Calibrating a CGE model with NTBs that Incorporates Standard Models of Modern Trade Theory*

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ABSTRACT: We propose a way to incorporate NTBs for the four workhorse models of the modern trade literature in computable general equilibrium models (CGEs). CGE models feature intermediate linkages and thus allow us to study global value chains (GVCs). We show that the Ethier-Krugman monopolistic competition model, the Melitz firm heterogeneity model and the Eaton and Kortum model can be defined as an Armington model with generalized marginal costs, generalized trade costs and a demand externality. As already known in the literature in both the Ethier-Krugman model and the Melitz model generalized marginal costs are a function of the amount of factor input bundles. In the Melitz model generalized marginal costs are also a function of the price of the factor input bundles. Lower factor prices raise the number of firms that can enter the market profitably (extensive margin), reducing generalized marginal costs of a representative firm. For the same reason the Melitz model features a demand externality: in a larger market more firms can enter. We implement the different models in a CGE setting with multiple sectors, intermediate linkages, non-homothetic preferences and detailed data on trade costs. We find the largest welfare effects from trade cost reductions in the Melitz model. We also employ the Melitz model to mimic changes in Non tariff Barriers (NTBs) with a fixed cost-character by analysing the effect of changes in fixed trade costs. While we work here with a model calibrated to the GTAP database, the methods developed can also be applied to CGE models based on the WIOD database.

Keywords: NTBs, Firm Heterogeneity, CGE Model, Demand Externality
JEL codes: F12, F14

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1 Introduction

There is a lively debate in the recent trade literature about the value added of firm heterogeneity in trade models. Arkolakis, et al. (2012) show that the welfare gains from trade can be expressed with two sufficient statistics, the domestic spending share and the trade elasticity. This holds in the Armington model, the Ricardian Eaton-Kortum model, the equal firms monopolistic competition Ethier-Krugman model and the firm heterogeneity Melitz model. The only difference is the interpretation of the trade elasticity. In Armington and Ethier-Krugman the trade elasticity is determined by the substitution elasticity between varieties, whereas in Eaton-Kortum and Melitz it is determined by productivity dispersion. Melitz and Redding (2013) instead show that trade cost reductions generate larger welfare gains in the Melitz firm heterogeneity model than in the equivalent model with homogeneous firms, the Ethier-Krugman model.

Firm heterogeneity has not been incorporated in a comprehensive way in multisector CGE models. Most important work in this respect is Balistreri (2012), who have included firm heterogeneity in one sector in a CGE model with other sectors characterized by an Armington setup. Allowing for firm heterogeneity in all sectors might be useful for various reasons. First, it can shed light on the discussion about the value added of firm heterogeneity in trade models by exploring the differences in modelling outcomes with other models. Second, various realistic microeconomic features can be modelled like the distinction of welfare effects into an intensive and extensive margin effect. Third, CGE models contain a large degree of sectoral detail, but are sometimes somewhat outdated in terms of modelling setup. With the incorporation of firm heterogeneity in all sectors, this drawback would disappear.

In this paper we map out a parsimonious representation of firm heterogeneity enabling incorporation in multisector CGE models. In particular, we show that both the Ethier-Krugman and the Melitz model can be defined as an Armington model by generalizing the expressions for iceberg trade costs and for marginal costs and by allowing for a demand externality in the Melitz model. In Ethier-Krugman generalized marginal costs are a function of the number of input bundles leading to so-called variety scaling (Francois (2013)). Variety scaling also props up in the Melitz model, but on top of that generalized marginal costs are also a function of the price of input bundles. The reason is that the extensive margin relative and the compositional margin are affected by the price of input bundles. With a lower price of input bundles more firms can sell profitably to the different destination markets generating a positive effect through the extensive margin (more varieties) and a negative effect through the compositional margin (lower average productivity because of the survival of the least productive firms as well). For the same reason there is a demand externality in the Melitz model: in a larger market with a higher price index more firms can survive, raising the extensive margin relative to the compositional margin. Generalized iceberg trade costs are a function of fixed and iceberg trade costs and of tariffs. We show theoretically that the Ethier-Krugman model is a special version of the Melitz model if the firm size distribution becomes granular. Granularity corresponds with a trade elasticity in Melitz equal to the substitution elasticity minus one. The reason is that under granularity the destination-varying component of the extensive margin cancels out against the compositional margin leaving only the intensive margin and the number of entrants-component of the extensive margin, the two channels also operative in Ethier-Krugman.

We implement the parsimonious representation of the different models in the multisector, multicountry, multifactor CGE model GTAP featuring intermediate linkages on non-homothetic preferences based on a detailed consistent dataset on output, trade flows, tariffs and transport services. Following Head and Mayer (2013) we decompose changes in trade flows in response to policy shocks into an intensive margin, an extensive margin and a compositional margin. It is shown with simulations that the destination-specific component of the extensive margin relative to the compositional margin rises when the firm size distribution becomes less granular In line with this finding we show that the welfare gains from reductions in trade costs are largest in the Melitz model and rise when the firm size distribution moves away from granularity.

We also examine the effect of a reduction in fixed trade costs at varying degrees of granularity. Since many non tariff barriers (NTBs) have a fixed trade cost character, we can use these results to interpret the effect of reductions in NTBs. So NTBs are paid once by firms to get access to a foreign market and can thus be mimicked by reductions in fixed trade costs in the Melitz model. Since the Armington and Ethier-Krugman model do not feature destination-specific fixed costs, the ability to analyse the effect of fixed cost-type NTBs is an important contribution of incorporating the Melitz firm heterogeneity model into the GTAP model. We find that the effect of reductions in fixed trade costs is larger with a lower degree of granularity of the firm size distribution with small firms being relatively more important in the distribution of firms.

Costinot and Rodriguez-Clare (2013) compare the welfare effects of trade and trade liberalization in the different trade models in different setups. They show that the expression for the price index in the most general model, the firm heterogeneity model, nests the expressions in the Armington and Ethier-Krugman model. Their exposition is different in several respects. First, they concentrate on welfare and thus only derive an expression for the price index. Second, they do not write the different models as special versions of an Armington economy with generalized marginal costs, generalized trade costs and a demand side externality. Third, they use exact hat algebra to derive their results on the welfare effects of trade liberalization.

