Numerical Magnitude Skills in 6-Years-Old Children: Exploring Specific Associations with Components of Executive Function

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Abstract
Little is known about how children learn to associate numbers with their corresponding magnitude and about individual characteristics contributing to performance differences on the numerical magnitude tasks within a relatively homogenous sample of 6-year-olds. The present study investigated the relationships between components of executive function and two different numerical magnitude skills in a sample of 162 kindergartners. The Symbolic Number Line was predicted by verbal updating and switching, whereas the Symbolic Magnitude Comparison was predicted by inhibition. Both symbolic tasks were predicted by visuo-spatial updating. Current findings suggest that visuo-spatial updating underlies young children’s retrieval and processing of numbers’ magnitude.

Keywords: executive function, numerical magnitude skills, preschool children, number line, magnitude comparison, number representation

1. Introduction
Preschool children acquire basic numerical knowledge long before they receive formal instruction in mathematics. Relying on their innate capacity to discriminate quantities and on the steady improvement of their counting skills during preschool and school age, children learn to link numbers with their corresponding magnitude (Dehaene, 2001; Krajewski & Schneider, 2009). During the development of their number skills children understand that each number (e.g., “five” or “5”) defines a quantity of units (e.g., “●●●●●”), independently of the objects described. They also become able to compare numbers and quantities and to classify them based on their magnitudes. These numerical magnitude skills develop with age and their accuracy has been shown to be a substantial predictor of later math achievement in general (Kolkman, Kroesbergen, & Leseman, 2013; Krajewski & Schneider, 2009) and later procedural fraction knowledge (Bailey et al., 2015).

Since numerical magnitude skills play an important role in the acquisition of mathematical knowledge, it is essential to investigate inter-individual variability in relation to these skills. Components of executive function, that is, “the control and the regulation of cognitive processes” (Miyake et al., 2000, p. 51), represent promising constructs for explaining differences in numerical magnitude skills (Friso-van den Bos, Kolkman, Kroesbergen, & Leseman, 2014; Kolkman, Hoitink, Kroesbergen, & Leseman, 2013). This is because components of executive function are related to other preschool numerical abilities, such as number recognition, counting and subitizing (Bull, Espy, & Wiebe, 2008; Bull, Espy, Wiebe, Sheffield, & Nelson, 2011; Bull & Scerif, 2001; Espy et al., 2004). Moreover, components of executive function predict mathematic achievement in primary school (for a review see Bull & Lee, 2014; for a meta-analysis see Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013).

1.1 Executive Function
To date, the most dominant framework conceptualizing components of executive function is Miyake et al.’s (2000) model postulating three distinct components of executive function: the inhibition of dominant responses (Inhibition), the switching between tasks or mental sets (Switching) and the storage and manipulation of representations in the working memory (Updating). However, the presence of a three-factor structure in children
is still discussed in the literature. While several studies support the existence of three distinct factors in children (Espy et al., 2004; Lehto, Juujarvi, Kooistra, & Pulkkinen, 2003; Wu et al., 2011), other studies suggest that executive functioning evolves from a single-factor to a three-factor structure during children’s development (Lee, Bull, & Ho, 2013; Shing, Lindenberger, Diamond, Li, & Davidson, 2010; Wiebe, Espy, & Charak, 2008). In the present study, we aimed at exploring the relations between numerical magnitude skills and the components of executive function in their diversity, therefore the three-factor structure was favored in our approach.

1.2 Numerical Magnitude Skills

Two classical tasks often used to measure numerical magnitude skills are the Symbolic Number Line (Siegler & Booth, 2004; Siegler & Opfer, 2003) and the Symbolic Magnitude Comparison (Dehaene, Bossini, & Giraux, 1993; De Smedt, Verschaffel, & Ghesquière, 2009; Durand, Hulme, Larkin, & Snowling, 2005). The former consists of an empty number line on which the participant has to place a given number. Typically, for preschool children the numbers to be placed range from 0 to 100, and their accuracy in a linear fit curve is calculated. The second task, Symbolic Magnitude Comparison, requires participants to indicate which of two presented digits is higher in magnitude.

While children gain experience with numbers, the shape of their estimates on the empty number line shifts from a logarithmic to a more accurate, linear pattern (Siegler & Opfer, 2003). Children showing a logarithmic pattern overestimate the distance between small numbers at the lower end towards the zero point and underestimate the distance between larger numbers at the end point of the line, which is usually 10 or 100. The shift from a logarithmic to a linear pattern occurs at different ages depending on the range of the number line. For number lines that range from 0 to 10, linear fits can be observed in kindergarten or even preschool, for number lines ranging from 0 to 100; children begin to show linear estimates between kindergarten and second grade (Booth & Siegler, 2008; Laski & Siegler, 2007; Siegler & Booth, 2004).

The linearity of children’s number line placements has been shown to be strongly associated with mathematical achievement (Siegler & Booth, 2004). At the same time, first and second graders with mathematical learning disability were significantly less accurate in their estimates on the number line than typically achieving children (Geary, Hoard, Nugent, & Byrd-Craven, 2008). Additionally, two trainings based on the number line were found to improve mathematical performance in both typically achieving 7-years-olds (Booth & Siegler, 2008) and in 8-10 years-old children with mathematical learning disability (Kucian et al., 2011). The mental number line is important for mathematical development because it is a dynamic structure: it begins with small whole numbers and expands over the course of development rightwards including larger whole numbers, leftwards including negative numbers and interstitially including fractions and decimals (Siegler & Lortie-Forgues, 2014). The mentioned studies suggest a crucial relation between the linearity of number line estimations and mathematical learning and achievement. It is therefore important to investigate the variables influencing the acquisition of linear estimates in children. By addressing the role of components of executive function in numerical magnitude skills, the present study makes an effort into this direction.

