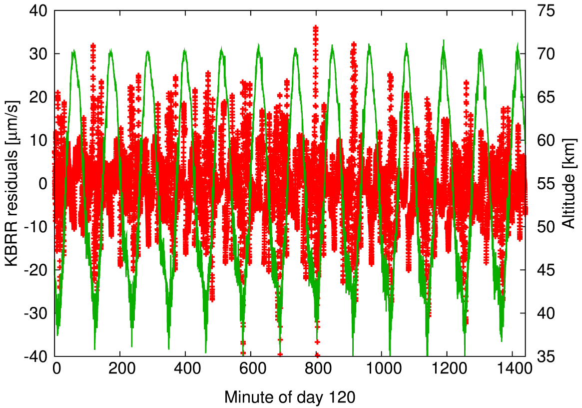
Day 120

GRAIL-A and
GRAIL-B in sunlight

Release 2: Allowing for a Ka-band time bias strongly reduces the KBRR residuals. But they are still quite large.

- Missing solar radiation pressure modelling?
- Further inconsistencies?

Orbit determination: Combined, release 2



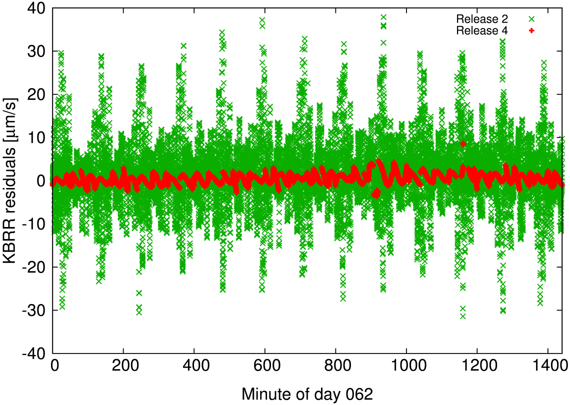
Day 120

Altitude of probes above lunar surface (LOLA topography)

Release 2: Allowing for a Ka-band time bias strongly reduces the KBRR residuals. But they are still quite large.

- Missing solar radiation pressure modelling?
- Further inconsistencies?

Orbit determination: Combined, release 4

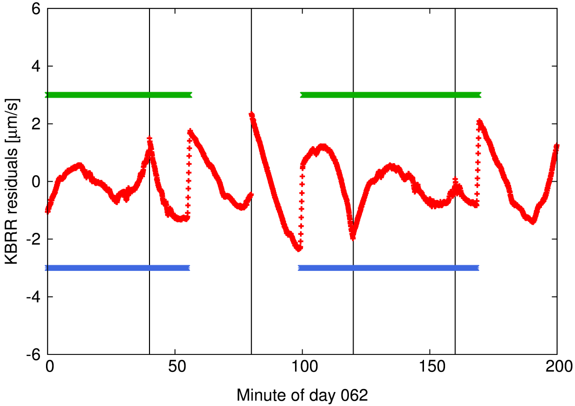


RMS 7.1 μm/s

RMS 1.3 μm/s

KBRR residuals when using release 2 data and GRGM660PRIM to $n = 660$ and release 4 data and GRGM900C to $n = 660$.

Orbit determination: Combined, release 4



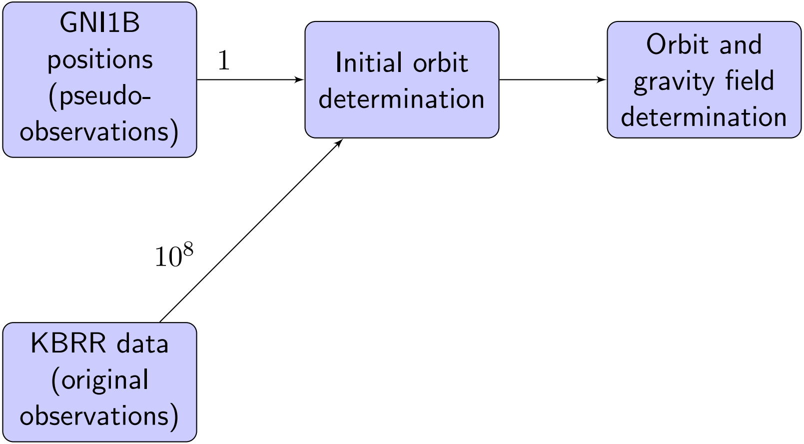
GRAIL-A and GRAIL-B in sunlight

The impact of the pseudo-stochstic pulses (every 40') of the currently adopted empirical orbit modeling and the solar radiation pressure becomes clearly visible.