2 Model

2.1 General Setup

Consider an economy with J countries. There are three groups of agents ag with demand for goods in sector r, private households p, government g and firms f. The group of agents ag in country j has demand q_j^{ag} with CES preferences over quantities of domestic and imported representative goods $q_j^{d,ag}$ and $q_j^{m,ag}$. We omit sector r subscripts as well as the derivation of demand for sector r goods and take this demand as our starting point:[1]

$$q_j^{ag} = \left(\left(e_j^d q_j^{d,ag} \right)^{\frac{\sigma-1}{\sigma}} + \left(e_j^m q_j^{m,ag} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

Quantities of imported and domestic varieties can be summed up to give total importer and domestic demand, q_j^s with $s = d, m$:

$$q_j^s = \sum_{ag \in \{p,g,f\}} q_j^{s,ag} \tag{2}$$

e_j^s is a demand side externality playing a role in the firm heterogeneity version of the model. The demand externality is identical for the different groups of agents. The reason is that upon paying fixed export costs for a destination country firms can serve all three groups of agents in the destination country and the zero cutoff profit condition is thus formulated over all three groups together. The externality is source-specific with the source domestic or importer, $s = d, m$. The reason is that we want to allow for different destination-specific taxes for imported goods and domestic goods.

Demand for $q_j^{s,ag}$ can be written as:

$$q_j^{s,ag} = \left(e_j^s \right)^{\sigma-1} \left(\frac{ta_j^{s,ag} p_j^s}{P_j^{ag}} \right)^{-\sigma} q_j^{ag} \tag{3}$$

$ta_j^{s,ag}$ is a group-importer specific import tariff, expressed in power terms. P_j^{ag} and p_j^s are respectively the price indices corresponding to q_j^{ag} and q_j^s defined below. For domestic goods equations (1)-(3) are the final equations generating total domestic demand q_j^d, but for imported goods, demand q_j^m consists of demand for goods from different sources i, q_{ij}:

$$q_j^m = \left(\sum_{i \neq j} \left(q_{ij} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

Solving for demand from source i, q_{ij}, gives:

$$q_{ij} = \left(\frac{p_{ij}}{p_j^m} \right)^{-\sigma} q_j^m \tag{5}$$

[1] Derivations and expressions for sectoral demand for the three groups of agents can be found in Hertel (1997) and also in Bekkers, et al. (2015).

p_{ij} is the price of the representative good traded from i to j. The different prices are defined as follows:

$$P_j^{ag} = \left(\left(\frac{ta_j^{d,ag} p_j^d}{e_j^d} \right)^{1-\sigma} + \left(\frac{ta_j^{m,ag} p_j^m}{e_j^m} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{6}$$

$$p_j^d = c_j b_j p_{Z_j} \tag{7}$$

$$p_j^m = \left(\sum_{i \neq j} (p_{ij})^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{8}$$

$$p_{ij} = ta_{ij} t_{ij} c_i \left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) \tag{9}$$

The price of the representative good, p_{ij}, in equation (9) is equal to cif-price calculated as the sum of the marginal cost times the price of input bundles in the exporting country, $b_i p_{Z_i}$, times the export subsidy applied to the fob-price plus the price of transport services p_{ij}^{tr} divided by a transport services technology shifter a_{ij}^{tr}, multiplied by generalized marginal costs in the exporting country, c_i, generalized iceberg trade costs t_{ij} and bilateral ad valorem tariffs, ta_{ij}, both expressed in power terms. Firms spend a fixed quantity share of sales on transport services. Technically, the cif-quantity traded o_{ij}^{cif} is a Leontief function of the quanity in fob-terms o_{ij}^{fob} and transport services tr_{ij}. The implication is that transport services work as a per unit trade cost and appear thus as an additive term to the fob price $te_{ij} b_i p_{Z_i}$. Equation (9) makes clear that the costs for transport services could be rewritten as ad valorem trade costs if the input bundles used in transport services would be identical to regular input bundles, since this would imply $p_{ij}^{tr} = p_{Z_i}$. So the reason that the costs for transport services operate as a per unit trade cost is that different input bundles are used.

The Armington model, the Krugman/Ethier model and the Melitz model can all be seen as special versions of the above structure, depending upon how the demand externality e_j^s in equation (1), generalized iceberg trade costs t_{ij}, and generalized marginal cost c_i in equation (9) are specified. In the subsections below we describe the main features of the different models, give the expressions for c_i, t_{ij}, e_j^s and provide the intuition of these expressions. In the appendix we give formal proofs that with the choices for c_i, t_{ij}, e_j^s the general setup-model is equivalent to the different models.

2.2 Armington Economy

Perfectly competitive firms in country i produce homogeneous country i varieties with marginal cost b_i. So, input bundles Z_i can be transformed into output x_i according to $x_i = \frac{Z_i}{b_i}$. With marginal cost pricing the price of output in country i, p_i^x, is given by, $p_i^x = b_i p_{Z_i}$. Firms face iceberg trade cost τ_{ij}. There is no demand externality in the Armington economy, so $e_j^s = 1$. Therefore, the Armington economy is characterized by equations (1)-(9) with the following expressions for c_i, t_{ij} and e_j^s:

$$c_i = 1 \tag{10}$$

$$t_{ij} = \tau_{ij} \tag{11}$$

$$e_j^s = 1 \tag{12}$$

2.3 Ethier-Krugman Economy

In the Ethier-Krugman economy, preferences are characterized by love for variety over varieties ω produced in different countries. Utility q_j^{ag} can thus be defined over physical quantities (output) $o(\omega)$ of varieties $\omega \in \Omega_{ij}$ shipped from all exporters i:

$$q_j^{ag} = \left(\sum_{i=1}^{J} \int_{\omega \in \Omega_{ij}} o^{ag}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{13}$$

The corresponding price index is defined over the prices of physical quantities of the varieties, $p^o(\omega)$:

$$P_j^{ag} = \left(\sum_{i=1}^{J} \int_{\omega \in \Omega_{ij}} p^{ag,o}(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}} \tag{14}$$

Firms in country i produce with an identical increasing returns to scale technology with fixed cost a_i and marginal cost b_i implying that each firm produces a unique variety. Increasing returns in combination with love for variety implies also that a larger number of input bundles leads to a more than proportional increase in utility since the number of varieties is larger. To capture this externality, generalized marginal costs c_i are falling in the number of varieties N_i and thus in the amount of input bundles Z_i. Employing the expressions for markup pricing, the free entry condition and factor market closure, c_i can be expressed as follows:[2]

$$c_i = \gamma_{ek} \left(\frac{\widetilde{Z}_i}{a_i} \right)^{\frac{1}{1-\sigma}} \tag{15}$$

