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$(\mathcal{O}_8, \mathcal{O}_8)$ contribution to $\bar{B} \to X_s\gamma\gamma$ at $O(\alpha_s)$

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In this analysis, we present the contribution associated with the chromomagnetic dipole operator $\mathcal{O}_8$ to the double differential decay width $d\Gamma/(ds_1 ds_2)$ for the inclusive process $\bar{B} \to X_s\gamma\gamma$. The kinematical variables s_1 and s_2 are defined as $s_i = (p_b - q_i)^2/m_b^2$, where p_b, q_1, q_2 are the momenta of b-quark and two photons. This contribution (taken at tree level) is of order α_s, like the recently calculated QCD corrections to the contribution of the operator $\mathcal{O}_7$. In order to regulate possible collinear singularities of one of the photons with the strange quark, we introduce a nonzero mass m_s for the strange quark. Our results are obtained for exact m_s, which we interpret as a constituent mass being varied between 400 and 600 MeV. Numerically it turns out that the effect of the $(\mathcal{O}_8, \mathcal{O}_8)$ contribution to the branching ratio of $\bar{B} \to X_s\gamma\gamma$ does not exceed $+0.1\,\%$ for any kinematically allowed value of our physical cutoff parameter c, confirming the expected suppression of this contribution relative to the QCD corrections to $d\Gamma_{77}/(ds_1\, ds_2)$ [1, 2].

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I. INTRODUCTION

Inclusive rare B-meson decays are known to be a unique source of indirect information about physics at scales of several hundred GeV. In the standard model (SM) all these processes proceed through loop diagrams and thus are relatively suppressed. In the extensions of the SM the contributions stemming from the diagrams with "new" particles in the loops can be comparable or even larger than the contribution from the SM. Thus getting experimental information on rare decays puts strong constraints on the extensions of the SM or can even lead to a disagreement with the SM predictions, providing evidence for some "new physics".

To make a rigorous comparison between experiment and theory, precise SM calculations for the (differential) decay rates are mandatory. While the branching ratios for $\bar{B} \to X_s\gamma$ [3] and $\bar{B} \to X_s\ell^+\ell^-$ are known today even to next-to-next-to-leading logarithmic (NNLL) precision (for reviews, see [4, 5] and [6] for recent updated predictions on radiative decay modes of B meson), other branching ratios, like the one for $\bar{B} \to X_s\gamma\gamma$ discussed in this paper, were only known to leading logarithmic (LL) precision in the SM [7–10]. As the process $\bar{B} \to X_s\gamma\gamma$ is expected to be measured at the planned Super B-factory in Japan (SuperKEKB) [11, 12], we recently completed first steps towards a next-to-leading logarithmic (NLL) result for this decay [1, 2], by working out QCD corrections to the numerically important $(\mathcal{O}_7, \mathcal{O}_7)$ contribution.

In this paper, we go one step further and provide the self-interference contribution to $\bar{B} \to X_s\gamma\gamma$ stemming from the chromomagnetic dipole operator $\mathcal{O}_8$ which starts at order α_s. Although a naive estimate suggests that this contribution is suppressed by a factor of $|C_8^{\rm eff} Q_d/C_7^{\rm eff}|^2 \sim 1/36$ relative to the QCD corrections to the $(\mathcal{O}_7,\, \mathcal{O}_7)$ interference, a more detailed investigation is in order: In both cases ($\mathcal{O}_7$ and $\mathcal{O}_8$), one of the two photons can be emitted from the strange quark in a collinear way, leading to contributions involving $\log(m_s/m_b)$ terms.[1] Concerning the other photon, the two cases differ, however. Unlike in the $\mathcal{O}_7$, the second photon can also be emitted from the s-quark in the $\mathcal{O}_8$ case. While a fully collinear emission of both photons is excluded by our cuts (see later), a leftover enhancement effect could still apply in the $\mathcal{O}_8$ case and thereby milder the naive suppression factor. As the average energies of the two photons are not very high, there might be a second effect related to the different infrared structure ($1/E_\gamma$-terms) of the two cases, which also potentially milders the naive suppression factor given above. We feel that these considerations motivate a detailed evaluation of the $(\mathcal{O}_8, \mathcal{O}_8)$-interference contribution.

The starting point of our calculation is the effective Hamiltonian, obtained by integrating out the heavy particles in the SM, leading to

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}}\, V_{ts}^\star V_{tb} \sum_{i=1}^{8} C_i(\mu)\mathcal{O}_i(\mu)\,, \qquad (1)$$

where we use the operator basis introduced in [13]:

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[1] We interpret m_s to be a constituent mass, varying it between 400 and 600 MeV.