The second task, Symbolic Magnitude Comparison, requires participants to indicate which of two presented digits is higher in magnitude. Similarly to the Symbolic Number Line, this task has been used to investigate participants’ number representation and understanding of magnitude (Dehaene, Dehaene-Lambertz, & Cohen, 1998). Furthermore, children’s accuracy on the Symbolic Magnitude Comparison was also shown to correlate positively with their mathematical achievement. Different studies with school aged children show that the ability to compare magnitudes predicts later mathematical achievement. In fact, when verbal ability is taken into account, these two skills explain 42 % of the variance of children’s arithmetic skills (Mundy & Gilmore, 2009; Durand et al., 2005). Rousselle and Noël (2007) also reported that second graders with mathematical learning disability were slower and less accurate than typical achieving children in comparing the magnitude of numbers. As with the Symbolic Number Line, the Symbolic Magnitude Comparison has been shown to be associated with mathematical achievement and mathematical learning disability during early school years. It is thus our aim to investigate which variables influence preschool numerical magnitude skills. Such variables might help to understand the origin of mathematical learning disability and enable us to develop suitable help targeting at children’s difficulties.

Although both tasks require basic numerical knowledge and show similar positive correlations to mathematical achievement, it remains unclear to what extent the two tasks measure a same construct (e.g., the child’s ability to mentally represent the magnitude of numbers). On the one side, in most studies both tasks showed only moderate positive correlations to each other (Friso-van den Bos, Kolkman et al., 2014; Kolkman, Kroesbergen, & Leseman, 2013; Sasanguie & Reynvoet, 2013). This suggests that they rely, at least partially, on different
abilities. On the other side, the Symbolic Number Line and the Symbolic Magnitude Comparison share the similarity of being unsatisfactorily accounted by the traditional dichotomy between symbolic and non-symbolic numerical skills. Symbolic numerical skills refer to one’s ability to manipulate number words and symbols, while non-symbolic numerical skills denote the ability to process non-symbolic quantities (Kolkman, Kroesbergen, & Leseman, 2013). Using confirmatory factor analysis to investigate the structure of children’s early numerical skills, both the Symbolic Number Line and the Symbolic Magnitude Comparison loaded on a separate factor, named mapping skills, distinct from other tasks loading on the symbolic and non-symbolic factors (Kolkman, Kroesbergen, & Leseman, 2013). The authors also showed that only this third factor based on the numerical magnitude skills positively predicted children’s mathematic achievement at 6 years. Friso-van den Bos, Kroesbergen, and van Luit (2014) found a similar structure in kindergartners: the scores of the Symbolic Number Line and of the Symbolic Magnitude Comparison loaded on both symbolic and non-symbolic factors. Thus, despite their fundamentally different characteristics, both tasks seem to measure a shared construct. Since both tasks demand a certain access to the magnitude of numbers, the numerical magnitude skills might rely on both symbolic and non-symbolic numerical processing, or also combine the symbolic and non-symbolic skills.

1.3 The Role of Executive Function in Numerical Magnitude Skills

Few studies have investigated the role of components of executive function in numerical magnitude skills. Although a positive correlation between children’s updating and their numerical magnitude skills has previously been documented in the literature (Geary et al., 2008; Kolkman, Kroesbergen, & Leseman, 2014; Krajewski & Schneider, 2009), only three recent studies compared the specific contributions of all three components of executive function. In the first one, Kolkman et al. (2013) explored in a sample of 47 kindergartners the role of verbal updating, inhibition and switching in three numerical magnitude tasks: the Symbolic Number Line, the Symbolic Magnitude Comparison and a categorization task, where children had to classify each number into one of five categories ranging from “really small” to “really big”. They also examined the relation between the components of executive function and the effect of a 6-session training that targeted the numerical magnitude skills. As expected, the authors found that children’s verbal updating was more strongly associated with each numerical task than with inhibition and switching. Moreover, the higher the children scored on the verbal updating task, the more they improved their estimations on the number line during the training. As for inhibition and switching, only low to moderate correlations were found between updating and the Symbolic Number Line and the Symbolic Magnitude Comparison. Since these were Bayesian analyses, allowing to contrasts the various correlations of the three components of executive function but not quantify each specific relation, the specific effects remain unclear.

A second study examined the correlations of three components of executive function, calculation, Symbolic Number Line and Non-Symbolic Magnitude Comparison in a sample of prep school children and children in third grade (White, Berthelsen, Walker, & Williams, 2015). The correlations differed between the two grades, in prep school only working memory was related to calculation, whereas in third grade inhibition and switching were both correlated with calculation and Magnitude Comparison. No relations were found for components of executive function and the number line estimates.

