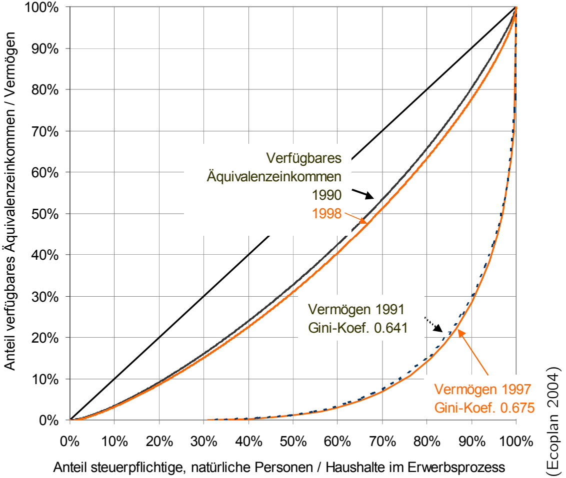
A new Stata command for computing and graphing percentile shares

Ben Jann

University of Bern, ben.jann@soz.unibe.ch

2015 UK Stata Users Group meeting
London, September 10–11, 2015


HAPPY
BIRTHDAY



- ▷ http://www.youtube.com/watch?v=slTF_XXoKAQ
- ▷ https://www.ted.com/talks/dan_ariely_how_equal_do_we_want_the_world_to_be_you_d_be_surprised

Outline

- Motivation
- Estimation of percentile shares
- The `pshare` command
- Examples using Bern tax data
- Small sample bias

Estimation of percentile shares

- Outcome variable of interest, e.g. income: Y
- Distribution function: $F(y) = \Pr\{Y \leq y\}$
- Quantile function: $Q(p) = F^{-1}(p) = \inf\{y | F(y) \geq p\}$, $p \in [0, 1]$
- Lorenz ordinates:

$$L(p) = \int_{-\infty}^{Q_p} y \, dF(y) \bigg/ \int_{-\infty}^{\infty} y \, dF(y)$$

- Finite population form:

$$L(p) = \sum_{i=1}^{N} Y_i \mathcal{I}\{Y_i \leq Q_p\} \bigg/ \sum_{i=1}^{N} Y_i$$

Estimation of percentile shares

- Percentile share: proportion of total outcome within quantile interval $[Q_{p_{\ell-1}}, Q_{p_\ell}]$, $p_{\ell-1} \leq p_\ell$

$$S_\ell = L(p_\ell) - L(p_{\ell-1})$$

- Percentile share "density":

$$D_\ell = \frac{S_\ell}{p_\ell - p_{\ell-1}} = \frac{L(p_\ell) - L(p_{\ell-1})}{p_\ell - p_{\ell-1}}$$

- Totals:

$$T_\ell = \sum_{i=1}^{N} Y_i \mathcal{I}\{Q_{p_{\ell-1}} < Y_i \leq Q_{p_\ell}\} = S_\ell \cdot \sum_{i=1}^{N} Y_i$$

- Averages:

$$A_\ell = \frac{T_\ell}{(p_\ell - p_{\ell-1}) \cdot N}$$

Estimation of percentile shares

- Estimation given sample of size n:

$$\widehat{S}_\ell = \widehat{L}(p_\ell) - \widehat{L}(p_{\ell-1})$$

$$\widehat{L}(p) = (1-\gamma)\widetilde{Y}_{j-1} + \gamma\widetilde{Y}_j \quad \text{where } \widehat{p}_{j-1} < p \leq \widehat{p}_j \text{ with } \widehat{p}_j = \frac{j}{n}$$

$$\widetilde{Y}_j = \sum_{i=1}^{j} Y_{(i)} \Bigg/ \sum_{i=1}^{n} Y_i \qquad \text{where } Y_{(i)} \text{ refers to ordered values}$$

$$\gamma = \frac{p - \widehat{p}_{j-1}}{\widehat{p}_j - \widehat{p}_{j-1}} \qquad \text{(linear interpolation)}$$

