FEM/FD Immersed Boundary FSI Simulations

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Introduction and Aim

Heart valves are an indispensable feature in the circulation of blood through a healthy organism. However, valves can develop dysfunctions. In particular, the Aortic Valve can suffer from stenosis due to calcification which leads to locally increased flow velocities, a higher pressure drop across the valve and regurgitation (backflow). We are developing a numerical tool that allows us to investigate the interaction of the blood flow with the surrounding tissue in hope of characterising it and explaining the cause of the mentioned pathologies in a quantitative and reproducible way

Immersed Boundary Method

Originally introduced by Charles S. Peskin[a] in the 1970's

Method allowing for fluid-structure interaction (FSI) without mesh manipulation

Velocities of the fluid in the Navier-Stokes equations are represented on Eulerian fields and displacements of the structure arerepresented by Lagrangian fields

No-slip boundary conditions are imposed by force density terms in the momentum equation

Forces and velocities are interpolated from and to structure nodes via interaction equations

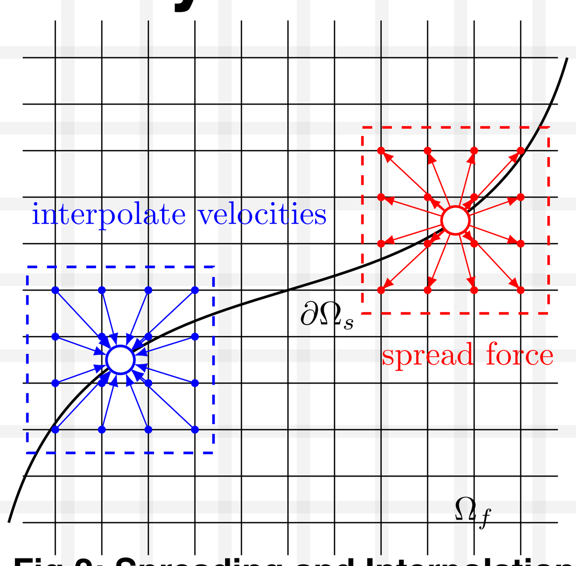

Fig.2: Spreading and Interpolation of fields

$$\rho\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\nabla p + \mu\Delta\mathbf{u} + \mathbf{f}, \qquad \nabla\cdot\mathbf{u} = 0$$

$$\mathbf{f}(\mathbf{x},t) = \int_{\partial\Omega_s} \mathbf{F}(\mathbf{s},t)\,\delta\left[\mathbf{x}-\mathbf{X}(\mathbf{s},t)\right]\mathrm{d}s$$

$$\frac{\partial\mathbf{X}(\mathbf{s},t)}{\partial t} = \mathbf{u}\left[\mathbf{X}(\mathbf{s},t),t\right] = \int_{\Omega_f} \mathbf{u}(\mathbf{x},t)\,\delta\left[\mathbf{x}-\mathbf{X}(\mathbf{s},t)\right]\mathrm{d}\mathbf{x}$$