The third study examined the effects of the components of executive function and counting skills on the shape of their number line estimates in children aged 4-8 years (Friso-van den Bos, Kolkman, Kroesbergen, & Leseman, 2014). Two number lines ranging from 0 to 10 and from 0 to 100, as well as two updating measures (a verbal and visuo-spatial updating task) were used. Regarding the number line ranging from 0 to 10, results suggested a unique contribution of inhibition predicting 19.7 % of the variance in the linearity of children’s number line estimates. Regarding the number line ranging from 0 to 100, results pointed to a dominant role of visuo-spatial updating and a marginal role of children’s counting skills. The two variables together explained 31.6 % of the variance in the linearity of children’s estimates. This last study provides first empirical evidence that the effects of components of executive function on the numerical magnitude skills might depend on the range of the number line and probably on children’s stage in learning the linearity of numbers. In their sample, most children reported linear placements on the shorter number line. They might thus have used their inhibitory ability in this task to restrain previous less mature strategies in favor of a more accurate linear placement. However, participants’ shift to linear estimates had not yet occurred on the large-scale number line. Here, the authors assumed that the substantial contribution of visuo-spatial updating reflects the high demand of the task on visuo-spatial skills. In order to choose an optimal position of a quantity on the number line, participants have not only to remember the scale of the number line, they might also mentally place other numbers as cues. For example, when trying to place the number 63, it may help to place mentally the number 50 at the center of the line (Friso-van den Bos et
A second important finding of this study was that when considering updating abilities in two different modalities (verbal and visuo-spatial), only the visuo-spatial updating predicted number line estimates. But to date, no study has evaluated whether this finding generalizes to other numerical magnitude skills.

1.4 Aims and Hypotheses

The main purposes of the current study were (1) to confirm the existing results regarding the role of components of executive function in the linearity of 6-years-old children’s number line estimates and (2) to explore the extent to which these results generalize to a second numerical magnitude skill, specifically, to the ability to compare the magnitude of numbers. In this regard, a first aim was to test the specific contributions of each component of executive function to the shape of children’s placements on a Symbolic Number Line task ranging from 0 to 100. Following Friso-van den Bos et al. (2014), we hypothesized that inter-individual differences in visuo-spatial updating are positively associated with the linearity of number line estimates, so that children with superior visuo-spatial updating skills would show more linear estimates. Regarding other components of executive function, we expected no specific contribution of verbal updating, inhibition and switching to the linearity of children’s number line estimates. Our second aim was to examine the specific influence of components of executive function on children’s capacity to compare magnitudes of numbers. We expected that inter-individual differences in visuo-spatial updating would be positively associated with participants’ accuracy on the Symbolic Magnitude Comparison. We further hypothesized that inhibition and switching would not substantially predict children’s accuracy on this task.

In order to discriminate the effects due to the use of numbers from the ones due to more general task demands, we also tested the effects of components of executive function on non-symbolic versions of the two tasks described above. In these versions, participants place or compare arrays of dots varying from 1 to 100 units, rather than numbers. These non-symbolic versions of the Number Line and the Magnitude Comparison tasks were shown to be less directly associated with mathematic achievement (Kolkman, Kroesbergen, & Leseman, 2013; Sasanguie, Gobel, Moll, Smets, & Reynvoet, 2013). The comparison of results obtained with the symbolic and the non-symbolic versions should help us to explore the role of different components of executive function in numerical magnitude skills. We assumed that if a component of executive function predicts both versions of a task significantly, the reasons of these associations are to be found in the task demand rather than in children’s use and understanding of numbers. In contrast, if a component of executive function predicts only individual differences in the symbolic version of a task, the reasons of this association are to be found in children’s use and understanding of numbers rather than in the task demands.

2. Method

2.1 Sample

The sample consisted of 162 kindergartners (51.9 % girls and 48.1 % boys) with a mean age of 6.45 years (SD = 0.37). They were recruited in 14 public kindergartens situated in different urban and rural areas of two German-speaking cantons of Switzerland. Concerning the language spoken at home 80.1 % of the children spoke Swiss German or German with at least one parent, 8.8 % spoke a foreign language with both parents (11.1% missing information). All participants were sufficiently fluent in German to understand task instructions. The participants came primarily from middle class families and the diversity in terms of parental education was representative for Switzerland.

2.2 Procedures

The research project was approved by the ethics review board of the University of Bern and written informed consent was obtained from children’s parents prior to study begin. The tests were completed during one session lasting about 45 minutes. Participants completed all tasks included in the present study individually on a laptop computer either in a separate room or in a quiet place of the classroom. The tests were conducted in early school lessons in the morning to make sure that children were not hungry or tired. The order of the various tasks was counterbalanced. A trained experimenter gave the instructions verbally for each task and was present to answer participants’ questions.
2.3 Measures

2.3.1 Inhibition
An adapted version of the Flanker Task (Eriksen & Eriksen, 1974) was used to measure inhibition. In each trial, a row of five identical red fish was presented on the screen. The fish presented in the centre appeared either looking in the same direction (congruent condition) or in the opposite direction (incongruent condition) as the four flanking fish. Participants were instructed to feed the central fish by responding to its orientation irrespective of the orientation of the flanking fish. They were instructed to press on the right response button when the central fish pointed to the right, and on the left response button when the central fish pointed to the left. Participants completed first a practice session of 6 trials, followed by two experimental blocks of 24 trials each. After each experimental block, children were given a mock and positive feedback about their performance. Because incongruent flanking stimuli increased inhibitory demand, percentage of accurate responses in the incongruent trials was taken as a measure of a child’s inhibition efficiency (for a more detailed description of the task see Roebers, Schmid, & Roderer, 2010).