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{s}_L \gamma_\mu T^a c_L)\,(\bar{c}_L \gamma^\mu T_a b_L)\,,\\
\mathcal{O}_2 &= (\bar{s}_L \gamma_\mu c_L)\,(\bar{c}_L \gamma^\mu b_L)\,,\\
\mathcal{O}_3 &= (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_q (\bar{q}\gamma^\mu q)\,,\\
\mathcal{O}_4 &= (\bar{s}_L \gamma_\mu T^a b_L) \textstyle\sum_q (\bar{q}\gamma^\mu T_a q)\,,\\
\mathcal{O}_5 &= (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \textstyle\sum_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho q)\,,\\
\mathcal{O}_6 &= (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \textstyle\sum_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho T_a q)\,,\\
\mathcal{O}_7 &= \tfrac{e}{16\pi^2}\,\left[\bar{s}\sigma^{\mu\nu}\,(\bar{m}_b\,R + \bar{m}_s\,L)\,F_{\mu\nu} b\right]\,,\\
\mathcal{O}_8 &= \tfrac{g_s}{16\pi^2}\,\left[\bar{s}\sigma^{\mu\nu}\,(\bar{m}_b\,R + \bar{m}_s\,L)\,T^a G^a_{\mu\nu} b\right]\,.
\end{aligned} \tag{2}$$

The symbols T^a ($a = 1, 8$) denote the $SU(3)$ color generators; g_s and e denote the strong and electromagnetic coupling constants. In Eq. (2), $\bar{m}_b$ and $\bar{m}_s$ are the running b and s-quark masses in the $\overline{\text{MS}}$-scheme at the renormalization scale μ. We keep the exact dependence on the strange-quark mass in our calculation. Further, as we are not interested in CP-violation effects in the present paper, we exploited the unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix and neglected $V_{ub}V_{us}^*$ (as $V_{ub}V_{us}^* \ll V_{tb}V_{ts}^*$) when writing Eq. (1).

While the Wilson coefficients $C_i(\mu)$ appearing in Eq. (1) have been known to sufficient precision at the low scale $\mu \sim m_b$ for a long time (see e.g. the reviews [4, 5] and references therein), the matrix elements $\langle s\gamma\gamma|\mathcal{O}_i|b\rangle$ and $\langle s\gamma\gamma\, g|\mathcal{O}_i|b\rangle$, which in a NLL calculation are needed to order g_s^2 and g_s, respectively, are only partially known now (see [1, 2] for the details of the provided contributions and [14] for a recent summary). Calculating the $(\mathcal{O}_i, \mathcal{O}_j)$-interference contributions for the differential distributions at order α_s is in many respects of similar complexity as the calculation of the photon energy spectrum in $\bar{B} \to X_s\gamma$ at order α_s^2 needed for the NNLL computation. There, the individual interference contributions, which all involve extensive calculations, were published in separate papers, sometimes even by two independent groups (see e.g. [15, 16]). It therefore cannot be expected that the NLL results for the differential distributions related to $\bar{B} \to X_s\gamma\gamma$ are given in a single paper. As a next step in the NLL enterprise, we derive in the present paper the $(\mathcal{O}_8, \mathcal{O}_8)$-interference contribution (which starts at order α_s) to the double differential decay width $d\Gamma/(ds_1 ds_2)$. The variables s_1 and s_2 are defined as $s_i = (p_b - q_i)^2/m_b^2$, where p_b and q_i denote the four-momenta of the b-quark and the two photons, respectively.

At order α_s there are only contributions to $d\Gamma_{88}/(ds_1 ds_2)$ with four particles (s-quark, two photons and a gluon) in the final state. These contributions correspond to specific cuts of the b-quark self energy at order $\alpha^2 \times \alpha_s$, involving twice the operator $\mathcal{O}_8$. As there are additional cuts, which contain for example only one photon, our observable cannot be obtained using the optical theorem, i.e., by taking the absorptive part of the b-quark self energy at three loops. We therefore calculate the mentioned contributions with four particles in the final state individually.

When calculating the contribution of $\mathcal{O}_8$ to $d\Gamma/(ds_1 ds_2)$, we restrict ourselves (as in refs. [1, 2]) to the region in the (s_1, s_2)-plane which is also accessible to three body decays $b \to s\gamma\gamma$ (associated e.g. with the tree-level contribution of $\mathcal{O}_7$), i.e.,

$$s_1 > x_4\,;\; s_2 > x_4\,;\; s_1 + s_2 < 1 + x_4\,;\; s_1 s_2 > x_4\,, \tag{3}$$

where $x_4 = (m_s/m_b)^2$. The energies E_1 and E_2 in the rest frame of the b-quark of the two photons are related to s_1 and s_2 in a simple way: $s_i = 1 - 2\,E_i/m_b$. As the energies E_i of the photons have to be away from zero in order to be observed, the values of s_1 and s_2 should be considered to be smaller than one. Furthermore, in order to see two separate photons, their invariant mass should also be away from zero. All these requirements can be implemented in terms of one physical cut parameter c ($c > 0$), by demanding[2]

$$s_1 \geq c\,,\;\; s_2 \geq c\,,\;\; 1 - s_1 - s_2 \geq c\,. \tag{4}$$

The kinematical region in the (s_1, s_2)-plane, which we take into account in this paper, therefore corresponds to the intersection of the regions given in eqs. (3) and (4). For explicit formulas representing this intersection, we refer to the appendix.