Improve force modelling to further reduce residuals!

Gravity field determination

Use initial orbits for a combined orbit and gravity field determination



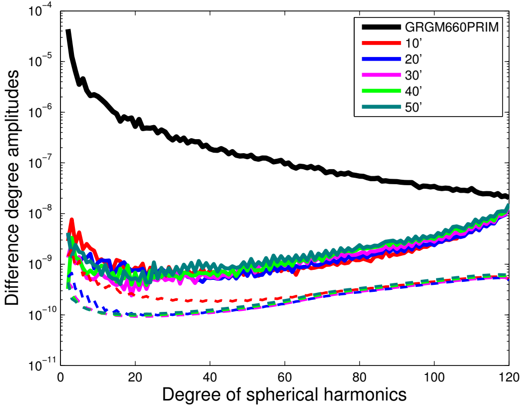
GNI1B positions (pseudo-observations)

Initial orbit determination

Orbit and gravity field determination

KBRR data (original observations)

Gravity field determination: Pulses

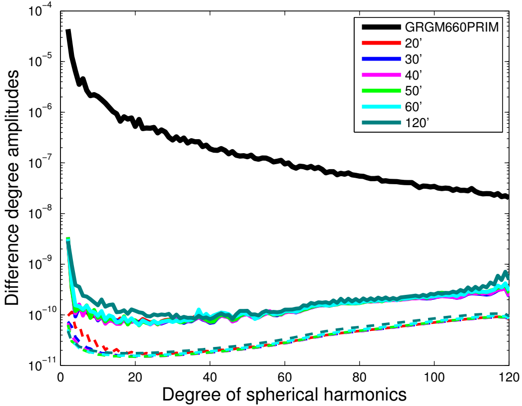


- Release 4 data
- Position and Ka-band
- A priori: GRGM660PRIM to $n = 120$

The number of pseudo-stochastic pulses should be carefully chosen:

- Enough to compensate for model deficiencies (missing RPR!)
- Not too many to absorb gravity signal
- Pulses every 40 minutes (constr. to $0.1\,\mathrm{mm/s}$) seems reasonable

Gravity field determination: Pulses

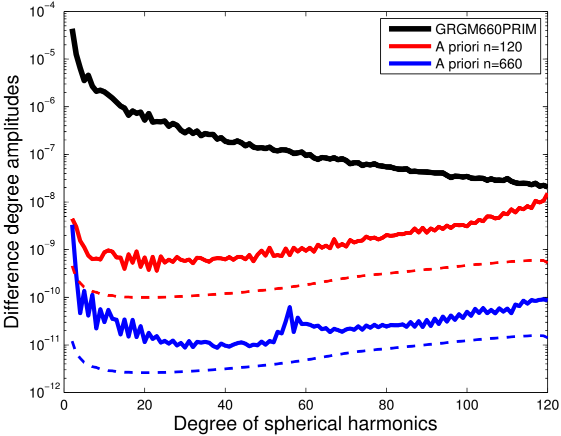


- Release 4 data
- Position and Ka-band
- A priori: GRGM660PRIM to $n = 660$

The number of pseudo-stochastic pulses should be carefully chosen:

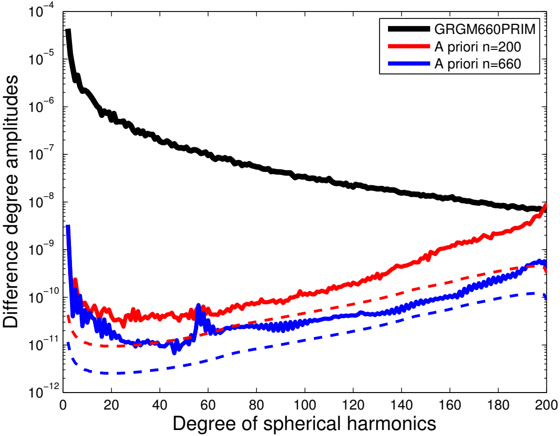
- Enough to compensate for model deficiencies (missing RPR!)
- Not too many to absorb gravity signal
- Pulses every 40 minutes (constr. to $0.1\,\mathrm{mm/s}$) seems reasonable

Gravity field determination: Up to $l_{\max} = 120$



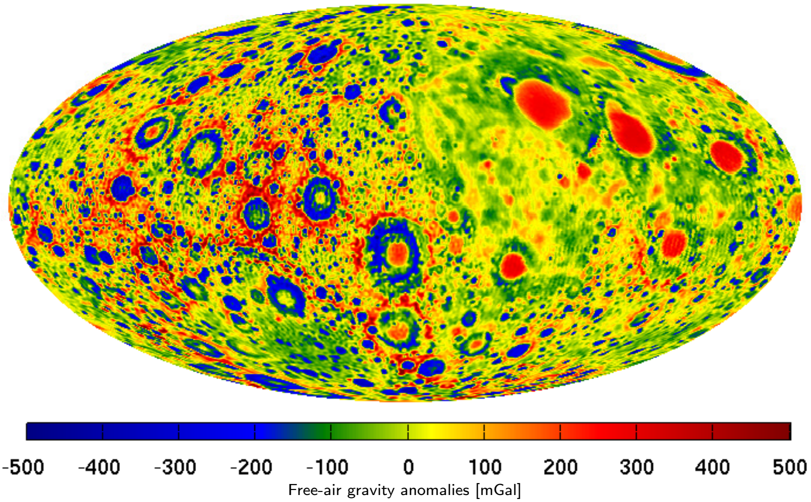
Gravity field estimated up to degree and order 120, when using GRGM660PRIM as a priori field up to $n = 120$ and $n = 660$ (to reduce omission error). No regularization is used. Position and KBRR observations are used with a relative weighting ratio of $1 : 10^8$.

Gravity field determination: Up to $l_{\text{max}} = 200$



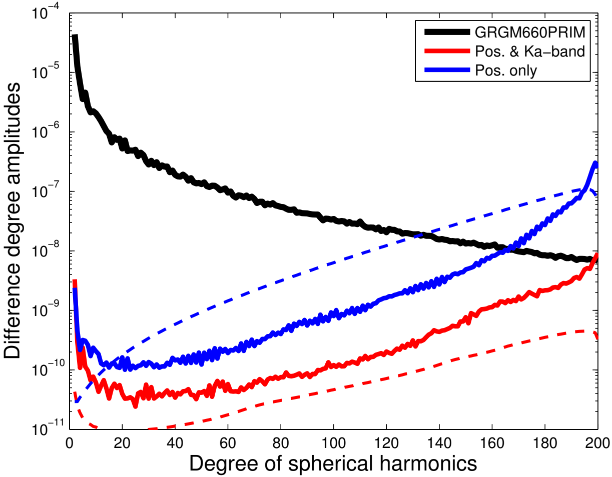
Gravity field estimated up to degree and order 200, when using GRGM660PRIM as a priori field up to $n = 200$ and $n = 660$ (to reduce omission error). No regularization is used. Position and KBRR observations are used with a relative weighting ratio of $1 : 10^8$.

Gravity field determination: Up to $l_{\max} = 200$



A priori field to $n = 200$, grid resolution: $0.5 \times 0.5°$.