Ω_{s2}

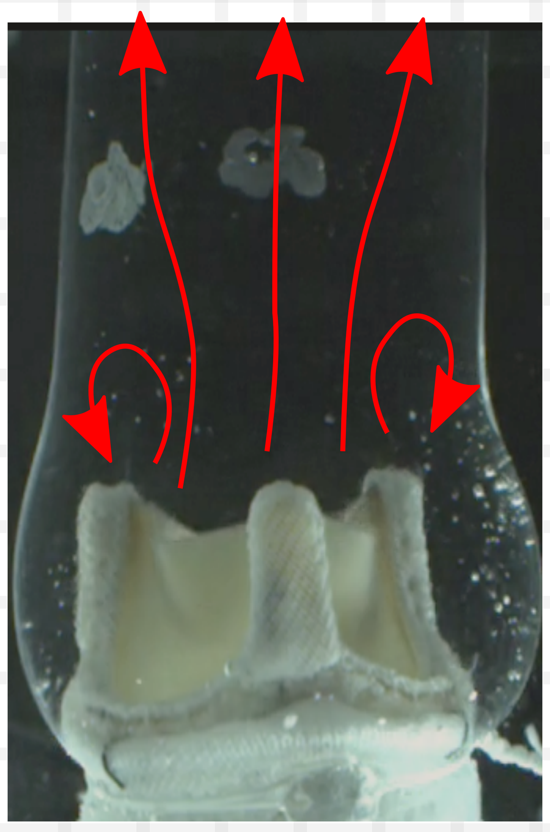

Fig.1: Bioprosthetic heart valve

Ω_{s1}

IMPACT Fluid Solver

Ω_{s3}

Numerical solver for **I**ncompressible (**T**urbulent) flows on **M**assively **PA**rallel **C**ompu**T**ers[b]

Massively parallel high-order Navier-Stokes solver developed at ETH Zurich

Finite differences up to 6th order (10th order compact) in space on staggered grids

Semi-implicit time integration with adaptive time-stepping: Crank-Nicolson for diffusive terms, low-storage 3rd order Runge-Kutta for convective terms

Iterative solver cascade with highly efficient commutation-based preconditioner for the pressure iteration

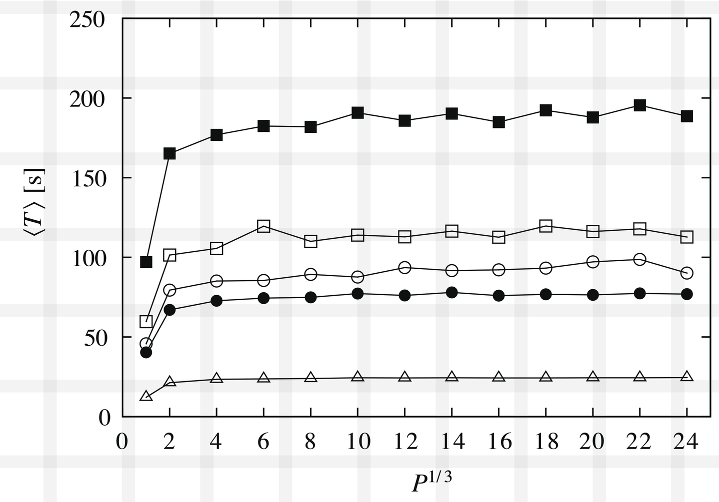

Fig.3: Weak scaling diagram [b]

Geometric data decompostion for low communication overhead and excellent scaling

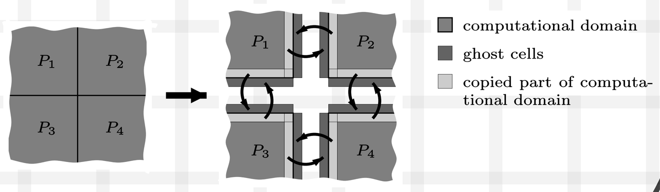

Fig.4: Geometric data decomposition [b]