Imposing these cuts, the photons do not become soft in our case, while one of them can become collinear with the strange quark. This implies that in the final result a single logarithm of m_s survives. The only source for such $\log(m_s)$ terms in our result is the mentioned collinear emission of the photons from the s quark. In particular, we emphasize that the $(\mathcal{O}_8, \mathcal{O}_8)$-contribution to the double differential decay width does not become singular when the gluon and the strange quark become collinear, since the gluon is emitted from the effective operator $\mathcal{O}_8$ directly and therefore there is no propagator denominator of the form $(p_s + p_g)^2$ which could become singular. In addition, soft-gluon related singularities also do not appear in this case (the matrix element associated with $\mathcal{O}_8$ even goes to zero when the gluon energy tends to 0). The absence of singularities generated by soft and/or collinear gluons is related to the fact that concerning QCD our observable (i.e. the triple or double differential decay width), based on the full effective Hamiltonian, is fully inclusive and therefore nonsingular. We also stressed this fact in [2], where the $(\mathcal{O}_7, \mathcal{O}_7)$-contribution was worked out. In this case there were gluon induced singularities in the virtual and bremsstrahlung corrections, but they canceled when combined as a consequence of the Kinoshita-Lee-Nauenberg (KLN) theorem. This means that the origin of $\log(m_s)$ terms is from collinear photon emission only. Note that concerning QED our observable

[2] The normalized invariant mass squared $s = (q_1 + q_2)/m_b^2$ of the two photons can be written as $s = 1 - s_1 - s_2 + s_3$, where s_3 is the normalized hadronic mass squared.

is not fully inclusive, because we want to observe exactly two photons in the final state; therefore $\log(m_s)$ terms remain. A further remark on the numerical m_s-dependence is in order: The $(\mathcal{O}_7, \mathcal{O}_7)$-contribution to the double differential decay width starts at order α_s^0. This leading contribution does not contain $\log(m_s)$ terms when applying the kinematical cuts discussed above. Only at order α_s^1 terms $\sim \log(m_s)$ appear, because one of the photons can become collinear with the strange quark. As a consequence, we expect the relative m_s-dependence of the $(\mathcal{O}_7, \mathcal{O}_7)$ contribution to be smaller than the corresponding dependence of the $(\mathcal{O}_8, \mathcal{O}_8)$ contribution, because the latter only starts at order α_s^1. In other words the m_s-dependence of the complete double differential decay width will be smaller than the one which is only based on the $(\mathcal{O}_8, \mathcal{O}_8)$ contribution discussed in this paper.

The main goal of this paper is to work out $d\Gamma_{88}/(ds_1 ds_2)$ as a further ingredient towards a systematic NLL prediction for the decay rate of $\bar{B} \to X_s\gamma\gamma$. For similar analysis for the case of $\bar{B} \to X_s\gamma$, one can see e.g. [17–21].

In this regard, we employ in our calculation a finite strange-quark mass m_s which we interpret to be of constituent type in the numerics. This approach has also been adopted previously, e.g. by Kaminski *et al.* in [22] and Asatrian and Greub in [2, 23]. The experience gained in these references shows that the constituent mass approach gives results which are similar to those when using fragmentation functions [23]. Therefore, we believe that this method is sufficient to obtain an estimate of the $(\mathcal{O}_8, \mathcal{O}_8)$-interference contribution. While the fragmentation approach seems better from the theoretical point of view, it is not clear that it leads to better final results in practice, because the fragmentation functions (for $s \to \gamma$ or $g \to \gamma$) suffer from experimental uncertainties, as pointed out in [23]. An alternative could be to look at the version with "isolated photons" a la Frixione [24] which corresponds, however, to a slightly different observable. Such an approach is beyond the scope of the present paper and is left for future studies.

Before moving to the detailed organization of our paper, we should mention that the inclusive double radiative process $\bar{B} \to X_s\gamma\gamma$ has also been explored in several extensions of the SM [8, 10, 25]. Also the corresponding exclusive modes, $B_s \to \gamma\gamma$ and $B \to K\gamma\gamma$, have been examined before, both in the SM [9, 26–34] and in its extensions [25, 30, 31, 35–43]. We should add that the long-distance resonant effects were also discussed in the literature (see e.g. [9] and the references therein). Finally, the effects of photon emission from the spectator quark in the B-meson were discussed in [26, 30, 44].

The remainder of this paper is organized as follows. In section II the calculation of the $(\mathcal{O}_8, \mathcal{O}_8)$-contribution to the double differential decay width $d\Gamma/(ds_1 ds_2)$ is presented. To regulate the configurations where photons are emitted from the s quark in a collinear way, a finite strange-quark mass m_s is introduced. This way the collinear singularities manifest themselves as $\log(m_s)$ terms in our final result, which reflects the feature for the photons having hadronic substructure. In section III we illustrate the numerical impact of the $(\mathcal{O}_8, \mathcal{O}_8)$-contribution to the double differential width and the total decay width (depending on a kinematical cut). The main text of our paper ends with a short summary in section IV. In the appendix V, we give the explicit formulas defining the four-particle phase-space region considered in this paper together with the explicit expressions for the master integrals (MIs) appearing in our calculation.