γ_{ek} is a function of the substitution elasticity σ:

$$\gamma_{ek} = \frac{\sigma - 1}{\sigma} \sigma^{\frac{1}{1-\sigma}} \tag{16}$$

And $\widetilde{Z}_i$ is a function of the number of input bundles, but also of the transport services and export subsidies paid.

$$\widetilde{Z}_i = Z_i - \frac{\sigma - 1}{\sigma} \left(\sum_{j=1}^{J} \frac{N_i \overline{r}_{ij}}{p_{Z_i} ta_{ij} \left(te_{ij} b_i + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}} \right)} - \frac{N_i \overline{r}_{ij}}{p_{Z_i} ta_{ij}} \right) \tag{17}$$

$\overline{r}_{ij}$ are the per-firm revenues divided by group-specific import tariffs. Henceforth, $N_i \overline{r}_{ij}$ represents the value of trade before group-specific import tariffs are paid. Generalized marginal cost does not fall proportionally in the amount of input bundles Z_i, as the number of varieties N_i does not increase proportionally with the amount of input bundles Z_i. N_i is calculated by combining factor market equilibrium and the free entry condition. Since transport services are sourced employing separate input bundles, they have to be subtracted in calculating the demand for input bundles from a specific country and sector. So an increase in transport costs leads to less labor demand for given zero-profit-revenues. As a resut higher transport costs raise the number of varieties for a given number of input bundles.[3]

Representative output x_i can be transformed into q_{ij} accounting for the iceberg trade costs τ_{ij}.There is no demand externality in the Ethier-Krugman economy, so we have:

$$t_{ij} = \tau_{ij} \tag{18}$$

$$e_j^s = 1 \tag{19}$$

So, the Ethier/Krugman economy is characterized by equations (1)-(9) with c_i, t_{ij} and e_j as defined in equations (15)-(19).

[2] Derivations in Appendix A

[3] An increase in transport costs raises input bundle demand also through the demand for transport services, but in the transport sector we assume perfect competition so there is no number of firms externality.

2.4 Melitz Economy

In the Melitz economy preferences are like in Ethier/Krugman characterized by love for variety over varieties produced by different firms from different countries as in equation (13)-(14). Goods are produced by firms with heterogeneous productivity. To start producing, firms can draw a productivity parameter φ from a distribution $G_i(\varphi)$ after paying a sunk entry cost en_i. The distribution of initial productivities is Pareto with a shape parameter θ and a size parameter κ_i:

$$G_i(\varphi) = 1 - \frac{\kappa_i^\theta}{\varphi^\theta} \tag{20}$$

A higher θ reduces the dispersion of the productivity distribution and a higher κ_i raises all initial productivity draws proportionally. We impose $\theta > \sigma - 1$ to guarantee that expected revenues are finite.

The productivity of firms stays fixed and firms face a fixed death probability δ in each period. Firms either decide to start producing for at least one of the markets or leave the market immediately. In equilibrium there is a steady state of entry and exit with a steady number of entrants drawing a productivity parameter, implying that the productivity distribution of producing firms is constant.

Firms produce with an increasing returns to scale technology with marginal cost equal to $\frac{1}{\varphi}$. We assume that productivity φ operates both on the costs of production and on the transport sector. This means that more productive firms also need less transport services, an assumption also made for iceberg trade costs τ_{ij}. If productivity would only operate on the cost of production in a setting where the costs for transport services operate as per unit trade costs, the model would become intractable in a multicountry, multisector setting. We would need this assumption of we would reformulate the model such that transport services would work as ad valorem instead of per unit trade costs. As explained in Section 2.1 this would be the case of input bundles used in transport services were identical to regular input bundles. Firms pay fixed costs f_{ij} for each market in which they sell. The fixed costs are paid partly in input bundles of the source country and partly in bundles of the destination country according to a Cobb Douglas specification with a fraction μ paid in source country input bundles. Upon paying the fixed entry costs for a destination market, firms can sell goods to all three groups of agents.

Since preferences are characterized by love for variety and production occurs with increasing returns to scale, an increase in the number of input bundles leads to a more than proportional increase in utility. To account for this externality, representative output is like in the Ethier/Krugman economy defined as variety scaled output.

Since productivity is heterogeneous, variety scaled output is also affected by input costs. Following Head and Mayer (2013) changes in costs lead to an adjustment in output along three margins, an intensive margin, an extensive margin and a compositional margin. Lower costs lead to more sales of firms already in the market, the intensive margin. This is a price effect and hence does not affect variety scaled output. Lower costs also raises the mass of firms that can produce profitably, the extensive margin. This leads to a rise in variety scaled output. And finally, lower costs reduces the average productivity of firms in the market, as more firms can survive, the compositional margin. This margin also affects variety scaled output. Accounting for the latter two margins, generalized marginal costs c_i can be written as:

$$c_i = \gamma_m \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta e n_i} \right)^{\frac{1}{1-\sigma}} p_{Z_i}^{\mu \frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{21}$$

The expression for $\widetilde{Z}_i$ is identical to the expression in the Ethier-Krugman model and is given in equation (17). γ_m is a function of σ and θ and an additional conversion parameter ψ for later use set equal to 1:

$$\gamma_m = \psi \left(\frac{\sigma}{\sigma - 1} \right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta - \sigma + 1} \tag{22}$$

x_i can be transformed into q_{ij} accounting for generalized iceberg trade costs, which are a function of iceberg trade costs τ_{ij}, fixed trade costs f_{ij}, import tariffs c_{ij} and the cif price $te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}$. Iceberg and fixed trade costs affect the transformation in the same way through the extensive and compositional margin as the price of input bundles p_{Z_i} affect generalized marginal costs.[4] We get the following expression for generalized iceberg trade costs:

$$t_{ij} = \left(\left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} \tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}} ta_{ij}^{\frac{\theta-\sigma+1}{\sigma-1} + \frac{\theta-\sigma+1}{(\sigma-1)^2}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \right) \tau_{ij} \tag{23}$$

The four terms between brackets represent the effects of the cif-price, tariffs, and iceberg and fixed trade costs through the extensive and compositional margin on converting fob variety

[4]Profits are calculated dividing revenues inclusive of tariffs by tariffs, $\pi = \frac{r}{1+ta} - cq - f$. Costinot and Rodriguez-Clare call this demand shifting. The alternative would be cost shifting with profits calculated as $\pi = r - c\,(1 + ta)\,q - f$. This makes it impossible to find an expression for the mass of firms as a function of market size, a problem also occuring in the Ethier/Krugman model.

scaled output into cif variety scaled output. Iceberg trade costs also have a direct effect through the intensive margin, represented by the last term outside of the brackets.