2.3.2 Switching
The assessment of switching was realized directly after the inhibition task and was added to the modified version of the Flanker Task (Diamond, Barnett, Thomas, & Munro, 2007; Röthlisberger, Neuenschwander, Cimeli, Michel, & Roebers, 2012). It consisted of two supplementary blocks following the same structure as the inhibition task with a practice session at the beginning and a feedback at the end of the experimental session. In the former block, participants were presented a row of five yellow fish similar to the ones in the inhibition task. In this switching task, children were instructed to feed the four flanking fish by responding to their orientation, irrespective of the orientation of the central fish. This first block aimed at familiarizing participants with this new and reversed rule. In the second block, rows yellow or red fish were randomly presented. Children were asked to respond to the orientation of the central fish in the case of red fish and to the orientation of the four flanking fish in the case of yellow fish. This block consisted of 8 practice trials and 40 test trials. Because accuracy in this last block depended on child’s ability to flexibly switch between the two rules, percentage of accurate responses in the incongruent trials was used as an indicator for switching ability.

2.3.3 Updating
A Backward Colour Span task (Zoelch, Seitz, & Schumann-Hengsteler, 2005) was used to assess verbal updating. This adaptation of the Digit Span Backward task was preferred in order to use a different material than in the numerical tasks. In each trial, participants saw a sequence of coloured buttons appearing successively on the screen. They were instructed to remember the colours presented and, at the end of the sequence, to recall them in the reversed order. Prior to the experimental session, children were asked to name the different colours used in the task to check for colour recognition. All colour names were monosyllabic. Children first completed a practice session of 3 trials. The experimental session started with 6 trials of two colours. The level of the task was then increased of one supplementary colour each time that the child solved at least 4 of the 6 trials correctly; otherwise, the task was interrupted. The number of correctly solved trials was used as an indicator for a child’s verbal updating.

An adaptation of the subtest Matrix from the Working Memory Test Battery for Children (WMTB-C; Pickering & Gathercole, 2001) was used to measure visuo-spatial updating. Children were presented a four by four matrix on the screen. During each trial, some squares of the matrix were successively coloured in black. Participants were asked to remember which squares became black and, then, to retrieve the sequence in the reversed order on an identical printed matrix placed in front of the laptop. Before starting the experimental session, two examples of trials were shown, then children and completed 3 practice trials under the supervision of an experimenter. The experimental session followed the same procedure as for the Backward Colour Span Task. It started with a span of two squares and the level was increased each time if at least 4 of the 6 trials were recalled correctly. The number of accurately solved trials was taken as score for child’s visuo-spatial updating.

2.3.4 Number Line
A computerized version of the Symbolic Number Line task (Friso-van den Bos et al., 2014; Siegler & Opfer, 2003) was used. In a first step, an empty horizontal number line was presented on the screen and participants were asked to place the numbers 1 and 100 at the left and right endpoints of the line. The two endpoints were then labelled for the rest of the task. In the following 22 test trials, a number appeared in a square situated above the number line. Children were instructed to place the presented number on the line. A linear fit score was then
calculated for each participant indicating the extent to which their estimations fitted a linear function. A linear fit score ranges from 0 (no fit at all) to 1 (exactly linear estimation). Internal consistency for this task had previously been reported to be good ($\alpha = .79$; Kolkman, Kroesbergen, & Leseman, 2013).

The non-symbolic version of the number line estimation task used dots instead of numbers. The presented magnitudes were identical to the ones of the symbolic version. To help children’s comprehension of the task, the dots represented gasoline drops and a car was positioned at the left endpoint of the line. Children were instructed that the car, when receiving a small number of drops (one drop of fuel), would drive to the left endpoint of the line and, when receiving many drops (100 drops), would drive to the right endpoint of the line. Both endpoints were then labelled with the corresponding number of drops. In all following test trials, a pattern of dots appeared in a square situated in the middle of the screen above the line and children were asked to estimate how far the car would drive with the presented number of drops. Similar to the symbolic version of the task, linear fit scores were computed. Kolkman, Kroesbergen, and Leseman (2013) reported a good internal consistency for this task ($\alpha = .73$).

2.3.5 Magnitude Comparison

The task was adopted from Kolkman, Kroesbergen, and Leseman (2013). In each trial, two numbers ranging between 1 and 100 were presented, and children were instructed to press the response button situated on the side of the highest number (left vs. right). Before children started the task, 6 practice trials with direct feedback on response accuracy were completed. The task’s score consisted in a participant’s percentage of correct responses in 21 test trials. A good internal consistency ($\alpha = .84$) had previously been found for this task (Kolkman, Kroesbergen, & Leseman, 2013).

In the non-symbolic version of the magnitude comparison task, children were presented two arrays of dots and were asked to press the response button situated on the side of the array containing more dots. To control for response bias due to the surface covered by the stimuli, the three conditions applied in Kolkman, Kroesbergen, and Leseman (2013) were here reproduced: (1) the covered surface was higher in the array containing less dots, (2) the covered surface was lower in the array containing less dots, (3) the covered surface were identical in both arrays. Participants first completed 6 practice trials with feedback on response accuracy followed by 85 test trials. The dependent variable was the percentage of correct responses during the test session. Kolkman, Kroesbergen, and Leseman reported a good internal consistency for this task ($\alpha = .84$).

3. Results

3.1 Statistics and Data Analysis

Due to schools’ time constraints, some participants were not able to complete all the study tasks. Four participants were thus excluded from the statistical analyses because of missing data on a predictor or dependent variable. Four supplementary participants were identified as multivariate outliers. These cases showed a Mahalanobis’ distance greater than the cut-off point proposed by Tabachnick and Fidell (2014) consisting in chi-square critical value with $df = k$ (number of included predictors) and $p < .001$ (in this case: $\chi^2(4) = 18.47$). All statistical analyses were carried out with and without these outliers and the four participants were consequently excluded, because their extreme values changed significantly the results of the analyses.