II. $(\mathcal{O}_8, \mathcal{O}_8)$ CONTRIBUTION TO THE DOUBLE DIFFERENTIAL SPECTRUM $d\Gamma/(ds_1 ds_2)$ AT $\mathcal{O}(\alpha_s)$

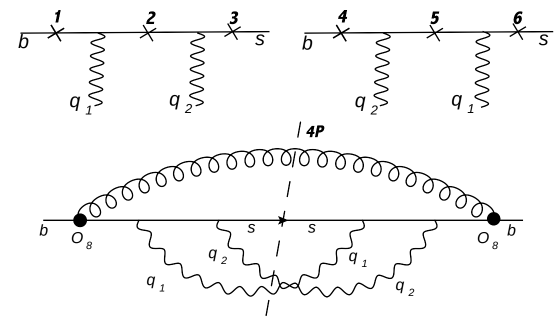

FIG. 1: On the first line the diagrams defining the $\mathcal{O}_8$ contribution to $b \to sg\gamma\gamma$ are shown at the amplitude level. The crosses in the graphs stand for the possible emission places of the gluon (emerging from the operator $\mathcal{O}_8$). On the second line the contribution to the decay width corresponding to the interference of diagram 1 with diagram 4 is illustrated. This sample interference diagram gives rise to $\log(m_s/m_b)$ terms due to collinear configurations of one of the photons with the s quark.

We now turn to the calculation of the $\mathcal{O}_8$ self-interference contribution to the decay width for $\bar{B} \to X_s\gamma\gamma$, which is based on the partonic process $b \to sg\gamma\gamma$, where g denotes a gluon. Although this is only a tree-level computation at order α_s, it is quite complicated because of the four particles in the final state, one of them being massive (the strange quark).

Before going into detail, we mention that the kinematical range of the variables $s_1 = (p_b - q_1)^2/m_b^2$ and $s_2 = (p_b - q_2)^2/m_b^2$ is larger in the $1 \to 4$ process considered in this section than the range given in Eq. (3), which corresponds to the $1 \to 3$ process $b \to s\gamma\gamma$. Nevertheless, we restrict ourselves to the range which corresponds to the intersection of the regions given in Eq. (3) and Eq. (4), as we also did in [1, 2] when considering virtual and bremsstrahlung corrections to the $\mathcal{O}_7$-contribution. For explicit formulas of the considered (s_1, s_2)-region, we refer to Eq. (8) in the appendix.

The diagrams defining the $\mathcal{O}_8$ contribution at the amplitude level are shown in the first line of Fig. 1. The amplitude squared, needed to get the (double differential) decay width, can be written as a sum of interferences of the different diagrams shown on the first line in Fig. 1. One such interference is shown on the second line of the same figure. The four-particle final state is described by five independent kinematical variables; s_1 and s_2 are just two of them.

In the present paper, we worked out in a first step the triple differential spectrum $d\Gamma_{88}/(ds_1\, ds_2\, ds_3)$, where $s_3 = (p_s + p_g)^2/m_b^2$ is the normalized hadronic mass squared and p_g is the final state gluon momentum. At this level, we computed the resulting MIs numerically for exact m_s (see section V for their explicit expressions). To get the double differential spectrum $d\Gamma_{88}/(ds_1\, ds_2)$ we then integrated over s_3 in its range $s_3 \in \left[m_s^2/m_b^2,\, s_1.s_2\right]$.

Last, as the various steps of the calculation are similar to those in Ref. [1], we refer to section 7 of that paper for more details on the techniques applied. Also, we refer to appendix B of Ref. [2] for a useful parametrization of the four-particle phase-space for the case where one of the particles is massive, which is based on the work in Ref. [45].

Parameter	Value
$\mathrm{BR}_{\mathrm{sl}}^{\mathrm{exp}}$	0.1049
$\mathrm{m_b}$	4.8 GeV
$\mathrm{m_c/m_b}$	0.29
$\mathrm{G_F}$	1.16637×10^{-5} GeV^{-2}
$\mathrm{V_{cb}}$	0.04
$\mathrm{V_{tb}V_{ts}^*}$	0.04
$\alpha_{(\mathrm{em})}{}^{-1}$	137

	$C^0_{8,eff}(\mu)$	$\alpha_s(\mu)$
$\mu = M_W$	-0.09739	0.1213
$\mu = 2\, m_b$	-0.13516	0.1818
$\mu = m_b$	-0.14905	0.2175
$\mu = m_b/2$	-0.16529	0.2714

TABLE I: **Upper:** Relevant input parameters used in this paper. **Lower:** The Wilson coefficient $C_{8,eff}(\mu)$ and $\alpha_s(\mu)$ at different values of the renormalization scale μ.