Finally, the demand externality does play a role under firm heterogeneity, again driven by the extensive and compositional margin. The following expression can be derived for the demand externality e_j:

$$e_j^s = \left(\frac{\sum\limits_{ag=\{s,p,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}} \right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}}{p_{Z_j}^{1-\mu}} \right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{24}$$

E_j^{ag} is expenditure by ag in country j. Both larger price indexes P_j^{ag}, larger market sizes E_j^{ag} and lower group-specific tariffs ta_j^{ag} for the different groups of agents ag raise the extensive margin relative to the compositional margin and thus reduce the price index P_j^{ag} and raise utility q_j^{ag}. A lower price of input bundles p_{Z_j} in the destination country also raises utility, as it raises welfare through the extensive margin relative to the compositional margin.

The Melitz economy is characterized by equations (1)-(9) with the expressions c_i, t_{ij} and e_j given in equations (21)-(24).

2.5 Eaton and Kortum

In the Eaton and Kortum economy preferences are CES over a continuum of varieties ω of mass 1:

$$q_j^{ag} = \left(\int_0^1 o_j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{25}$$

All countries can potentially produce all goods o_j in country j with a productivity χ. There is perfect competition in the product market and to ship goods from i to j export taxes, iceberg trade costs and transport services have to be paid. The price of goods shipped from country i to j is thus given by $ta_j^{s,ag} p_{ij}^o(\omega) = \frac{ta_j^{s,ag} ta_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\chi(\omega)}$. As in the Melitz model we assume that productivity operates both on production and transport services.

Productivity χ is drawn in each country from a country-specific Frechet distribution function with $T_i(Z_i)$ a measure of absolute advantage of country i and ρ a (inverse) measure of the strength comparative advantage:

$$G_i(\chi) = 1 - \exp\left(-\frac{T_i(Z_i)}{\chi^\rho}\right) \tag{26}$$

Consumers buy each good ω from the country with the lowest price, inclusive of trade costs. This implies a distribution of prices for each country j, from which an expression for the price index follows. The probability that country i delivers a good to country j for group ag is equal to:

$$\pi_{ij} = \frac{T_i(Z_i)\left(ta_{ij}\left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{-\rho}}{\sum_{k=1}^{J} T_k(Z_k)\left(ta_{kj}\left(te_{kj}p_{Z_k} + \frac{p_{kj}^{tr}}{a_{kj}^{tr}}\right)\right)^{-\rho}} \tag{27}$$

It can be shown that the price distribution of goods bought from country i in country j is equal to the general distribution of prices in country j, $G_j(\varphi)$. This implies that average expenditure in country j does not vary by source as pointed out by Eaton and Kortum. This implication thus also holds for quantity and thus the quantity sold from i to j is equal to the share of goods bought from i in equation (27):

$$q_{ij} = \frac{T_i(Z_i)\left(ta_{ij}\tau_{ij}\left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{-\rho}}{\sum_{k=1}^{J} T_k(Z_k)\left(ta_{kj}\tau_{kj}\left(te_{kj}p_{Z_k} + \frac{p_{kj}^{tr}}{a_{kj}^{tr}}\right)\right)^{-\rho}} \sum_{ag\in\{p,g,f\}} q_j^{ag} \tag{28}$$

Finally, the price index follows from calculating the expected price and substituting the result into the expression for the price index corresponding to utility in equation (25):

$$P_j^{ag} = \gamma_{eako}\left(\sum_{k=1}^{J} T_k(Z_k)\left(ta_j^{s,ag} ta_{kj}\left(te_{kj}p_{Z_k} + \frac{p_{kj}^{tr}}{a_{kj}^{tr}}\right)\right)^{-\rho}\right)^{-\frac{1}{\rho}} \tag{29}$$

With $s' = d$ if $i = j$ and $s' = m$ if $i \neq j$ and $ta_{ii} = t_{ii} = te_{ii} = 1$ and $\frac{p_{ii}^{tr}}{a_{ii}^{tr}} = 0$ and $\gamma_{eako} = \left(\Gamma\left(\frac{\rho-\sigma+1}{\rho}\right)\right)^{\frac{1}{1-\sigma}}$.

Following Ramondo (2014) we can assume that technology T_i increases proportionally with the number of input bundles:

$$T_i = \phi_i Z_i \tag{30}$$

ϕ_i is a measure of innovation intensity. As pointed out by Ramondo (2014) the specification in equation (30) follows from a setting where productivity of a technology is drawn from a Frechet distribution with dispersion parameter ρ and location (absolute advantage) parameter ϕ_i as in the baseline model. On top each good can be produced with more than one technology with the number of technologies per good equal to the number of input bundles Z_i. The best technology of a good is then Frechet distributed with absolute advantage parameter $\phi_i Z_i$.

Comparing the expressions for quantity demanded and the price index in the Eaton and Kortum model in equations (28)-(29) and in the general setup-model in equations (A.1)-(A.2) implies the following expressions for c_i, t_{ij} and e_j^s, together with $\sigma = \rho$ in the demand equations and $\sigma = \rho + 1$ in the price index equations:

$$c_i = \gamma_{eako} \left(T_i \left(Z_i \right) \right)^{-\frac{1}{\rho}} \tag{31}$$

$$t_{ij} = \tau_{ij} \tag{32}$$

$$e_j^s = 1 \tag{33}$$

So the Eaton and Kortum model is equivalent to the Armington model with two differences. First, productivity T_i can be assumed to be a function of the number of input bundles Z_i and second, the estimated tariff elasticity implies a different trade elasticity in the two models, as will be discussed in Section 4 on parameter estimation. If productivity T_i rises proportionally with Z_i, the scale effect works as in the Ethier-Krugman and Melitz model.

2.6 Nesting

From the expressions in the previous 3 subsections it follows directly that Krugman/Ethier is a special case of Melitz up to a constant and Armington is a special case of both.