- Standard errors
  - using estimating equations approach as proposed by Binder and Kovacevic (1995)
  - supports complex survey data

The `pshare` command

- `pshare estimate`
  - estimates the percentile shares and their variance matrix
  - arbitrary cutoffs for the percentile groups
  - joint estimation across multiple outcome variables or subpopulations
  - shares as proportions, densities, totals, or averages
  - etc.
- `pshare contrast`
  - computes contrasts between outcome variables or subpopulations
  - differences, ratios, or log ratios
- `pshare stack`
  - displays percentile shares as stacked bar chart
- `pshare histogram`
  - displays percentile shares as histogram

Examples

- Tax data from canton of Bern, Switzerland, 2012
- individual level data from personal tax forms, 20% sample
- information on income components, deductions, assets, etc.

Examples

```
. use taxdata
(Some tax data)

. describe

Contains data from taxdata.dta
  obs:       119,939                          Some tax data
 vars:             3                          27 Jun 2015 23:49
 size:     1,079,451
--------------------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
--------------------------------------------------------------------------------
agecat          byte    %9.0g      agecat     Age group
income          float   %9.0g                 Total income
wealth          float   %9.0g                 Net wealth
--------------------------------------------------------------------------------
Sorted by:

. help pshare
```

Quintile shares (the default)

```
. pshare estimate income
Percentile shares (proportion)                  Number of obs     =    119,939
```

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
0-20	.029269	.0002685	109.02	0.000	.0287428	.0297952
20-40	.1048592	.0004634	226.30	0.000	.103951	.1057674
40-60	.1645584	.0006001	274.24	0.000	.1633823	.1657345
60-80	.2365146	.0008311	284.59	0.000	.2348856	.2381435
80-100	.4647989	.0018814	247.05	0.000	.4611113	.4684864

Interpretation: The top 20% percent of the population get 46.5% of all income; the bottom 20% only get 2.9% of all income etc.

Decile shares

```
. pshare estimate income, nquantiles(10)
```

Percentile shares (proportion) Number of obs = 119,939

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
0-10	.0045258	.0000935	48.42	0.000	.0043426	.004709
10-20	.0247432	.0001932	128.07	0.000	.0243645	.0251219
20-30	.0435825	.0002279	191.23	0.000	.0431359	.0440292
30-40	.0612766	.0002527	242.45	0.000	.0607812	.061772
40-50	.0752199	.0002815	267.19	0.000	.0746681	.0757717
50-60	.0893385	.0003267	273.48	0.000	.0886982	.0899788
60-70	.1065221	.0003835	277.74	0.000	.1057704	.1072739
70-80	.1299924	.0004627	280.95	0.000	.1290855	.1308993
80-90	.1654318	.0005818	284.32	0.000	.1642914	.1665722
90-100	.2993671	.0023598	126.86	0.000	.2947419	.3039922

Bottom 50%, Mid 40%, and Top 10%

```
. pshare estimate income wealth, percentiles(50 90)
```

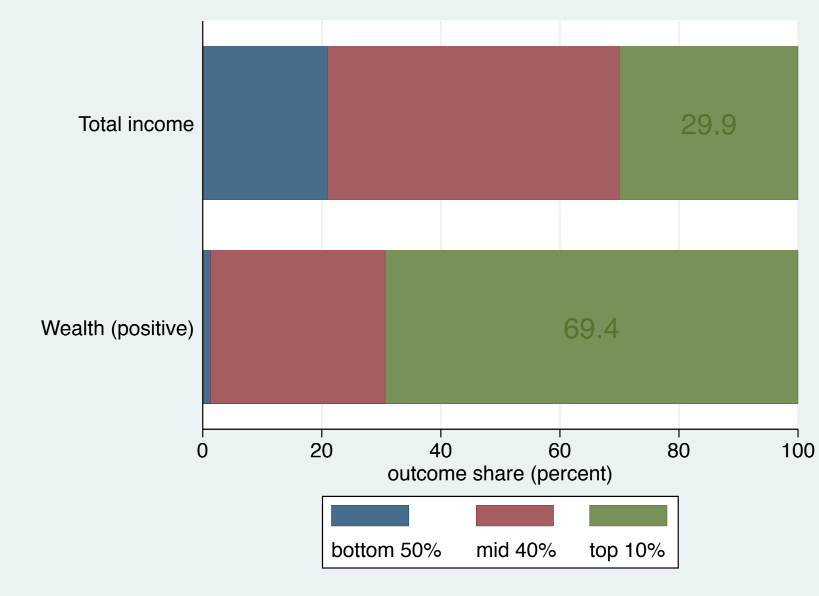
Percentile shares (proportion) Number of obs = 119,939