III. NUMERICAL ILLUSTRATIONS

In the previous section we described the calculation for the $(\mathcal{O}_8, \mathcal{O}_8)$ contribution to the double differential decay width for $\bar{B} \to X_s\gamma\gamma$ at NLL precision.

The Wilson coefficient $C_{8,eff}(\mu)$ at the low scale[3]

$$C_{8,eff}(\mu) = C^0_{8,eff}(\mu_b)$$

has been known for a long time (see Ref. [13] and references therein). Numerical values for the input parameters and for this Wilson coefficient at various values for the scale μ, together with the numerical values of $\alpha_s(\mu)$, are given in upper and lower panels of Table I, respectively.

To stress that the $(\mathcal{O}_8, \mathcal{O}_8)$-contribution to $d\Gamma/(ds_1 ds_2)$ only starts at the NLL level, we write

$$\frac{d\Gamma_{88}}{ds_1 ds_2} = \frac{d\Gamma_{88}^{(1)}}{ds_1 ds_2} \tag{5}$$

where $d\Gamma_{88}^{(1)}/(ds_1 ds_2)$ has the form

$$\frac{d\Gamma_{88}^{(1)}}{ds_1\, ds_2} = \frac{\alpha^2\, \bar{m}_b^2(\mu)\, m_b^3\, |C_{8,eff}(\mu)|^2\, G_F^2\, |V_{tb}V_{ts}^*|^2\, Q_d^4}{1024\, \pi^5} \times \frac{\alpha_s}{4\pi}\, C_F\; \kappa_{88}^{(1)}(s_1, s_2, m_s/m_b)\,. \tag{6}$$

The function $\kappa_{88}^{(1)}(s_1, s_2, m_s/m_b)$, which encodes the dependence on s_1, s_2 and on m_s/m_b, is too lengthy to be displayed explicitly. We note that we will keep the exact m_s dependence in our numerics.

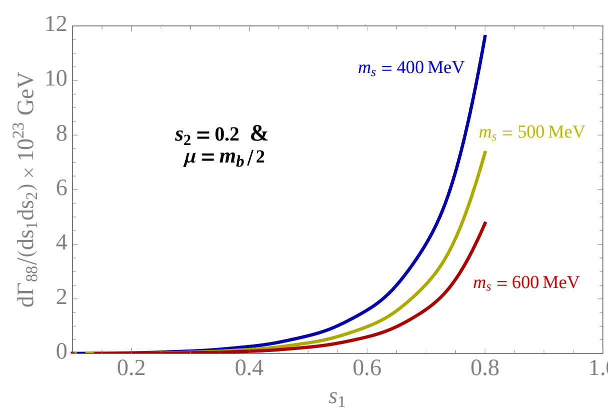

FIG. 2: $d\Gamma_{88}/(ds_1\, ds_2)$ [as given in eqs. (5) and (6)] as a function of s_1 for s_2 fixed at 0.2, $\mu = m_b/2$ and m_s varied between 400 and 600 MeV. The blue(top), yellow(middle) and red(bottom) lines show the width when choosing m_s to be 400, 500 and 600 MeV, respectively.

In Table II, the impact of $\frac{d\Gamma_{88}^{(1)}}{ds_1\, ds_2}$ on the branching ratio for $\bar{B} \to X_s\gamma\gamma$ is presented for various choices of m_s, c and the scale μ. It is seen that this contribution is much smaller than the corresponding numbers for the $(\mathcal{O}_7, \mathcal{O}_7)$ contribution (see Table 4 of Ref. [2] for comparison).

To obtain the values for the branching ratio in Table II as a function of the cutoff parameter c defined in Eq. (4), we integrate the double differential spectrum over the corresponding ranges in s_1 and s_2 [see Eq. (8)], divide by the semileptonic decay width and multiply with the measured semileptonic branching ratio. For illustrative

[3] At NLL precision, $C_{8,eff}(\mu)$ is needed only up to order α_s^0, because the square of the matrix element $\langle sg\gamma\gamma|\mathcal{O}_8|b\rangle$ starts at order α_s^1. Furthermore, for our current purpose we identify the $\overline{\mathrm{MS}}$ mass $\bar{m}_b(\mu)$ with the corresponding pole mass.

purposes, it is sufficient to take the lowest order formula for the semileptonic decay width [see e.g. Eq. (6.2) in Ref. [2]].

In Fig. 2 we plot $d\Gamma_{88}/(ds_1\, ds_2)$, calculated in this paper, as a function of s_1, while s_2 is kept fixed at $s_2 = 0.2$. The renormalization scale is chosen to be $\mu = m_b/2$ and m_s is varied between 400 and 600 MeV. This figure shows that $d\Gamma_{88}/(ds_1\, ds_2)$ is orders of magnitude smaller in size than $d\Gamma_{77}/(ds_1\, ds_2)$ (for comparison see Fig. 7 of Ref. [2] which is an extended analysis of the work in Ref. [1]). For other choices of the scale μ, the behavior of the spectrum is similar, but even smaller in size.