Fluid-Structure Interaction and MOONoLith

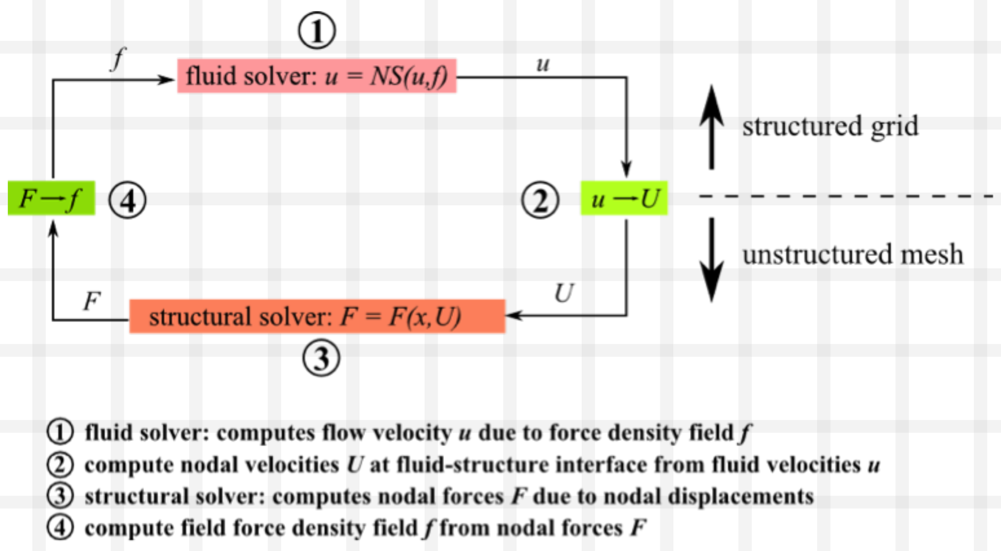

Fig.5: Cyclic coupling of solvers

Fluid and structure are coupled in a cyclic manner in which the computed velocity field is passed to the strucural solver which in turn returns the obtained structural forces

We use the massively parallel algorithmic framework[c] within the MOONoLith library for the variational transfer ([pseudo] L^2-projections, also compatible with classical interpolation) of discrete fields between arbitrarily distributed meshes (FEM, FVM, FDM, spectral)

It includes a parallel search strategy, output dependent load balancing, computation of element intersections and parallel assembly of the algebraic transfer operators

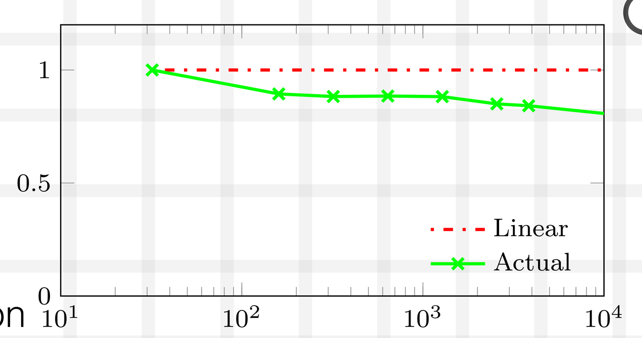

Fig.7: Weak scaling diagram [c]

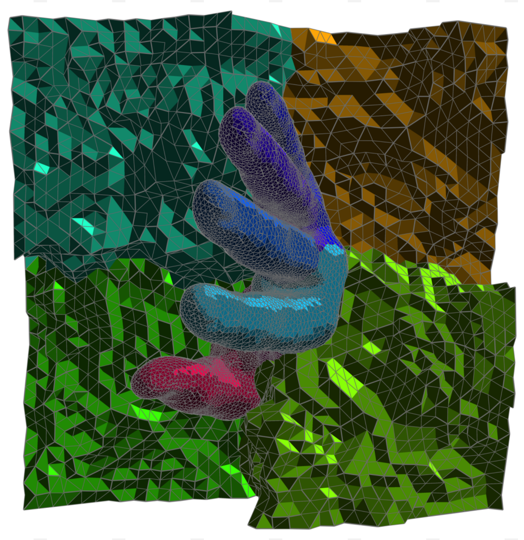

Fig.6: Transfer between arbitrary domains, colors represent processes [c]

Assembly of the transfer operators is done by identifying all pairs of intersecting elements and for each pair constructing the intersection polytope after which quadrature points are generated on the new grid and the local marix elements computed via quadrature

Parallel intersection detection is done by a tree-search algorithm wherein in a first phase intersections are searched for within bounding volumes of geometric data, this is then refined by performing breadth-first traversals of the tree simultaneously and creating a look-up table for each partition to map it to all processes on which this partition is not empty until in the last phase a list of intersecting element pairs is obtained

Knowing the element pairs we construct tuples with one slave element and its master elements each, then these tuples are assigned a weight that is proportional to a specific set of parameters, with which we control the process assignment of the tuples