Melitz can be converted into an Ethier/Krugman model by setting θ equal to $\sigma - 1$, the size parameter of the productivity distribution κ_i equal to the inverse of marginal cost $\frac{1}{b_i}$, sunk entry costs times the death probability δen_i divided by the size parameter of the productivity distribution κ_i, $\delta en_i / \kappa_i$ equal to the fixed cost a_i and the conversion parameter ψ in equation (22) as follows:

$$\psi = \left(\frac{\sigma}{\sigma - 1} \right)^{\theta - \sigma + 2} \sigma^{\frac{\theta}{\sigma - 1}} \left(\theta - \sigma + 1 \right) \tag{34}$$

$\theta = \sigma - 1$ implies that the demand externality e_j^s is 1. It can be easily verified that the expressions for c_i and t_{ij} in equations (21)-(23) become equal to the price of the representative good in the Ethier/Krugman economy in equations (15)-(18). Ethier/Krugman can be converted into Armington by setting the marginal cost parameter c_i equal to 1 and thus dropping the variety scaling.

The intuition for why $\theta = \sigma - 1$ implies that Melitz leads to Krugman/Ethier is the following. As pointed out above a change in trade costs generates a change in trade flows along three margins, an intensive margin of already exporting firms, an extensive margin representing an increase in the mass of varieties and a compositional margin representing the change in average productivity of firms exporting. If trade costs fall, trade rises with an elasticity of $\sigma - 1$ along the intensive margin and with an elasticity θ along the extensive margin. It falls along the compositional margin with an elasticity $\sigma - 1$. So, if $\theta = \sigma - 1$, the extensive and compositional margin cancel out and only the intensive margin remains. Therefore, the model with heterogeneous firms works out identically as a model with homogeneous firms.

The conversion factor ψ in moving from Melitz to Ethier/Krugman is necessary. Without this conversion factor utility would become infinite in Melitz with $\theta = \sigma - 1$. The reason is that $\theta = \sigma - 1$ would imply that average productivity would become infinite. Still, when θ approaches $\sigma - 1$ the effect of changes in trade costs will be identical to the effect in an Ethier/Krugman economy. So, we can see the Ethier/Krugman model as a limiting case of the Melitz model.

3 Margin Decomposition of Trade in Melitz Model

Total trade flows as measured in cif-terms, inclusive of bilateral import tariffs, but exclusive of group-specific importer tariffs, can be written as:

$$V_{ij} = N_{ij}\widetilde{\overline{r}}_{ij} = N_{ij}\frac{1}{1 - G\left(\varphi_{ij}^*\right)}\int_{\varphi_{ij}^*}^{\infty} \overline{r}_{ij}\left(\varphi\right) g\left(\varphi\right) d\varphi \tag{35}$$

Log differentiating equation (35) on the RHS and LHS wrt to the endogenous variables gives:

$$\begin{aligned} d\ln V_{ij} = d\ln N_{ij} + N_{ij}\frac{1}{1 - G\left(\varphi_{ij}^*\right)}\int_{\varphi_{ij}^*}^{\infty} d\ln \overline{r}_{ij}\left(\varphi\right)\frac{r_{ij}\left(\varphi\right)}{r_{ij}\left(\widetilde{\varphi}\right)} g\left(\varphi\right) d\varphi \\ + \frac{\partial \ln\left(1 - G\left(\varphi_{ij}^*\right)\right)}{\partial \ln \varphi_{ij}^*} d\ln \varphi_{ij}^* \left(\frac{\overline{r}_{ij}\left(\varphi_{ij}^*\right)}{\overline{r}_{ij}\left(\widetilde{\varphi}\right)} - 1\right) \end{aligned} \tag{36}$$

The first term represents the extensive margin, EM, the second term the intensive margin, IM, and the third term the compositional margin, CM. To elaborate on these expressions, we first log differentiate the expression for φ^*_{ij} in equation (B.7):

$$\widehat{\varphi^*_{ij}} = \frac{\mu}{\sigma-1}\widehat{p_{Z_i}} + \frac{1-\mu}{\sigma-1}\widehat{p_{Z_j}} + \left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} + \widehat{\tau_{ij}} + \widehat{te_{ij}p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}} + \frac{1}{\sigma-1}\widehat{f_{ij}}$$
$$-\frac{1}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p^s_j q^{s,ag}_j}{\sum\limits_{ag'=\{s,p,f\}} p^s_j q^{s,ag'}_j}\left((\sigma-1)\widehat{P^{ag}_j} + \widehat{E^{ag}_j} - \sigma\widehat{ta^{ag}_j}\right) \tag{37}$$

We can elaborate on the extensive margin, employing the expression for N_{ij} and NE_i in equations (B.17)-(B.18) and the expression for $\widehat{\varphi^*_{ij}}$ in equation (37):

$$EM = d\ln N_{ij} = -\theta\widehat{\varphi^*_{ij}} + \widehat{NE_i} \tag{38}$$

We can elaborate on the intensive margin, IM, employing the expression for $r^{ag}_{ij}(\varphi)$ and $p^{ag}_{ij}(\varphi)$ in equations (B.3)-(B.4) and summing over the three income groups:

$$IM = \frac{\theta-\sigma+1}{\theta}$$
$$*\int\limits_{\varphi^*_{ij}}^{\infty} d\ln\left(\left(\frac{\sigma}{\sigma-1}\frac{ta_{ij}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}\right)}{\varphi}\right)^{1-\sigma}\sum_{ag=\{s,p,f\}}\left(ta^{ag}_j\right)^{-\sigma}\left(P^{ag,e}_j\right)^{\sigma-1}E^{ag}_j\right)\frac{g(\varphi)}{1-G\left(\varphi^*_{ij}\right)}d\varphi$$
$$=(1-\sigma)\left(\widehat{\tau_{ij}}+\widehat{ta_{ij}}+\left(\widehat{te_{ij}p_{Z_i}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\right)\right)+\sum_{ag=\{s,p,f\}}\frac{p^s_j q^{s,ag}_j}{\sum\limits_{ag'=\{s,p,f\}} p^s_j q^{s,ag'}_j}\left((\sigma-1)\widehat{P^{ag}_j}+\widehat{E^{ag}_j}-\sigma\widehat{ta^{s,ag}_j}\right)$$
$$\tag{39}$$