To test our hypotheses, four linear regression analyses were conducted. Because some of our regression models did not fully respect the assumption of normally distributed errors, we computed new confidence intervals based on $B = 2000$ bootstrapped samples with the Bias Corrected accelerated method (BCa; see Carpenter & Bithell, 2000). Bootstrap-based confidence intervals are assumed to be more accurate than standard intervals in case of distributions not fully respecting parametrical assumptions (DiCiccio & Efron, 1996). However, the results obtained with bootstrap intervals did not differ from the ones with standard confidence intervals. Because generalization did not seem to be affected by our distributions of errors, we consequently decided to present the results produced with the standard confidence intervals.

3.2 Descriptive Statistics

Table 1 displays the means, standard deviations and ranges of all variables. The distribution of the inhibition measure showed a negative skewness of -2.12 (SE = .20) and a very high kurtosis of 4.49 (SE = .39) due to a strong ceiling effect. Indeed, 64.9% of the participants solved all the incongruent trials with one or no mistake at all. The distributions of the two non-symbolic magnitude tasks showed also high kurtosis of 2.02 (SE = .39) for the Non-symbolic Number Line and of 2.26 (SE = .39) for the Non-Symbolic Magnitude Comparison. The skewness and kurtosis of all the other variables did not deviate significantly from the normal distribution.
Table 1. Descriptive statistics (mean, standard deviation and range)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inhibition (%)</td>
<td>89.27</td>
<td>14.46</td>
<td>29-100</td>
</tr>
<tr>
<td>Switching (%)</td>
<td>84.57</td>
<td>12.09</td>
<td>41-100</td>
</tr>
<tr>
<td>Verbal Updating</td>
<td>9.31</td>
<td>4.16</td>
<td>0-18</td>
</tr>
<tr>
<td>Visuo-spatial Updating</td>
<td>4.95</td>
<td>3.62</td>
<td>0-14</td>
</tr>
<tr>
<td>Number Line S</td>
<td>0.45</td>
<td>0.28</td>
<td>.00-.97</td>
</tr>
<tr>
<td>Number Line NS</td>
<td>0.68</td>
<td>0.19</td>
<td>.00-.95</td>
</tr>
<tr>
<td>Magnitude Comparison S (%)</td>
<td>68.16</td>
<td>17.74</td>
<td>37-100</td>
</tr>
<tr>
<td>Magnitude Comparison NS (%)</td>
<td>64.90</td>
<td>7.81</td>
<td>31-83</td>
</tr>
</tbody>
</table>

*Note.* S = Symbolic; NS = Non-symbolic.

Median estimates were computed with the placements of all children on the Symbolic Number Line. These were then fitted to a linear and to a logarithmic function in order to compare the data found in our sample with the ones previously reported in the literature. The median estimates as well as their linear and logarithmic fit scores are shown in Figure 1. The linear fit ($R^2 = .82$) of the median estimates was descriptively lower than the logarithmic fit ($R^2 = .96$), but the difference between the two estimates was not significant, t(21) = -.90, p > .05. Our findings were comparable to the median estimates of 6-year-old children previously found in the literature. Reported linear fit scores ranged between .69-.91 and logarithmic fit scores ranged between .87-.95 (Friso-van den Bos, Kolkman et al., 2014; Laski & Siegler, 2007; Siegler & Booth, 2004).
3.3 Correlations

The correlations between all variables are presented in Table 2. Pearson correlations are displayed below the diagonal, while partial correlations controlling for age are presented above the diagonal. The minimal changes in magnitude between the two types of correlations indicate that those associations were not due to differences in age. The four tasks for the different components of executive function exhibited low to moderate positive correlations to each other. Inhibition and switching showed the highest correlation sharing 23% of common variance, while inhibition and visuo-spatial updating did not significantly correlate at all. Interestingly, the two different updating variables correlated only moderately, sharing 7.3% of common variance. Another interesting pattern emerged in the associations between the numerical tasks. The highest positive correlation was exhibited by the two symbolic tasks that shared 23% of their variance. The two non-symbolic tasks correlated also positively but to a lower extent, sharing only 5.3% of common variance. The Non-symbolic Magnitude Comparison task showed substantially lower positive correlations to the different components of executive function than the other numerical tasks.

Table 2. Pearson correlations and partial correlations controlling for age

<table>
<thead>
<tr>
<th>Measure</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Inhibition</td>
<td></td>
<td>.51</td>
<td>.34</td>
<td>.16</td>
<td>.28</td>
<td>.21</td>
<td>.31</td>
<td>.13</td>
</tr>
<tr>
<td>2. Switching</td>
<td>.48</td>
<td></td>
<td>.18</td>
<td>.25</td>
<td>.34</td>
<td>.24</td>
<td>.25</td>
<td>.17</td>
</tr>
<tr>
<td>3. Verbal Updating</td>
<td>.33</td>
<td>.18</td>
<td></td>
<td>.26</td>
<td>.35</td>
<td>.30</td>
<td>.21</td>
<td>.01</td>
</tr>
<tr>
<td>4. Visuo-spatial Updating</td>
<td>.16</td>
<td>.25</td>
<td>.27</td>
<td></td>
<td>.39</td>
<td>.21</td>
<td>.24</td>
<td>.15</td>
</tr>
<tr>
<td>5. Number Line S</td>
<td>.27</td>
<td>.35</td>
<td>.35</td>
<td>.41</td>
<td></td>
<td>.37</td>
<td>.47</td>
<td>.21</td>
</tr>
<tr>
<td>6. Number Line NS</td>
<td>.19</td>
<td>.22</td>
<td>.31</td>
<td>.22</td>
<td>.37</td>
<td></td>
<td>.38</td>
<td>.26</td>
</tr>
<tr>
<td>8. Magnitude Comparison NS</td>
<td>.13</td>
<td>.15</td>
<td>.00</td>
<td>.12</td>
<td>.17</td>
<td>.23</td>
<td>.12</td>
<td></td>
</tr>
</tbody>
</table>