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
income						
0-50	.2093481	.0008937	234.24	0.000	.2075964	.2110998
50-90	.4912848	.0016618	295.64	0.000	.4880278	.4945419
90-100	.2993671	.0023598	126.86	0.000	.2947419	.3039922
wealth						
0-50	-.0237426	.0010954	-21.67	0.000	-.0258896	-.0215956
50-90	.3042619	.0062104	48.99	0.000	.2920896	.3164343
90-100	.7194807	.0057992	124.07	0.000	.7081143	.730847

Stacked bars plot

```
. generate wealth0 = cond(wealth<0, 0, wealth)

. label variable wealth0 "Wealth (positive)"

. pshare estimate income wealth0, percentiles(50 90) percent

Percentile shares (percent)                  Number of obs     =    119,939

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
income       |
        0-50 |   20.93481   .0893717   234.24   0.000     20.75964    21.10998
       50-90 |   49.12848   .1661772   295.64   0.000     48.80278    49.45419
      90-100 |   29.93671   .2359796   126.86   0.000     29.47419    30.39922
-------------+----------------------------------------------------------------
wealth0      |
        0-50 |   1.314179    .029754    44.17   0.000     1.255862    1.372496
       50-90 |   29.32997   .5773461    50.80   0.000     28.19838    30.46156
      90-100 |   69.35585   .6038125   114.86   0.000     68.17239    70.53931
------------------------------------------------------------------------------

. pshare stack, plabels("bottom 50%" "mid 40%" "top 10%") ///
>     values mlabsize(zero) p3(mlabsize(large))
```

Stacked bars plot



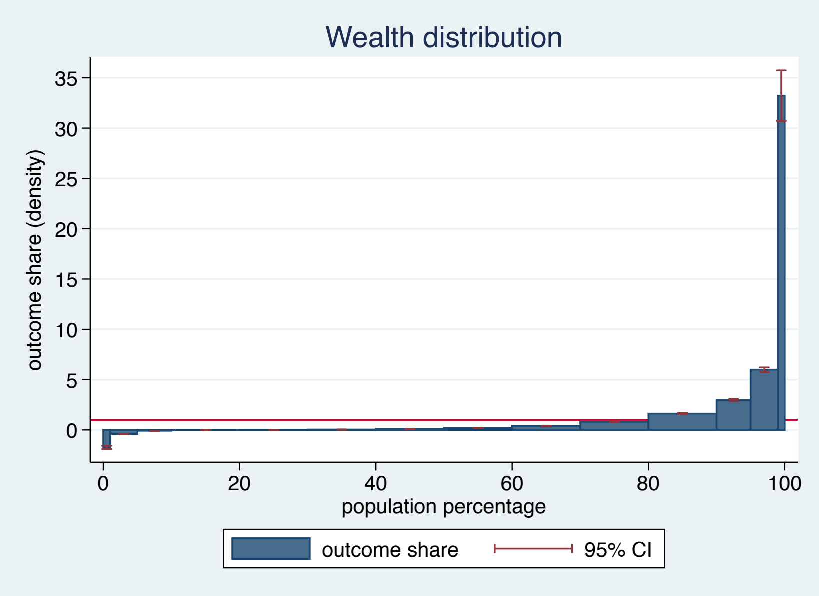
Histogram of densities

```
. pshare estimate wealth, p(1 5 10(10)90 95 99) density
Percentile shares (density)                     Number of obs     =    119,939
```