Finally, we can express the compositional margin, CM, as follows, using the distribution function of the Pareto distribution in equation (20) and the expression for $r_{ij}(\varphi)$ in equation (B.3):

$$CM = -\theta\widehat{\varphi^*_{ij}}\left(\frac{\theta-\sigma+1}{\theta}-1\right) = (\sigma-1)\widehat{\varphi^*_{ij}} \tag{40}$$

Adding up the three margins, we can express the overall margin thus as follows:

$$
\begin{aligned}
d\ln V_{ij} &= TM = EM + IM + CM \\
&= -\frac{\theta-\sigma-1}{\sigma-1}\mu\widehat{p_{Z_i}} - (1-\mu)\frac{\theta-\sigma-1}{\sigma-1}\widehat{p_{Z_j}} + \widehat{NE}_i - \left(\theta + \frac{\theta-\sigma-1}{\sigma-1}\right)\widehat{ta_{ij}} - \theta\widehat{\tau_{ij}} \\
&- \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\right) - \frac{\theta-\sigma-1}{\sigma-1}\widehat{f_{ij}} + \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p^s_j q^{s,ag}_j}{\sum\limits_{ag'=\{s,p,f\}} p^s_j q^{s,ag'}_j}\left((\sigma-1)\widehat{P^{ag}_j} + \widehat{E^{ag}_j} - \sigma\widehat{ta^{s,ag}_j}\right)
\end{aligned}
\tag{41}
$$

4 Parameter Estimation

In the Armington and Ethier-Krugman model we only need estimates of the substitution elasticity, whereas the firm heterogeneity model requires estimates of both the substitution elasticity σ and the shape parameter θ of the productivity distribution. In the Eaton and Kortum model we need estimates of the dispersion parameter of the productivity distribution ρ. We write down the gravity equation of our general model to reveal which parameters can be identified by estimating a gravity equation. The value of sales from country i to country j in cif-terms, v_{ij}, follows from equation (5). Since p_{ij} is the price inclusive of bilateral tariffs ta_{ij}, we have to divide $p_{ij}q_{ij}$ by ta_{ij} to get the value of trade in cif-terms:

$$
v_{ij} = \frac{p_{ij}q_{ij}}{ta_{ij}} = \frac{p^{1-\sigma}_{ij}}{ta_{ij}}\left(p^m_j\right)^{-\sigma} q^m_j = ta^{-\sigma}_{ij}\left(t_{ij}c_i\left(te_{ij}b_i p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}\right)\right)^{1-\sigma}\left(p^m_j\right)^{\sigma} q^m_j \tag{42}
$$

Since we have observable values for tariffs ta_{ij}, we employ estimates of the tariff elasticity in the different models to identify the parameters.[5] Equation (42) shows that σ is equal to the tariff elasticity in the Armington and Ethier-Krugman model, where t_{ij} is equal to 1. In the Melitz model instead t_{ij} is a function of bilateral tariffs ta_{ij} implying that the tariff elasticity is not equal to σ. Substituting the expression for t_{ij} in equation (23) into the general gravity equation (42) gives:

$$
v_{ij} = ta_{ij}^{-\left(\theta+1+\frac{\theta-\sigma+1}{\sigma-1}\right)}\left(te_{ij}b_i p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}\right)^{-\theta}\tau^{-\theta}_{ij} f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}} c_i \left(p^m_j\right)^{\sigma} q^m_j \tag{43}
$$

[5] Some papers in the recent quantitative trade models literature concentrate estimation of the trade elasticity, the elasticity of trade values with respect to iceberg trade costs. In some models the trade elasticity is equal to the tariff elasticity. Since we do not have values for iceberg trade costs and since the trade elasticity deviates from the tariff elasticity in the Melitz model, we do not focus on the trade elasticity. The trade elasticity is equal to $\sigma-1$, $\sigma-1$, θ and ρ in respectively the Armington, Ethier-Krugman, Melitz and Eaton-Kortum model.

The tariff elasticity is determined by both σ and θ, so additional information is required to identify both parameters. The reason why the tariff elasticity is not identical to the trade elasticity θ is twofold. First, we estimate the gravity equation employing cif-values and therefore have to divide by the power of the tariff ta_{ij} implying a tariff elasticity $\theta + 1$. Second, in the Melitz model tariffs affect trade flows also through the cutoff productivity. Higher tariffs reduce trade flows because less firms can enter the market profitably (the extensive margin relative to compositional margin effect), responsible for the second part $(\frac{\theta-\sigma+1}{\sigma-1})$ of the elasticity. As discussed in Appendix B this additional effect occurs with tariffs based on the landed price (revenue shifting). Since iceberg trade costs τ_{ij} and export taxes te_{ij} are based on the cost-price (cost-shifting), the additional effect through the cutoff productivity is absent in the elasticities of these variables.

We discuss three possibilities to identify both parameters in the Melitz model in combination with the tariff elasticity $\theta + 1 + \frac{\theta-\sigma+1}{\sigma-1}$. First, we can try to find observable trade costs that are proportional with iceberg trade costs τ_{ij} or fixed trade costs f_{ij}. Although fixed trade cost measures are available such as the World Bank cost of doing business data, we do not have information to determine whether these measures are exactly or more or less than proportional with fixed trade costs. Therefore, this is a not a viable option. Second, we can use information on the international transport margin to identify θ. Therefore, we rewrite equation (43) as follows:

$$v_{ij} = ta_{ij}^{-\left(\theta+1+\frac{\theta-\sigma+1}{\sigma-1}\right)} \left(te_{ij} b_i p_{Z_i}\right)^{-\theta} \left(1 + itm_{ij}\right)^{-\theta} \tau_{ij}^{-\theta} f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}} c_i \left(p_j^m\right)^{\sigma} q_j^m$$

With itm_{ij} the international transport margin defined as the value of payments to international transport services $vits_{ij}$ divided by the fob-value of trade, v_{ij}^{fob}, $itm_{ij} = \frac{p_{ij}^{tr} tr_{ij}}{te_{ij} b_i p_{Z_i} q_{ij}^{fob}}$. The coefficient on one plus the international transport margin thus enables us to identify θ and with the tariff elasticity we can then obtain σ. We can use data on the international transport margin from the GTAP dataset. Third, we can use the fact that a productivity distribution with shape parameter θ implies a firm size distribution with a shape parameter equal to $\theta/(\sigma-1)$. So we can estimate $\theta/(\sigma-1)$ from log-firm-size-log-rank regressions (Axtell (2001), di Giovanni and Levchenko (2012)). We can estimate the firm-size shape parameter at the sectoral level using American firm level data provided by BEA. As an alternative we can follow Helpman, et al. (2004) and calculate the standard deviation of log firm sales from the US Census of

Manufacturing, which is equal to $\theta - (\sigma - 1)$ and thus also gives us estimates of σ for given θ.