Note. Pearson correlations are displayed below the diagonal; partial correlations are displayed above the diagonal. S = Symbolic; NS = Non-symbolic. Correlations higher than .17 are significant at p < .05; correlations higher than .21 are significant at p < .01; correlations higher than .27 are significant at p < .001.

3.4 Predictions of the Symbolic Numerical Tasks

To test our hypotheses that inter-individual differences in visuo-spatial updating would positively predict the linearity of children’s estimates on the Symbolic Number Line while the other components of executive function do not contribute to this prediction, we carried out a linear regression with children’s linear fit scores as dependent variable by entering the four tasks measuring the components of executive function as predictors simultaneously. The results of this analysis are displayed in Table 3.

The prediction was highly significant (F(4,149) = 14.52, p < .001) explaining 28.1% of the variance in children’s linear fit score. As expected, visuo-spatial updating showed the strongest contribution by explaining 7.1% of the variance. Contrary to our hypotheses, verbal updating and switching predicted also significantly the linearity of children’s placements. Verbal updating explained additional 4% of the variance. Similarly, switching explained 3.4% of the variance. Only inhibition did not contribute significantly to this prediction.

A second linear regression analysis was then conducted on the Symbolic Magnitude Comparison to examine the specific associations of the components of executive function with children’s ability to compare the magnitude of numbers. Again, we expected that inter-individual differences in visuo-spatial updating would positively predict children’s accuracy on the Symbolic Magnitude Comparison and that the other components of executive function would not show significant effects. Therefore, we entered the four tasks measuring three components of executive function as predictors and children’s accuracy on the Symbolic Magnitude Comparison as dependent variable in the regression analysis. The results of this analysis are also presented in Table 3.
Again, the prediction was highly significant ($F(4,149) = 6.69, p < .001$) and explained 15.2% of the variance in children’s accuracy on the Symbolic Magnitude Comparison. The highest specific association was found again concerning visuo-spatial updating that explained 2.9% of the variance. Inhibition was an additional significant predictor explaining 2.7% of the variance. Switching and verbal updating did not significantly contribute in this analysis.

### Table 3. Linear regression analyses predicting children’s score on the symbolic number line and on the symbolic magnitude comparison

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Symbolic Number Line</th>
<th>Symbolic Magnitude Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B$</td>
<td>$r^2$</td>
</tr>
<tr>
<td>Inhibition</td>
<td>.017</td>
<td>.002</td>
</tr>
<tr>
<td>Switching</td>
<td>.066**</td>
<td>.034</td>
</tr>
<tr>
<td>Verbal Updating</td>
<td>.062**</td>
<td>.040</td>
</tr>
<tr>
<td>Visuo-spatial Updating</td>
<td>.080***</td>
<td>.071</td>
</tr>
<tr>
<td>Total $R^2$</td>
<td>.281***</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* *p* < .05; **p* < .01; ***p* < .001.

### 3.5 Predictions of the Non-Symbolic Numerical Tasks

To try to differentiate the effects of the components of executive function related to the use of numbers from the ones related to general task demands, we carried out explorative regression analyses on the non-symbolic versions of the Number Line and Magnitude Comparison tasks.

In both analyses, the same predictor variables were simultaneously entered in the model as was done before for the symbolic numerical tasks. The results of these analyses are presented in Table 4.

Regarding the Non-symbolic Number Line, the prediction was highly significant ($F (4,149) = 5.76, p < .001$) and explained 13.4% of the variance in children’s linear fit scores. Verbal updating was, however, the only significant predictor and explained specifically 5% of the variance.

In contrast to all previous analyses, the prediction of the Non-symbolic Magnitude Comparison was not significant ($F(4,149) = 1.45, p > .05$). None of the components of executive function could significantly predict participants’ ability to compare non-symbolic magnitudes.

### Table 4. Linear regression analyses predicting children’s score on the non-symbolic number line and on the non-symbolic magnitude comparison

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Non-Symbolic Number Line</th>
<th>Non-Symbolic Magnitude Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B$</td>
<td>$r^2$</td>
</tr>
<tr>
<td>Inhibition</td>
<td>.006</td>
<td>.000</td>
</tr>
<tr>
<td>Switching</td>
<td>.028</td>
<td>.013</td>
</tr>
<tr>
<td>Verbal Updating</td>
<td>.046**</td>
<td>.050</td>
</tr>
<tr>
<td>Visuo-spatial Updating</td>
<td>.021</td>
<td>.011</td>
</tr>
<tr>
<td>Total $R^2$</td>
<td>.134***</td>
<td></td>
</tr>
</tbody>
</table>

*Note.* **p* < .01; ***p* < .001.
4. Discussion

The present study investigated the associations between the three components of executive function and children’s numerical magnitude skills using two different numerical tasks: the Symbolic Number Line and the Symbolic Magnitude Comparison. According to previous findings, we hypothesized that children with higher visuo-spatial updating would perform better on these two numerical tasks. Regarding inhibition, switching and verbal updating, we did not expect other specific relations with the numerical tasks. In order to help the interpretation of our results, we further analysed associations of components of executive function with non-symbolic versions of the two numerical tasks. Symbolic and non-symbolic versions only differed in the modality in which the magnitudes were presented (numbers vs. arrays of dots). Therefore, we assumed that if a factor of executive function predicted only the symbol versions of the tasks, the reasons of its contribution were to be found in children’s use and understanding of numbers.

