## Title

alphawgt — Cronbach's alpha for weighted data

## **Methods and Formulas**

Let  $x_i$ , i = 1, ..., k, denote the variables over which  $\alpha$  ist to be calculated. If no weights are specified, calculations are done as described in [R] **alpha**. If weights are specified, the average correlation  $\bar{r}$ , average covariance  $\bar{c}$ , and average variance  $\bar{v}$  are calculated as

$$\bar{r} = \frac{\sum_{i=2}^{k} \sum_{j=1}^{i-1} s_i s_j W_{ij} r_{ij}}{\sum_{i=2}^{k} \sum_{j=1}^{i-1} W_{ij}}, \quad \bar{c} = \frac{\sum_{i=2}^{k} \sum_{j=1}^{i-1} s_i s_j W_{ij} c_{ij}}{\sum_{i=2}^{k} \sum_{j=1}^{i-1} W_{ij}}, \quad \text{and} \quad \bar{v} = \frac{\sum_{i=1}^{k} W_{ii} c_{ii}}{\sum_{i=1}^{k} W_{ii}}$$

where  $r_{ij}$  and  $c_{ij}$  are the correlation and covariance between  $x_i$  and  $x_j$ ,  $c_{ii}$  is the variance of  $x_i$ ,  $s_i$  is the sign with which  $x_i$  enters the scale, and  $W_{ij}$  and  $W_{ii}$  are the sum of weights over the observations used to calculate  $r_{ij}$ ,  $c_{ij}$ , and  $c_{ii}$  (the difference to the standard formulas is the usage of the sum of weights W instead of the number of observations n; note: if option casewise is specified then  $W_{ij} = W_{ii}$  for all i, j).

Additionaly, the calculation of  $r_{ij}$ ,  $c_{ij}$ , and  $c_{ii}$  is different from the standard case. Let l = 1, ..., n be an indicator for the observations of  $x_i$  and  $w_l$  be the weights. In the case of frequency weights (fweights) the variance  $c_{ii}$  and covariance  $c_{ij}$  are calculated as

$$c_{ii} = \frac{1}{W_{ii} - 1} \sum_{l=1}^{n_{ii}} w_l (x_{il} - \bar{x}_i)^2 \quad \text{and} \quad c_{ij} = \frac{1}{W_{ij} - 1} \sum_{l=1}^{n_{ij}} w_l (x_{il} - \bar{x}_i) (x_{jl} - \bar{x}_j)$$

and in the case of analytical weights (aweights) as

$$c_{ii} = \frac{1}{W_{ii} - W_{ii}/n_{ii}} \sum_{l=1}^{n_{ii}} w_l (x_{il} - \bar{x}_i)^2 \quad \text{and} \quad c_{ij} = \frac{1}{W_{ij} - W_{ij}/n_{ij}} \sum_{l=1}^{n_{ij}} w_l (x_{il} - \bar{x}_i) (x_{jl} - \bar{x}_j)$$

where  $\bar{x}_i$  is the weighted mean of  $x_i$ , i.e.  $\bar{x}_i = \frac{1}{W_{ii}} \sum_{l=1}^{n_{ii}} w_l x_{il}$  in the case of the variance and  $\bar{x}_i = \frac{1}{W_{ij}} \sum_{l=1}^{n_{ij}} w_l x_{il}$  in the case of the covariance. The calculation of  $r_{ij}$  is

$$r_{ij} = \frac{\sum_{l=1}^{n_{ij}} w_l(x_{il} - \bar{x}_i)(x_{jl} - \bar{x}_j)}{\sqrt{\sum_{l=1}^{n_{ij}} w_l(x_{il} - \bar{x}_i)^2 \sum_{l=1}^{n_{ij}} w_l(x_{jl} - \bar{x}_j)^2}}.$$

## Also See

Background: [R] alpha, [U] 14.1.6 weight