As equation (43) shows, iceberg and fixed trade costs enter together in multiplicative form in the expression for trade flows and for import shares. This implies that we can use the conventional approach for Armington CGE-models and calibrate the combination of iceberg and fixed trade costs such that the trade shares in the baseline simulation are equal to the trade shares in the data. Therefore, we do not need information on the value of fixed trade costs separately. Balistreri (2012) estimate the source- and destination-specific components of fixed trade costs structurally from the model, but add a bilateral residual term to obtain a perfect fit between actual and fitted trade flows. We do not follow this route, since it is unclear to what extent source- and destination-specific components of fixed trade costs obtained in this way really represent fixed trade costs instead of iceberg trade costs, given that iceberg and fixed trade costs enter as a combined term in the theoretical gravity equation. So possible simulations on the effects of reductions in source- and destination-specific components of fixed trade costs do not properly inform us about the effects of reductions in fixed trade costs. Moreover, we think it is more interesting to include observable variables in the gravity equation and subsequently also in the CGE model to evaluate the effect of changing observable variables instead of unobservable source- and destination-specific components of fixed trade costs.

In the Eaton and Kortum model the value of trade is given by the same expression as the quantity of trade, except for the fact that the quantity demanded is replaced by the value demanded:

$$v_{ij} = \frac{p_{ij} q_{ij}}{ta_{ij}} = \frac{ta_{ij}^{-(\rho+1)} T_i \left(Z_i\right) \left(ta_{ij} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{-\rho}}{\sum\limits_{k=1}^{J} T_k \left(Z_k\right) \left(ta_{kj} \tau_{kj} \left(te_{kj} p_{Z_k} + \frac{p_{kj}^{tr}}{a_{kj}^{tr}}\right)\right)^{-\rho}} \sum_{ag \in \{p,g,f\}} E_j^{ag} \tag{44}$$

Equation (44) shows that the tariff elasticity employing the cif-value of trade in the gravity equation is equal to $\rho + 1$ in the Eaton and Kortum model.

5 Evaluating the Effect of Trade Cost Measures

6 Simulation Results

We implemented the changes to the GTAP model as described in Appendix D. We present simulation results of a model with 10 countries/regions and 10 sectors. We explored the ef-

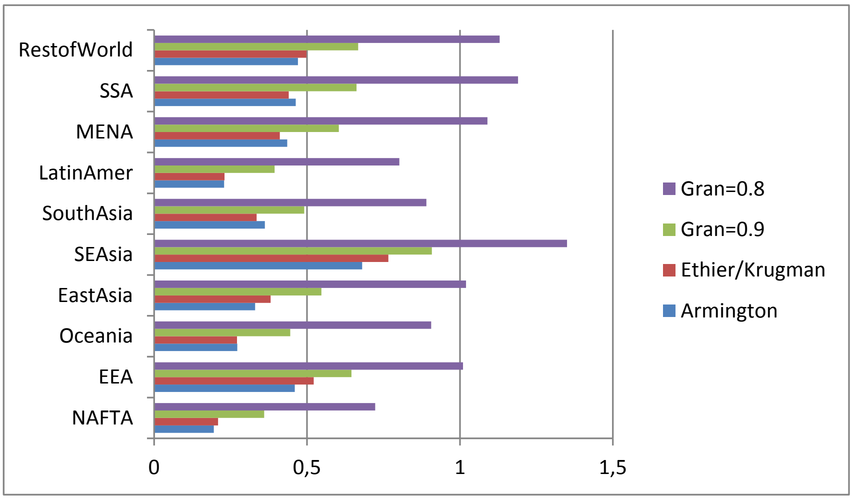

Figure 1: Effect of one percent reduction in iceberg trade costs on regional utility in percentage changes

fects of a reduction in iceberg trade costs by 1% in all sectors in the Armington model, the Ethier-Krugman model and the Melitz model, varying the degree of granularity of the firm size distribution in the latter. Figures (1)-(3) shows the effects on regional utility, world trade volumes, and world prices. The figures convey three clear messages. First, the positive welfare effects rise in the degree of granularity and the effects are larger in the Melitz model than in the Armington and Ethier-Krugman models. Second, changes in trade volumes do not vary much across the models. This can be explained from the fact that the supply-side and demand-side externalities also operate on domestic sales and thus do not lead to an extra incentive to trade internationally in the Ethier-Krugman and Melitz model in comparison to the Armington model. Third, the differences between the Armington and Ethier-Krugman model are small. The likely reason for this is that Ethier-Krugman scale effects operate in all sectors. So economies cannot benefit much from scale effects, since increasing resources in one industry imply reduced resources in other industries.

We also examined the effect of a reduction in fixed trade costs at varying degrees of granularity. Figures (4)-(6) display the effect of a 10% reduction in fixed trade costs on regional utility, world trade volumes, and world prices. The figures show that the welfare, trade volume and price effects are all stronger with a less granular firm size distribution where small firms are

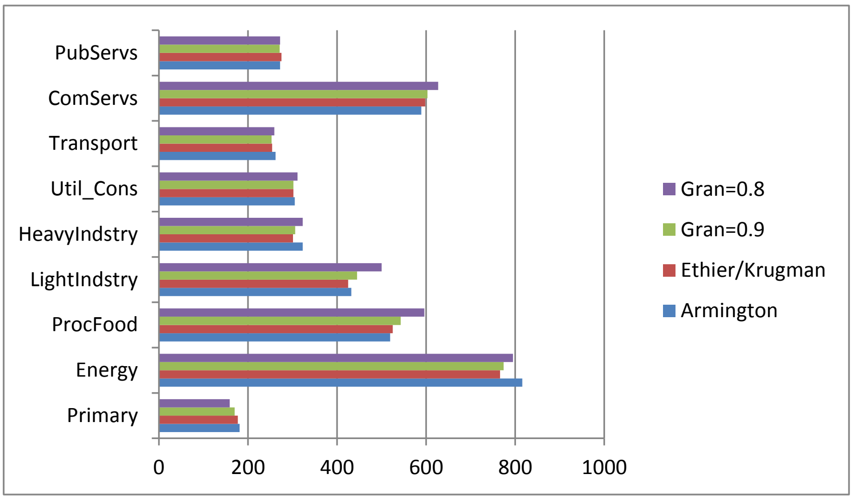

Figure 2: Effect of one percent reduction in iceberg trade costs on world trade volumes in percentage changes