Our expectations that inter-individual differences in visuo-spatial updating are substantially linked with children’s numerical magnitude skills were met. Of all components of executive function, visuo-spatial updating showed the strongest associations with both symbolic numerical tasks. Moreover, visuo-spatial updating was the only variable predicting individual differences in both tasks. Thus, our results corroborated previous findings underlining the important role of updating in numerical magnitude skills (Kolkman et al., 2013).

However, contrary to our hypotheses, the other components of executive function also showed significant associations to the symbolic numerical tasks. Verbal updating and switching were found to be predictive of the linearity of children’s estimates on the Symbolic Number Line, while inhibition predicted significantly participants’ accuracy on the Symbolic Magnitude Comparison. Our findings did not only support the dominant role of visuo-spatial updating which can be found in the literature, they also revealed meaningful associations of the other components of executive function with numerical skills.

Concerning the analyses of the non-symbolic versions of the tasks, verbal updating was the only factor of executive function predicting the Non-symbolic Number Line, while no factor of executive function could predict children’s performance on the Non-symbolic Magnitude Comparison. Comparing the results obtained with both versions of each task we are tempted to make the following suggestions: (1) the associations of visuo-spatial updating with both numerical tasks seem to depend on the use of numbers; (2) the Number Line is associated with verbal updating irrespectively of the magnitudes modality; (3) the relations of inhibition and switching with the numerical magnitude skills appear less consistent; (4) participants’ ability to compare non-symbolic magnitudes seems to rely on other processes than executive functioning. These four suggestions are discussed more in detail in the following paragraphs.

Of all the components of executive function, visuo-spatial updating was the only one that consistently predicted both symbolic numerical tasks. Moreover, inter-individual differences in visuo-spatial updating were not associated with participants’ performance anymore once the magnitudes were presented as arrays of dots instead of numbers. We therefore suggest that visuo-spatial updating is involved in numerical magnitude skills, because children need to process the magnitude of numbers. According to Dehaene’s (1992) Triple-Code Model, numbers are stored in three different formats: a verbal representation (“fifty-two”), an Arabic representation (“52”), and an analogue representation of numbers’ magnitude. Dehaene conceptualized the latter, the analogue representation, as a mental number line on which abstract magnitudes are spatially distributed in an increasing order. In his model, the retrieval of this analogue representation consists in the diffuse activation of a magnitude on the mental number line. In the past two decades, a large amount of empirical evidence at the behavioural and neuropsychological levels supported the existence of an analogue numbers’ representation and its spatial organization in the form of a mental number line (for a review see de Hevia, Vallar, & Girelli, 2008).

In our study, the participants were supposed to retrieve the analogue representation of numbers in order to place them correctly on the Symbolic Number Line or to compare their magnitudes. We assume that the access to this spatially organized analogue representation rely substantially on children’s visuo-spatial updating. Several studies already emphasized the importance of visuo-spatial skills in numerical magnitude skills (Bachot, Gevers, Fias, & Roeyers, 2005; Friso-van den Bos et al., 2014; Krajewski & Schneider, 2009) and in early math achievement (e.g., De Smedt et al., 2009). Moreover, other studies revealed that, while the processing of numbers in older children and adults prompted the activation of their magnitudes automatically, the access to the analogue representation in younger children did not occur in an automatic way (Girelli, Lucangeli, & Butterworth, 2000; Rubin, Henik, Berger, & Shahar-Shalev, 2002). It is likely that the retrieval of the analogue representation of numbers in younger children relies on executive functioning and that visuo-spatial
Consistent with this explanation, Unsworth and Engle (2007) postulated that participants’ performance on updating tasks results from their efficiency in two kinds of processes: attentional processes, engaged in maintaining representations active in a primary memory, and retrieval processes, engaged in searching and recalling information stored in a secondary memory. Thus, participants higher on visuo-spatial updating might more efficiently access their representations of magnitude and better succeed to maintain these representations vivid, while comparing numbers or placing them on a horizontal line. Thus, children showing better visuo-spatial updating processes have an advantage in performing the Symbolic Number Line or the Symbolic Magnitude Comparison.

Inversely, children having difficulties to manipulate numbers’ magnitude might benefit from training programs targeting the accessibility of their spatial representation of magnitude. For example, Kucian et al. (2011) developed a computer-based training program that was argued to help automatization of the processing of numbers’ magnitude in children aged 8-10 years. After the training, participants’ performance was improved and they seemed to rely less on executive functioning to solve the Symbolic Number Line, as shown by a reduction of their brain activation in the frontal areas. Because of the association between the numerical magnitude skills and later mathematical achievement, it might be important to support children showing deficits in visuo-spatial updating in developing an accurate and easily accessible representation of numbers’ magnitude.