wealth	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
0-1	-1.740562	.0843581	-20.63	0.000	-1.905903	-1.575222
1-5	-.3963251	.0106253	-37.30	0.000	-.4171505	-.3754998
5-10	-.0789102	.0027628	-28.56	0.000	-.0843253	-.0734951
10-20	-.0017146	.0001861	-9.21	0.000	-.0020794	-.0013497
20-30	.0062028	.0002404	25.80	0.000	.0057316	.0066741
30-40	.0363781	.0008941	40.69	0.000	.0346257	.0381306
40-50	.0937488	.002118	44.26	0.000	.0895975	.0979
50-60	.2023709	.0044304	45.68	0.000	.1936873	.2110544
60-70	.4100251	.0087438	46.89	0.000	.3928874	.4271628
70-80	.811196	.0168879	48.03	0.000	.778096	.8442961
80-90	1.619027	.0328938	49.22	0.000	1.554556	1.683499
90-95	2.951972	.0589668	50.06	0.000	2.836398	3.067546
95-99	5.990715	.114097	52.51	0.000	5.767087	6.214344
99-100	33.22535	1.281602	25.92	0.000	30.71343	35.73727

```
. pshare histogram, yline(1) ylabel(0(5)35, angle(hor)) ti(Wealth distribution)
```

Histogram of densities

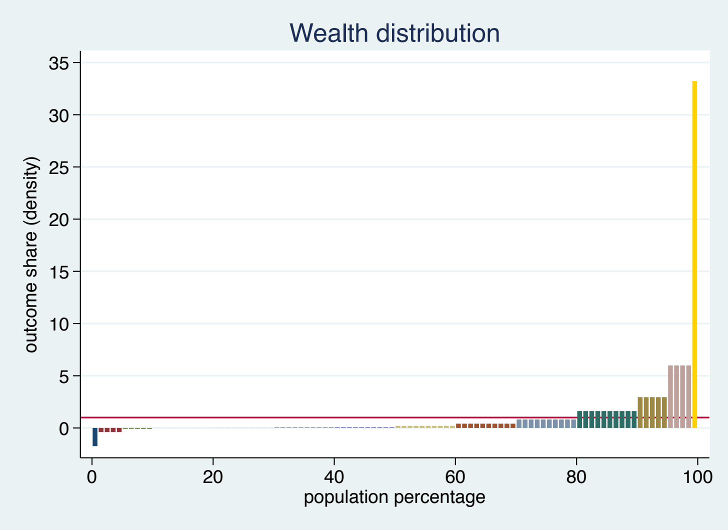


Histogram of densities

- Interpretation
  - Take 100 dollars and divide them among 100 people who line up along the x-axis.
  - The heights of the bars shows you how much each one gets.
  - If all get the same, then everyone would get one dollar (red line).
  - However, according to the observed distribution, the rightmost person (i.e. the richest) would get 33 (!) of the 100 dollars, the next four would get 6 dollar each, and so on.
  - At the bottom, there are also some people you would have to take away some money (e.g., you would have to take away 1.74 dollars from the rightmost person).

Using spikes and group-specific styles

```
. pshare hist, yline(1) ylabel(0(5)35, angle(hor)) ti(Wealth distribution) ///
>     spikes(100) lw(*3) psep legend(off)
```



