Abstract

A complete analysis of the essential spectrum of matrix-differential operators $\mathcal{A}$ of the form

$$\begin{pmatrix} -\frac{\mathrm{d}}{\mathrm{d}t}p\frac{\mathrm{d}}{\mathrm{d}t}+q & -\frac{\mathrm{d}}{\mathrm{d}t}b^*+c^* \\ b\frac{\mathrm{d}}{\mathrm{d}t}+c & D \end{pmatrix} \quad \text{in} \quad L^2((\alpha,\beta)) \oplus \left(L^2((\alpha,\beta))\right)^n \tag{1}$$

singular at $\beta \in \mathbb{R} \cup \{\infty\}$ is given; the coefficient functions p, q are scalar real-valued with $p > 0$, b, c are vector-valued, and D is Hermitian matrix-valued. The so-called "singular part of the essential spectrum" $\sigma_{\mathrm{ess}}^{\mathrm{s}}(\mathcal{A})$ is investigated systematically. Our main results include an explicit description of $\sigma_{\mathrm{ess}}^{\mathrm{s}}(\mathcal{A})$, criteria for its absence and presence; an analysis of its topological structure and of the essential spectral radius. Our key tools are: the asymptotics of the leading coefficient $\pi(\cdot,\lambda) = p - b^*(D-\lambda)^{-1}b$ of the first Schur complement of (1), a scalar differential operator but non-linear in λ; the Nevanlinna behaviour in λ of certain limits $t \nearrow \beta$ of functions formed out of the coefficients in (1).The efficacy of our results is demonstrated by several applications; in particular, we prove a conjecture on the essential spectrum of some symmetric stellar equilibrium models.