The symbolic and non-symbolic versions of the Number Line were both linked with children’s verbal updating. The relationship between verbal updating and the Symbolic Number Line has already been reported twice, but exclusively from studies operating with a single verbal measure of updating (Geary et al., 2008; Kolkman et al., 2013). Only Friso van-den Bos et al. (2014) previously examined the specific effects of both forms of updating on the Symbolic Number Line and they did not report any influence of participants’ verbal updating in the linearity of their estimates. The authors suggested that when both updating abilities are assessed, only the visuo-spatial measure of updating predicts number line estimates.

A possible explanation of this inconsistency might be that the absence of effect of verbal updating in Friso van-den Bos et al. (2014) was not due to the inclusion of the visuo-spatial measure of updating, but due to other variables that we did not include in our study. Indeed, the authors included a measure of participants’ counting skills that was substantially correlated to the measure of verbal updating, and that probably weakened the influence of the latter in the analyses. Children higher on verbal updating have already been shown to perform better on counting tasks (Noël, 2009). Moreover, counting skills were shown to be associated with the shape of children’s estimates on the Symbolic Number Line (Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008). Therefore, it is possible that verbal updating only indirectly influences the linearity of children’s placements on the Symbolic and Non-symbolic Number Lines in fostering counting skills. Consistently with this explanation, Kolkman, Kroesbergen, and Leseman (2013) found, on the basis of structural equation modeling, that preschoolers’ symbolic numerical skills, when measured with a number recognition task and a counting task, predicted participants’ non-symbolic numerical skills and numerical magnitude skills measured one year later. Further investigations addressing this issue should therefore also include measures of participants’ symbolic numerical skills in order to clarify the role of verbal updating in children’s numerical development.

An intriguing finding, which was not reported in Friso van-den Bos et al. (2014) who had tested a similar population, was the role of switching and inhibition in the numerical magnitude skills. Both components of executive function were associated with only one of the two symbolic numerical tasks and did not contribute to the predictions of the non-symbolic tasks. Interestingly, ambiguous findings regarding the role of switching and inhibition in numerical magnitude skills and in mathematical achievement were already reported (Bull & Lee, 2014; Kolkman et al., 2013). This might be imputable to the specific demands of each symbolic numerical task. The Symbolic Number Line not only demands participants to access the magnitude of an Arabic number, but also demands participants to depict the number on a horizontal line. It is possible that this whole translation from an Arabic format of input to the representation of its magnitude on a concrete support require participants to shift attention between different formats of numbers. In 6-years-old children, this processing might rely, at least partially, on their switching ability. This interpretation also explains why switching did not predict the non-symbolic version of the task, in which the input and output magnitudes are both expressed as physical quantities. Conversely, in the Symbolic Magnitude Comparison, participants need to rapidly compare the magnitude of two numbers and children may apply unsuitable strategies, like selecting a number because of a high digit in the units’ position. Although the task seems trivial for adults, the level of accuracy in our study
points out its complexity for kindergartners. In this sense, better inhibition skills might help children to inhibit maladaptive strategies and increase their accuracy on the task.

Our finding that no factor of executive function predicted children’s ability to compare arrays of dots might indicate that the Non-symbolic Magnitude Comparison relies on more basic processes than executive functioning. Consistently, Dehaene (2001) suggested that humans’ ability to discriminate non-symbolic magnitudes is mostly biologically determined and is also partly present in infants and animals. Indeed, a whole body of research provides support that animals are able to discriminate small and large quantities above chance level even after controlling for non-numerical parameters (for a review see Boysen & Capaldi, 1993). Therefore, it is probable that the Non-symbolic Comparison relies largely on innate automatic processes and thus does not require executive functioning.

Regrettably, the cross-sectional nature of the data precludes causal interpretations. The results from this study provide a spotlight on individual differences at one given time in development. Therefore, more longitudinal studies are needed to clarify the causal processes linking executive functioning, the numerical magnitude skills and later mathematic achievement and to illuminate the course of these associations along a child’s development. It is probable that the associations between executive functioning and the numerical magnitude skills change as children’s retrieval of numbers’ magnitude gets more automatic. Indeed, older children and adults were shown to access their analogue representation of numbers automatically (Girelli et al., 2000; Rubinsten et al., 2002). Moreover, several studies suggested that more automatic skills rely less on executive functioning (e.g., Altemeier, Jones, Abbott, & Berninger, 2006). Therefore, longitudinal studies would not only allow drawing stronger conclusions about the causal relations between executive functioning and the numerical magnitude skills, they would also help to clarify how these associations evolve along children’s numerical development.

The present study went beyond previous research by investigating the associations of various components of executive function with both symbolic and non-symbolic versions of the Number Line and the Magnitude Comparison. In this way, we aimed at differentiating the associations that occurred specifically when using numbers from the ones due to general task demand. Our results support previous researchers suggesting that the processing of non-symbolic quantities relies on a separate cognitive system, which might be mostly supported by biologically determined processes (Dehaene, 2001). Another key finding of this study was that children with better visuo-spatial updating skills performed better on a symbolic version of the Number Line and that this finding extends to other numerical magnitude skills, namely the symbolic Magnitude Comparison. This result supports previous findings showing that the retrieval of numbers’ magnitude relies on visuo-spatial processes (e.g., de Hevia et al., 2008). Moreover, it also suggests that executive functioning plays a substantial role in children’s early numerical development. The present study emphasizes the importance of an accurate and easily accessible representation of numbers’ magnitude—a skill that could be supported in case of mathematical difficulties.

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