Ω_{s4}

Results

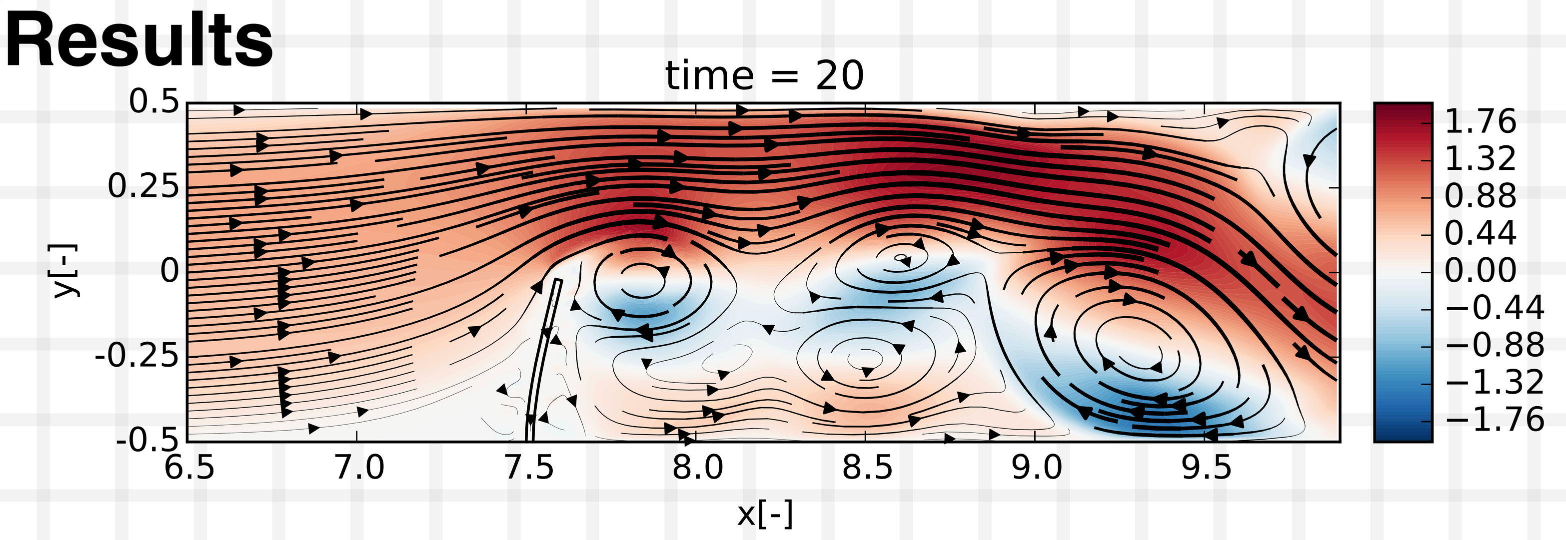

Fig.9: Horizontal velocity component with streamlines

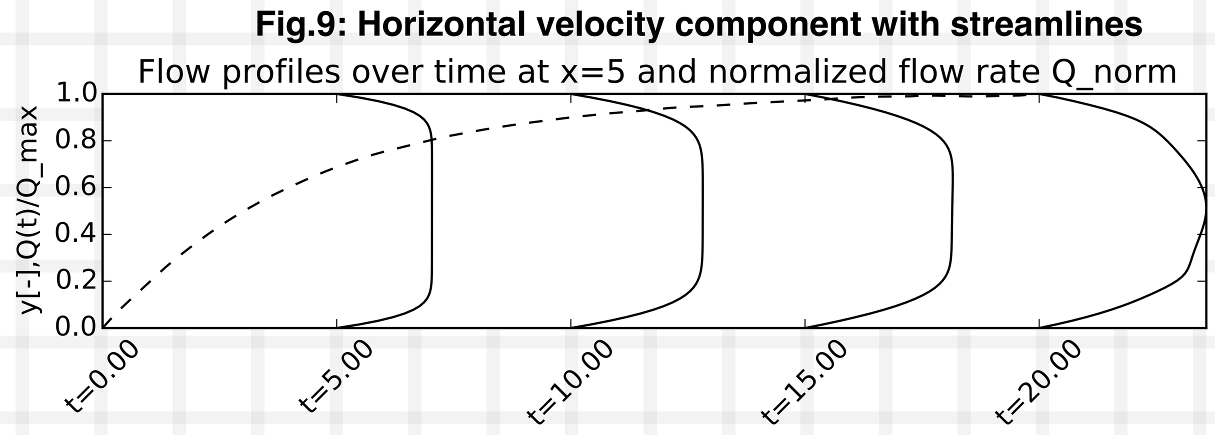

Fig.10: Impinging flow profiles and flow rate

Ω_{s6}

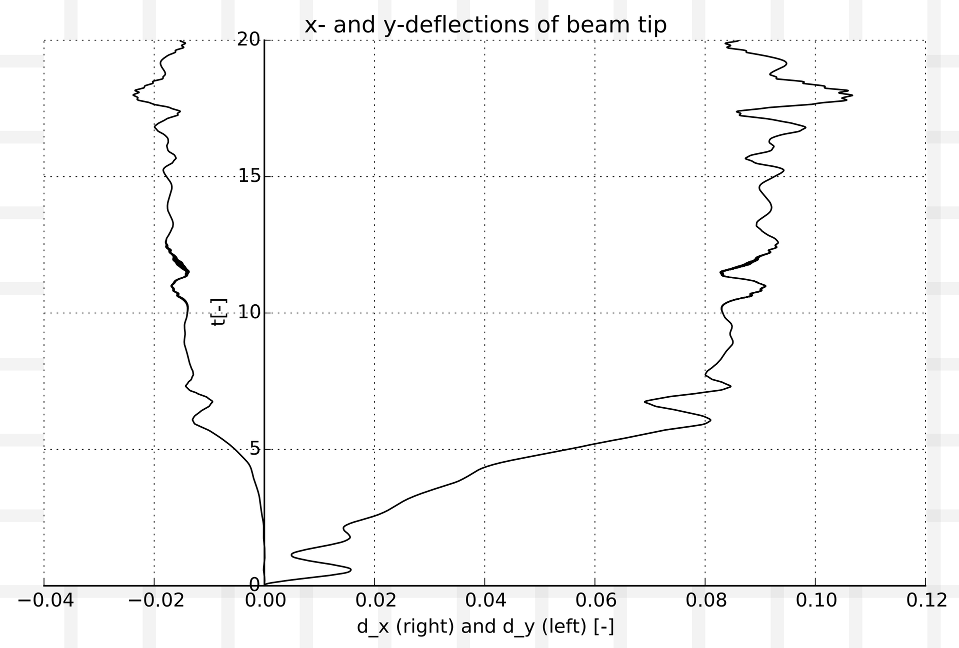

Fig.11: Beam tip deflection

As a simple test case we simulated a linearly elastic beam with elastic modulus of E=5MPa inside a channel at Re=1500

The beam was discretised with FEM using 126 constant strain triangles, the fluid was discretized on a grid of 64x448 gridpoints with FDM

The transfer of fields was performed according to the classical Immersed Boundary Method

The contour plot shows the horizontal velocity component with superimposed streamlines scaled with the local velocity magnitude

Structure with APTS in MOOSE

Ω_{s5}

MOOSE is a parallel, modular FEM framework under development at Idaho National Laboratory since 2008 that features a high-level interface to many solver libraries

We develop and implement an **A**dditive **P**arallel **T**rust-region **S**olution-method for non-linear solid mechanics (e.g. biological) within this framework

The global non-linear problem ist subdivided into smaller non-linear subproblems that are solved in parallel and then recombined again in one iteration

