## Title

alphawgt - Cronbach's alpha for weighted data

## Methods and Formulas

Let $x_{i}, i=1, \ldots, k$, denote the variables over which $\alpha$ ist to be calculated. If no weights are specified, calculations are done as described in $[\mathrm{R}]$ alpha. If weights are specified, the average correlation $\bar{r}$, average covariance $\bar{c}$, and average variance $\bar{v}$ are calculated as

$$
\bar{r}=\frac{\sum_{i=2}^{k} \sum_{j=1}^{i-1} s_{i} s_{j} W_{i j} r_{i j}}{\sum_{i=2}^{k} \sum_{j=1}^{i-1} W_{i j}}, \quad \bar{c}=\frac{\sum_{i=2}^{k} \sum_{j=1}^{i-1} s_{i} s_{j} W_{i j} c_{i j}}{\sum_{i=2}^{k} \sum_{j=1}^{i-1} W_{i j}}, \quad \text { and } \quad \bar{v}=\frac{\sum_{i=1}^{k} W_{i i} c_{i i}}{\sum_{i=1}^{k} W_{i i}}
$$

where $r_{i j}$ and $c_{i j}$ are the correlation and covariance between $x_{i}$ and $x_{j}, c_{i i}$ is the variance of $x_{i}, s_{i}$ is the sign with which $x_{i}$ enters the scale, and $W_{i j}$ and $W_{i i}$ are the sum of weights over the observations used to calculate $r_{i j}, c_{i j}$, and $c_{i i}$ (the difference to the standard formulas is the usage of the sum of weights $W$ instead of the number of observarions $n$; note: if option casewise is specified then $W_{i j}=W_{i i}$ for all $i, j$ ).

Additionaly, the calculation of $r_{i j}, c_{i j}$, and $c_{i i}$ is different from the standard case. Let $l=1, \ldots, n$ be an indicator for the observarions of $x_{i}$ and $w_{l}$ be the weights. In the case of frequency weights (fweights) the variance $c_{i i}$ and covariance $c_{i j}$ are calculated as

$$
c_{i i}=\frac{1}{W_{i i}-1} \sum_{l=1}^{n_{i i}} w_{l}\left(x_{i l}-\bar{x}_{i}\right)^{2} \quad \text { and } \quad c_{i j}=\frac{1}{W_{i j}-1} \sum_{l=1}^{n_{i j}} w_{l}\left(x_{i l}-\bar{x}_{i}\right)\left(x_{j l}-\bar{x}_{j}\right)
$$

and in the case of analytical weights (aweights) as
$c_{i i}=\frac{1}{W_{i i}-W_{i i} / n_{i i}} \sum_{l=1}^{n_{i i}} w_{l}\left(x_{i l}-\bar{x}_{i}\right)^{2} \quad$ and $\quad c_{i j}=\frac{1}{W_{i j}-W_{i j} / n_{i j}} \sum_{l=1}^{n_{i j}} w_{l}\left(x_{i l}-\bar{x}_{i}\right)\left(x_{j l}-\bar{x}_{j}\right)$
where $\bar{x}_{i}$ is the weighted mean of $x_{i}$, i.e. $\bar{x}_{i}=\frac{1}{W_{i i}} \sum_{l=1}^{n_{i i}} w_{l} x_{i l}$ in the case of the variance and $\bar{x}_{i}=\frac{1}{W_{i j}} \sum_{l=1}^{n_{i j}} w_{l} x_{i l}$ in the case of the covariance. The calculation of $r_{i j}$ is

$$
r_{i j}=\frac{\sum_{l=1}^{n_{i j}} w_{l}\left(x_{i l}-\bar{x}_{i}\right)\left(x_{j l}-\bar{x}_{j}\right)}{\sqrt{\sum_{l=1}^{n_{i j}} w_{l}\left(x_{i l}-\bar{x}_{i}\right)^{2} \sum_{l=1}^{n_{l j}} w_{l}\left(x_{j l}-\bar{x}_{j}\right)^{2}}} .
$$

## Also See

Background: $\quad[R]$ alpha, $[U]$ 14.1.6 weight