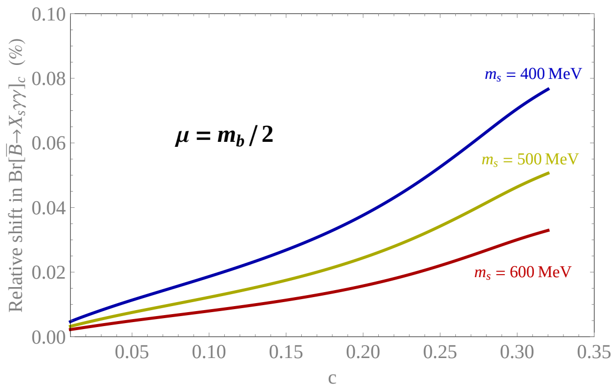

FIG. 3: Relative shift ($\frac{\mathrm{Br}[\bar{\mathrm{B}}\to \mathrm{X_s}\gamma\gamma]_{\mathrm{c}}^{88}}{\mathrm{Br}[\bar{\mathrm{B}}\to \mathrm{X_s}\gamma\gamma]_{\mathrm{c}}}$) of the branching ratio for $\bar{\mathrm{B}} \to \mathrm{X_s}\gamma\gamma$ (in percent) due to the $(\mathcal{O}_8\,,\mathcal{O}_8)$ contribution as a function of the cut parameter c for $\mu = m_b/2$. The blue(top), yellow(middle) and red(bottom) lines show the relative shifts when setting $m_s = 400$ MeV, 500 MeV and 600 MeV, respectively. For other choices of the scale μ the relative change is even smaller.

In Fig. 3 we investigate the numerical impact of the $(\mathcal{O}_8\,,\mathcal{O}_8)$ contribution on the branching ratio of $\bar{B} \to X_s\gamma\gamma$ (see the discussion in the third paragraph of the introduction). More precisely, we worked out the relative shift

$$\frac{\mathrm{Br}[\bar{\mathrm{B}} \to \mathrm{X_s}\gamma\gamma]_{\mathrm{c}}^{88}}{\mathrm{Br}[\bar{\mathrm{B}} \to \mathrm{X_s}\gamma\gamma]_{\mathrm{c}}} \tag{7}$$

of the branching ratio due to the $(\mathcal{O}_8, \mathcal{O}_8)$ contribution, as a function of the kinematical cut parameter c. Fig. 3 clearly shows that this contribution is below 0.1% in the full (s_1, s_2)-range considered in this paper. We mention that in $\bar{B} \to X_s\gamma$ the situation concerning the $\mathcal{O}_8$ contribution is different. As pointed out in refs. [17, 20], in this decay mode the contribution of $\mathcal{O}_8$ is non-negligible, in particular, for values of $E_\gamma < 1.1$ GeV. On the other hand, in the double radiative decay, the effects described in the references just mentioned are also present in the $\mathcal{O}_7$ contribution; as a consequence the effect of the $\mathcal{O}_8$ contribution stays small in the full phase space.

IV. CONCLUDING REMARKS

In the present work we calculated the set of the $O(\alpha_s)$ corrections to the decay process $\bar{B} \to X_s\gamma\gamma$ originating from diagrams involving the chromomagnetic dipole operator $\mathcal{O}_8$. To perform this calculation, it was necessary to work out diagrams with four particles (s quark, two photons and a gluon) in the final state. From the technical point of view, the calculation was made possible by the use of the Laporta algorithm [46] to identify the needed master integrals. We then solved the resulting MIs numerically, keeping the exact dependence on the strange-quark mass m_s, which we varied between 400 and 600 MeV in the numerical illustrations.

We conclude that the numerical impact of the self-interference contribution of the chromomagnetic dipole operator $\mathcal{O}_8$ to the decay rate is minor when compared to the self-interference effect of the electromagnetic dipole operator $\mathcal{O}_7$.

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	Branching ratios for $\bar{B} \to X_s\gamma\gamma$					
	$c = 1/50$			$c = 1/100$		
	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2m_b$
NLL_1	1.57	1.03	0.71	1.79	1.17	0.80
NLL_2	0.96	0.63	0.43	1.09	0.71	0.49
NLL_3	0.59	0.39	0.27	0.67	0.44	0.30

TABLE II: Branching ratios (in units of 10^{-11}) for $\bar{B} \to X_s\gamma\gamma$ when only considering the $(\mathcal{O}_8\,,\mathcal{O}_8)$ contribution calculated in this paper. The left half of the table corresponds to the results when choosing $c = 1/50$, while in the right half c is set to be $c = 1/100$. The rows labeled with NLL_1, NLL_2 and NLL_3 give the result of this specific NLL contribution when setting $m_s = 400$ MeV, $m_s = 500$ MeV and $m_s = 600$ MeV, respectively. See the text for details.