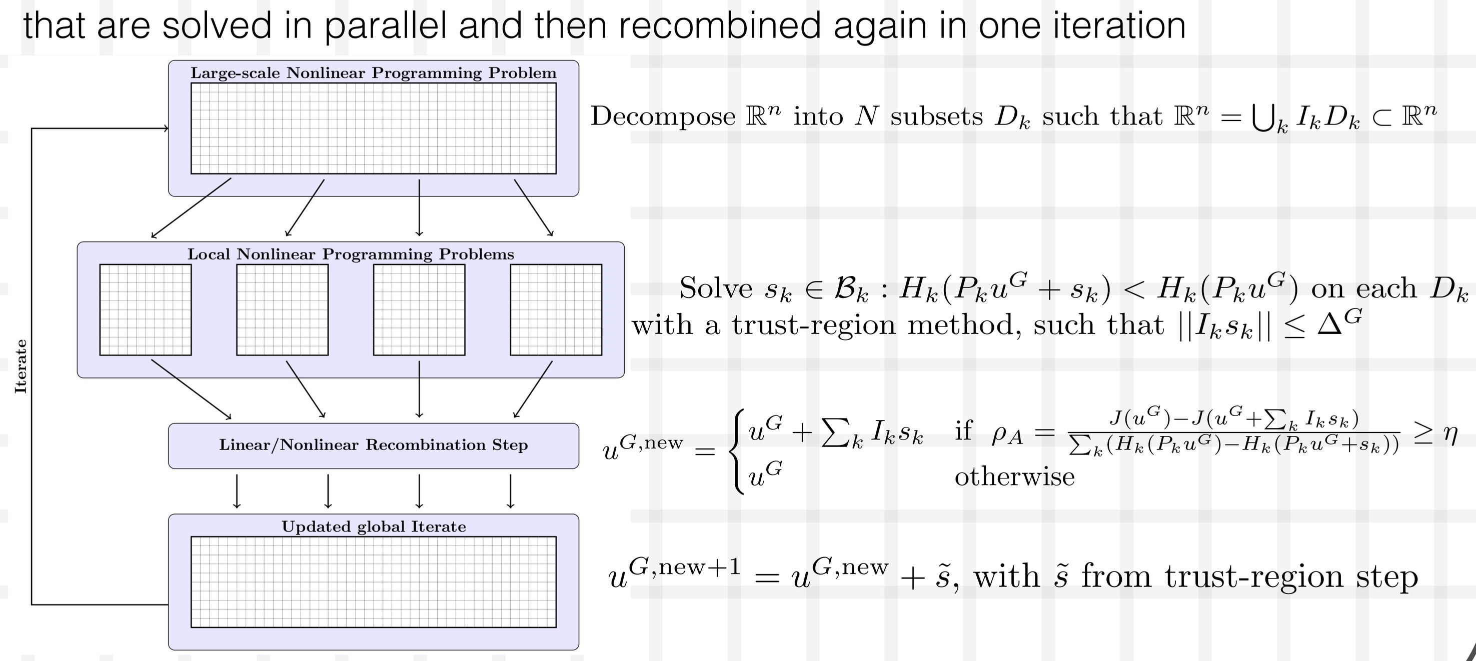

Fig.8: Parallel decomposition of nonlinear problem

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Ω_{s7}

Ω_f

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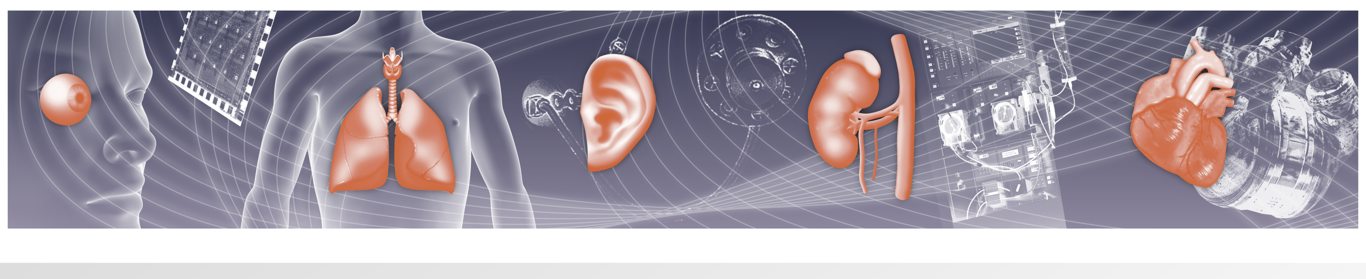
[a] C.S. Peskin, The Fluid Dynamics of Heart Valves: Experimental, Theoretical, and Computational Methods, Ann. Rev. Fluid Mech., 1982, 14
[b] R. Henniger, D. Obrist, L. Kleiser, High-order accurate solution of the incompressible Navier-Stokes equations on massively parallel computers, J. Comp. Phys., 2010, 229
[c] R. Krause, P. Zulian, A Massively Parallel Approach to the Variational Transfer of Discrete Fields Between Arbitrarily Distributed Finite Element Meshes, submitted to SISC



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