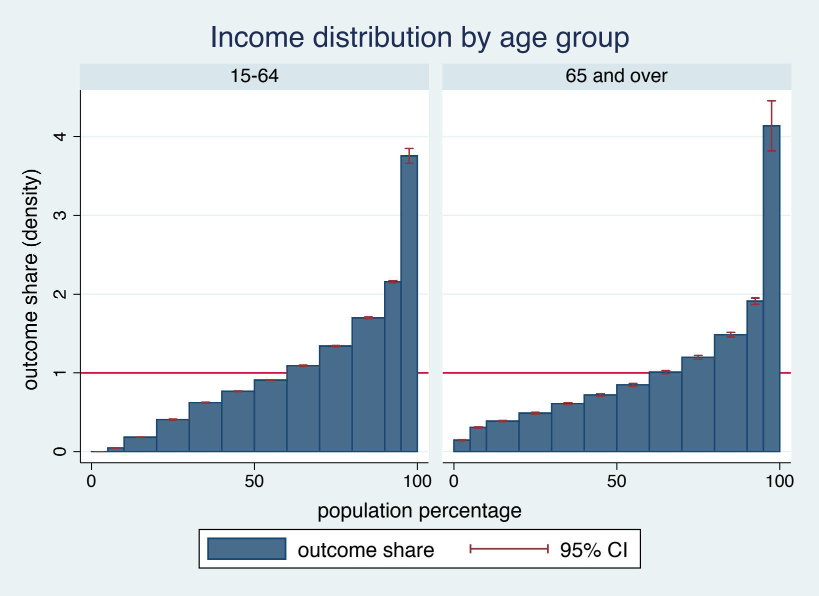
Analysis of subpopulations

```
. pshare estimate income, p(5 10(10)90 95) over(agecat) density
Percentile shares (density)                    Number of obs     =    119,939

          15: agecat = 15-64
          65: agecat = 65 and over
```

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
15						
0-5	.0002108	.0000446	4.73	0.000	.0001234	.0002982
5-10	.0477019	.001519	31.40	0.000	.0447247	.050679
10-20	.1839598	.0019449	94.59	0.000	.1801479	.1877717
20-30	.4072382	.0030794	132.25	0.000	.4012026	.4132738
30-40	.62253	.0026311	236.60	0.000	.617373	.627687
40-50	.7665669	.0027358	280.20	0.000	.7612048	.7719291
50-60	.9087896	.0031534	288.20	0.000	.9026091	.9149701
60-70	1.091312	.0037406	291.75	0.000	1.08398	1.098643
70-80	1.340543	.0045022	297.75	0.000	1.331719	1.349367
80-90	1.698174	.0056207	302.13	0.000	1.687158	1.70919
90-95	2.157966	.0076957	280.41	0.000	2.142882	2.173049
95-100	3.755896	.0478808	78.44	0.000	3.66205	3.849741
65						
0-5	.1460041	.0039658	36.82	0.000	.1382311	.153777
5-10	.3081252	.0037889	81.32	0.000	.300699	.3155513
10-20	.3879465	.0043662	88.85	0.000	.3793888	.3965043
20-30	.4893443	.0056362	86.82	0.000	.4782975	.5003911
30-40	.6099115	.006742	90.46	0.000	.5966972	.6231257
40-50	.7204667	.0078164	92.17	0.000	.7051468	.7357867
50-60	.8488989	.0091605	92.67	0.000	.8309446	.8668533
60-70	1.009523	.0107486	93.92	0.000	.9884558	1.03059
70-80	1.19784	.0126163	94.94	0.000	1.173113	1.222568
80-90	1.484796	.0155048	95.76	0.000	1.454407	1.515185
90-95	1.911148	.0203231	94.04	0.000	1.871315	1.950981
95-100	4.137269	.1622893	25.49	0.000	3.819184	4.455353

```
. pshare histogram, yline(1) byopts(ti(Income distribution by age group))
```