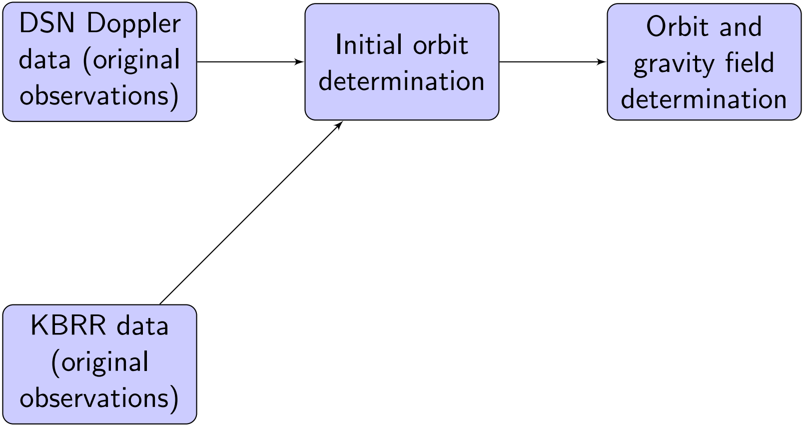
Gravity field determination: Up to $l_{\max} = 200$



The solution is dominated by the GNI1B positions only at the lowest degrees. However, DSN data analysis is a **must** to obtain fully independent results also for the long wavelengths.

Gravity field determination: DSN data

Goal: Replace GNI1B positions by original DSN Doppler observations.



DSN Doppler data (original observations)

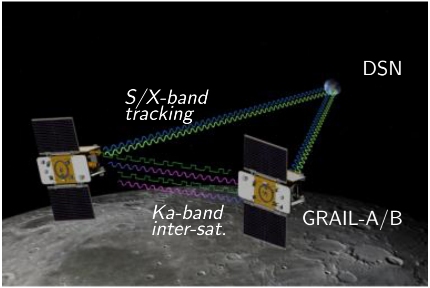
Initial orbit determination

Orbit and gravity field determination

KBRR data (original observations)

Status of DSN Doppler data processing

Idea of DSN Doppler tracking:



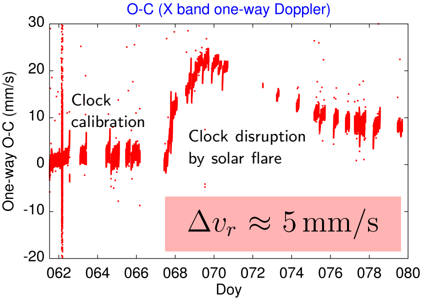
- X-band one-way link (8.4 GHz, 1 sec integration time)
- S-band two-way link (2.3 GHz, 10 sec integration time)
- Emitted: f_0
- Received: f_R
- Ground station:
  Measure $f(t) = f_R - f_0$

Observed $\mathcal{D}_O = \frac{\Delta N}{T_c}$: Number of cycles of $f(t)$ in time interval T_C

Analytical model of the measured quantity:

Computed $\mathcal{D}_C = f_0 \frac{\Delta\rho}{T_c}$: Range rate from GRAIL to DSN antennas

Status of DSN Doppler data processing



Model based on:

- GRAIL orbit from GNI1B,
- DSN Earth-fixed coordinates,
- Earth rotation IERS2010,
- planetary ephemeris DE421.

$\longleftarrow$ Assess $v_r = c\frac{\mathcal{D}}{f_0}$

Analytical model of one-way and two-way Doppler observations accurate at mm/s level (still to be improved).

NEXT STEP:

Set up orbit determination process (currently being implemented).

Conclusion & Outlook

Conclusion & Outlook

- Due to availability of GNI1B positions the adaption of the CMA from GRACE to GRAIL is feasible without DSN data analysis. But the latter is a must for independent solutions.
- Pseudo-stochastic orbit parametrization allows for “Bernese” lunar gravity fields without sophisticated background models.
- We reach the μm/s level for KBRR residuals. But radiation pressure modelling is crucial to further improve the solutions.
- Significant differences between release 2 and release 4 of the data (not only Ka-band time bias has been fixed by the GRAIL science team).
- We have achieved a good understanding of the DSN Doppler observable and are working towards orbit determination using this data.