V. APPENDIX

In this appendix, we give the explicit formulas defining the four-particle phase-space region considered in this paper as a result of the intersection of regions given in Eq. (3) and Eq. (4). Further, we give the explicit forms of the master integrals appearing in our calculation of the $(\mathcal{O}_8, \mathcal{O}_8)$-contribution to the decay width for $\bar{B} \to X_s\gamma\gamma$.

A. Explicit formulas for the range in the (s_1, s_2)-plane

The kinematical conditions on the phase-space variables s_1 and s_2, as implicitly formulated in Eq. (3) and Eq. (4), can easily be converted to explicit ranges. There are the following three cases (using $x_4 = m_s^2/m_b^2$):

$$\begin{gathered}
\underline{\mathbf{(i)}\ \text{if}\ \mathbf{x_4 \le c^2}} \\
c < s_1 < 1 - 2\,c\ ;\ c < s_2 < 1 - s_1 - c \\
\underline{\mathbf{(ii)}\ \text{if}\ \mathbf{c^2 < x_4 < c\,(1-2\,c)}} \\
c < s_1 < \tfrac{x_4}{c}\ ;\ \tfrac{x_4}{s_1} < s_2 < 1 - s_1 - c \\
\text{or} \\
\tfrac{x_4}{c} < s_1 < 1 - 2\,c\ ;\ c < s_2 < 1 - s_1 - c \\
\underline{\mathbf{(iii)}\ \text{if}\ \mathbf{x_4 \ge c\,(1-2\,c)}} \\
{s_1}^- < s_1 < {s_1}^+\quad ;\quad \tfrac{x_4}{s_1} < s_2 < 1 - s_1 - c \\
\text{with} \\
{s_1}^\pm = \left(1 - c \pm \sqrt{(1-c)^2 - 4\,x_4}\right)/2\,.
\end{gathered} \tag{8}$$

Case (ii) is understood to be the sum of the two possibilities written in Eq. (8). Further, it can be seen from the same equation that if one puts $m_s = 0$, one would simply end up with case (i), as previously considered in [1, 2]. As an example, in Fig. 4 we give the geometrical representation of case (i) of Eq. (8).

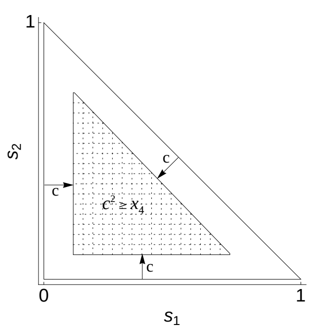

FIG. 4: The shaded area shows the (s_1, s_2) phase-space region for the case $c^2 \ge x_4$.

B. Explicit forms for the Master Integrals

In a first step, we managed to write the triple differential decay width $d\Gamma_{88}/(ds_1 ds_2 ds_3)$ as a linear combination of five independent MIs.

The full four-particle phase space can be parametrized in terms of five independent variables. According to the procedure described in Appendix B.2 of Ref. [2], three of the five variables can be chosen to be s_1, s_2 and s_3. The MIs are therefore given in terms of integrals over two variables called λ_4 and λ_5, running in the interval $[0, 1]$.

Since we regulated possible collinear singularities by keeping m_s exact and since soft photons are excluded by the cuts imposed through Eq. (4), we can work in $d = 4$ dimensions; this considerably simplifies the expressions in Appendix B.2 of Ref. [2].

The MIs, defined at the level of the triple differential decay width, depend on s_1, s_2, s_3 and x_4. We denote them by $B^{\nu_1\nu_2}_{\text{set}_i}(s_1, s_2, s_3, x_4)$, where ν_1, ν_2 stand for the powers of the propagators in the MIs and i defines the set (propagator structure) where they belong. Our parametrized MIs are of the form ($\lambda_{4,5} \in [0, 1]$)

$$B^{\nu_1\nu_2}_{\text{set}_i}(s_1, s_2, s_3, x_4) = \mathcal{N}_{\text{ps}} \int_{\lambda_4}\int_{\lambda_5} d\lambda_4 d\lambda_5\, \frac{P_{1,i}^{-\nu_1} P_{2,i}^{-\nu_2}}{\sqrt{(1-\lambda_5)\,\lambda_5}} \tag{9}$$

where $\mathcal{N}_{\text{ps}}$ is the phase-space factor with $\mathcal{N}_{\text{ps}} = \frac{s_3 - x_4}{2048\pi^6\, s_3}$, and the propagators $P_{1,i}, P_{2,i}$ are understood to be expressed in terms of the integration variables λ_4, λ_5 and the variables s_1, s_2, s_3, following the parametrization used in [2]. Based on these considerations, we have the following expressions for the MIs:

$$\begin{aligned}
&\underline{\text{set}_1}: \\
&\underline{P_1 = (p_g - p_b + q_1)^2 - m_s^2,\ P_2 = (p_g - p_b)^2 - m_s^2} \\
B^{00}_{\text{set}_1} &= \tfrac{s_3 - x_4}{2048\pi^5\, s_3} \\
B^{10}_{\text{set}_1} &= \int_{\lambda_4}\int_{\lambda_5} d\lambda_4 d\lambda_5\, \tfrac{I^{10}_{\text{set}_1}(\lambda_4,\lambda_5)}{\sqrt{(1-\lambda_5)\lambda_5}} \\
&= \tfrac{\log(s_3/x_4)}{2048\pi^5\,(s_1 - s_3)} \\
B^{01}_{\text{set}_1} &= \int_{\lambda_4}\int_{\lambda_5} d\lambda_4 d\lambda_5\, \tfrac{I^{01}_{\text{set}_1}(\lambda_4,\lambda_5)}{\sqrt{(1-\lambda_5)\lambda_5}} \\
B^{11}_{\text{set}_1} &= \int_{\lambda_4}\int_{\lambda_5} d\lambda_4 d\lambda_5\, \tfrac{I^{11}_{\text{set}_1}(\lambda_4,\lambda_5)}{\sqrt{(1-\lambda_5)\lambda_5}} \\
&\underline{\text{set}_2}: \\
&\underline{P_1 = (p_g - p_b + q_1)^2 - m_s^2,\ P_2 = (p_g - p_b + q_2)^2 - m_s^2} \\
B^{11}_{\text{set}_2} &= \int_{\lambda_4}\int_{\lambda_5} d\lambda_4 d\lambda_5\, \tfrac{I^{11}_{\text{set}_2}(\lambda_4,\lambda_5)}{\sqrt{(1-\lambda_5)\lambda_5}}
\end{aligned} \tag{10}$$

where the respective integrands explicitly read

$$I^{10}_{\text{set}_1} = \mathcal{N}_{\text{ps}} \frac{s_3}{(s_1-s_3)(s_3(1-\lambda_4)+x_4\lambda_4)}\,, \quad (11)$$

$$\begin{aligned} I^{01}_{\text{set}_1} &= \mathcal{N}_{\text{ps}}\, s_1 (s_1-s_3)\, s_3\, [s_1 \{s_3 (s_1 + (s_2-2)\, s_3 \\ &- (s_1+s_2-s_3)\, x_4 + x_4) - (s_1 (s_1+s_2) \\ &- (s_1 - s_2 + 2)\, s_3)\, \lambda_4 (s_3 - x_4)\} \qquad (12) \\ &-2 f_{\text{root}} (s_1-1)(s_1-s_3)(2\lambda_5-1)(s_3-x_4)]^{-1}\,, \end{aligned}$$

$$\begin{aligned} I^{11}_{\text{set}_1} &= -\mathcal{N}_{\text{ps}}\, s_1 s_3^2\, [(s_3(\lambda_4-1) - x_4\lambda_4) \qquad (13) \\ &\{s_1 (s_3 (s_1 + (s_2-2)\, s_3 - (s_1+s_2-s_3)\, x_4 + x_4) \\ &- (s_1(s_1+s_2) - (s_1-s_2+2)\, s_3)\, \lambda_4 (s_3-x_4)) \\ &-2 f_{\text{root}} (s_1-1)(s_1-s_3)(2\lambda_5-1)(s_3-x_4)\}]^{-1}\,, \end{aligned}$$

$$\begin{aligned} I^{11}_{\text{set}_2} &= -\mathcal{N}_{\text{ps}}\, s_1 s_3^2\, [(\lambda_4 (x_4 - s_3) + s_3)\{2(2\lambda_5-1) \\ &(s_1-1)(s_1-s_3)\, f_{\text{root}} (s_3-x_4) + s_1 (\lambda_4 (s_3 (s_3 - s_2 \\ &+2) - s_1 (s_2+s_3))(x_4-s_3) + s_3 ((s_1+s_2-s_3 \\ &-1)\, x_4 - s_1 s_2 + s_3))\}]^{-1}\;; \qquad (14) \end{aligned}$$

$$f_{\text{root}} = \sqrt{\frac{s_1^2 (s_1+s_2-s_3-1)(s_1 s_2 - s_3) s_3 (\lambda_4-1)\lambda_4}{(s_1-1)^2 (s_1-s_3)^2}}\,.$$

In Eq. (10), the integrations involved in $B^{00}_{\text{set}_1}$ were trivial to perform. For $B^{10}_{\text{set}_1}$, an analytical solution is possible, using the differential equation (DE) method. For the remaining MIs, as the corresponding integrands $I^{\nu_1\nu_2}_{\text{set}_i}(\lambda_4, \lambda_5)$ develop complicated structures, we performed these integrations numerically for exact m_s.

As can be understood from their propagator structures, two of the MIs, $B^{01}_{\text{set}_1}$ and $B^{11}_{\text{set}_2}$, are symmetric under the exchange of $s_1 \leftrightarrow s_2$.

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