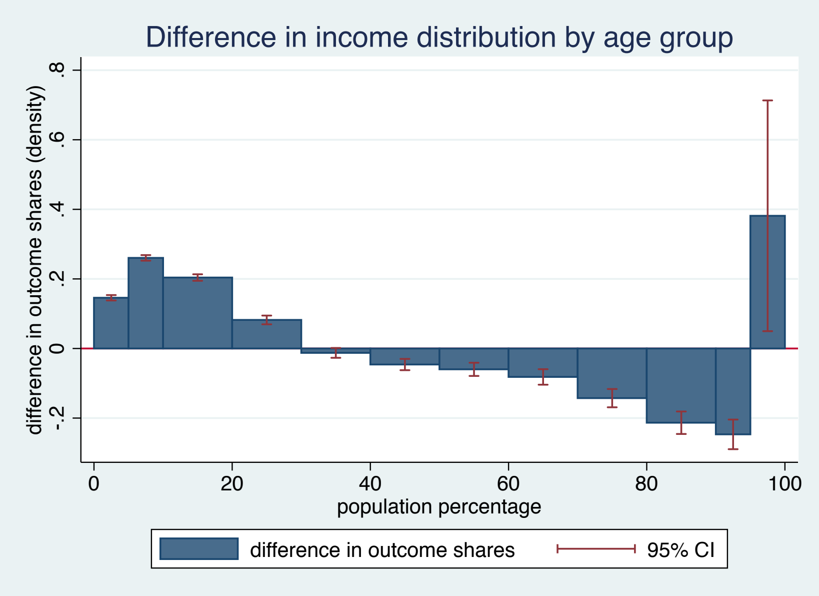
Analysis of subpopulations



Subpopulation contrasts

```
. pshare contrast

Differences in percentile shares (density)      Number of obs     =    119,939

          15: agecat = 15-64
          65: agecat = 65 and over

------------------------------------------------------------------------------
      income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
65           |
         0-5 |   .1457933   .0039661    36.76   0.000     .1380198    .1535667
        5-10 |   .2604233    .004082    63.80   0.000     .2524226     .268424
       10-20 |   .2039867   .0047798    42.68   0.000     .1946184     .213355
       20-30 |   .0821061   .0064225    12.78   0.000      .069518    .0946941
       30-40 |  -.0126185   .0072372    -1.74   0.081    -.0268034    .0015664
       40-50 |  -.0461002   .0082813    -5.57   0.000    -.0623314    -.029869
       50-60 |  -.0598907    .009688    -6.18   0.000     -.078879   -.0409023
       60-70 |  -.0817888   .0113809    -7.19   0.000    -.1040952   -.0594824
       70-80 |  -.1427025   .0133956   -10.65   0.000    -.1689575   -.1164474
       80-90 |   -.213378   .0164921   -12.94   0.000    -.2457023   -.1810537
       90-95 |  -.2468179   .0217313   -11.36   0.000    -.2894109   -.2042248
      95-100 |   .3813731   .1692052     2.25   0.024     .0497337    .7130125
------------------------------------------------------------------------------
(contrasts with respect to preceding subpopulation)

. pshare hist, yline(0) ti(Difference in income distribution by age group)
```

Subpopulation contrasts



Bivariate analysis: Wealth by income group

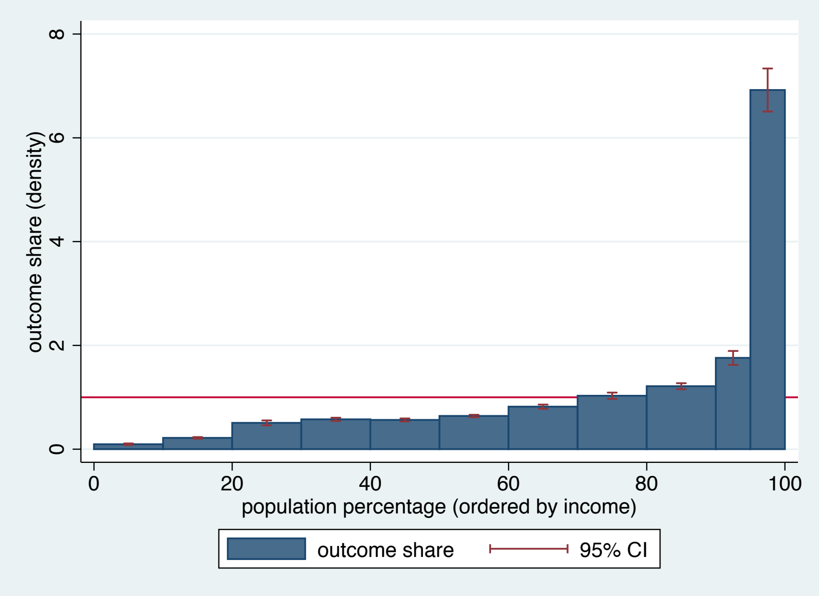
```
. pshare estimate wealth, p(10(10)90 95) pvar(income) density vce(boot)
(running pshare on estimation sample)

Bootstrap replications (50)
----+--- 1 ---+--- 2 ---+--- 3 ---+--- 4 ---+--- 5
..................................................    50

Percentile shares (density)                 Number of obs     =    119,939
                                            Replications      =         50
```

wealth	Observed Coef.	Bootstrap Std. Err.	z	P>|z|	Normal-based [95% Conf.	Interval]
0-10	.094965	.0082328	11.54	0.000	.0788291	.1111009
10-20	.2175728	.0082347	26.42	0.000	.201433	.2337126
20-30	.506814	.0242173	20.93	0.000	.4593491	.554279
30-40	.5747378	.0162314	35.41	0.000	.5429247	.6065508
40-50	.5637113	.014835	38.00	0.000	.5346353	.5927874
50-60	.6399473	.0116937	54.73	0.000	.6170281	.6628664
60-70	.8189444	.0208025	39.37	0.000	.7781722	.8597166
70-80	1.029533	.0311481	33.05	0.000	.9684839	1.090582
80-90	1.213529	.0293399	41.36	0.000	1.156024	1.271034
90-95	1.75912	.0684403	25.70	0.000	1.624979	1.893261
95-100	6.921371	.2116263	32.71	0.000	6.506591	7.336151

```
(percentile groups with respect to income)

. pshare histogram, yline(1)
```

The results show that the top income households are also the ones among which most of the wealth is accumulated.

Bivariate analysis: Wealth by income group

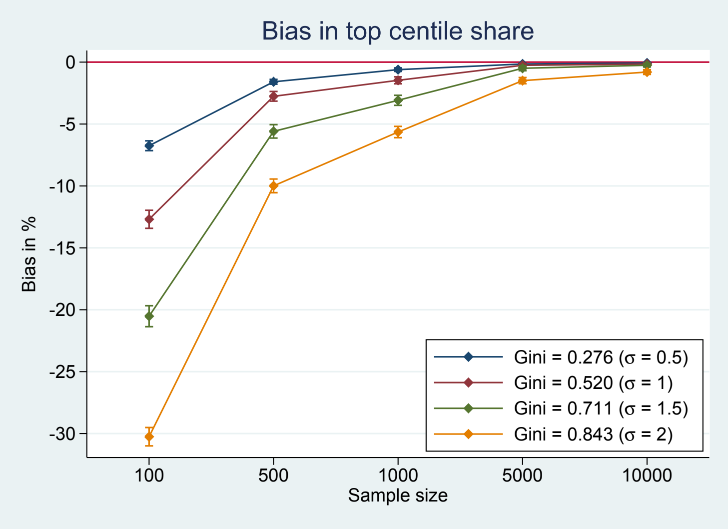


Small sample bias

- Percentile shares are affected by small sample bias.
- The top percentile share is typically underestimated.
- The problem is difficult to fix.
  - Corrections could be derived based on parametric assumptions.
  - Smoothing out the data by adding random noise can be an option, but this also requires parametric assumptions.
  - I evaluated a non-parametric small-sample correction using a bootstrap approach: the bias in bootstrap samples is used to derive correction factors for the main results.
  - This works very well in terms of removing bias (unless the distribution is extremely skewed).
  - **However:** MSE increases compared to uncorrected results!
  - No idea how to improve on this.

Small sample bias: How bad is the problem?

- Simulation: relative bias in top 1% share using a log-normal distribution



Software and paper

- Software:

```
. ssc install pshare
. ssc install moremata
. mata mata mlib index
```

- Paper:
  - Jann, Ben. 2015. Assessing inequality using percentile shares. University of Bern Social Sciences Working Papers No. 13. https://ideas.repec.org/p/bss/wpaper/13.html

References

- Ecoplan (2004). Verteilung des Wohlstands in der Schweiz. Bern: Eidgenössische Steuerverwaltung.
- Binder, D. A., M. S. Kovacevic (1995). Estimating Some Measures of Income Inequality from Survey Data: An Application of the Estimating Equations. Survey Methodology 21(2): 137-145.