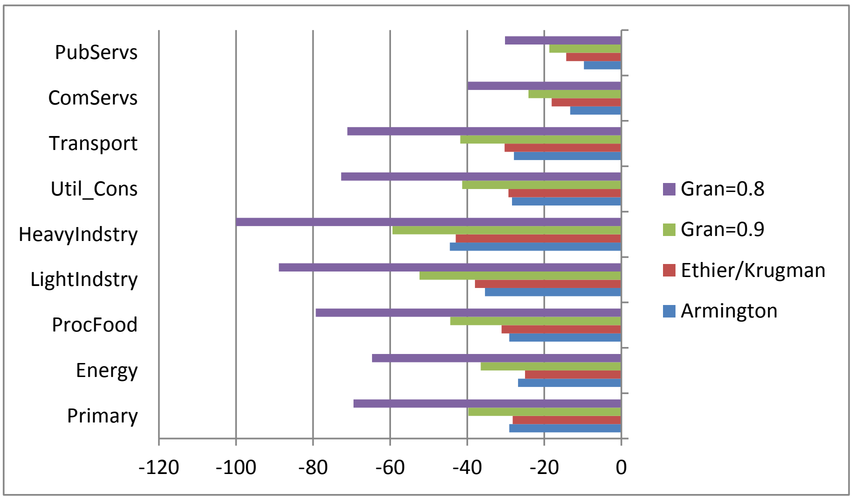

Figure 3: Effect of one percent reduction in iceberg trade costs on world trade prices in percentage changes

relatively more important. This reflects that the extensive margin relative to the compositional margin becomes more important as the firm size distribution becomes less granular. With a bigger role for small firms, fixed trade costs matter more. In contrast to reductions in iceberg trade costs, trade volumes also rise more with a less granular firm size distribution. The effects of iceberg trade cost reductions do not rise with a reduction in granularity. This is clear from the decomposition in Section 3, showing that the overall effect of reductions in τ is a function solely of θ, whereas the coefficient on fixed trade costs is $\frac{\theta-\sigma+1}{\sigma-1}$ and thus falls in the degree of granularity.

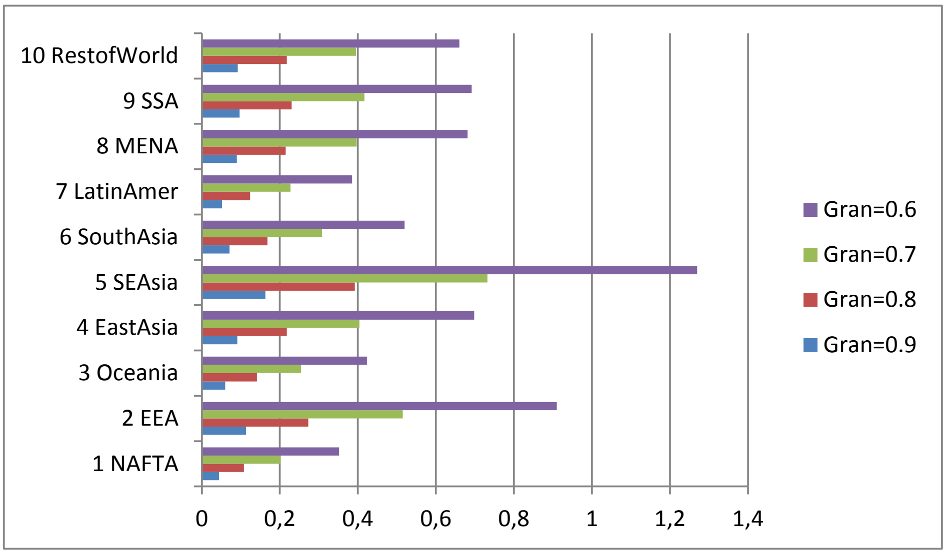

Figure 4: Effect of ten percent reduction in fixed trade costs on regional utility in percentage changes

7 Concluding Remarks

We have shown that both the Ethier-Krugman monopolistic competition model and the Melitz firm heterogeneity model can be defined as an Armington representative agent model. This representation of these two models also makes clear that the Melitz model generates the same equilibrium outcome as the Ethier-Krugman model when the firm size distribution is granular.

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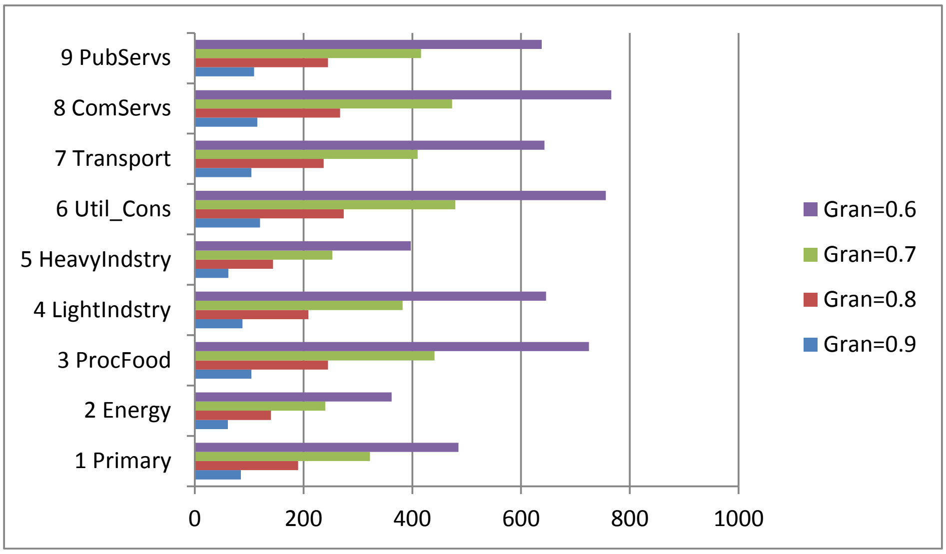

Figure 5: Effect of ten percent reduction in fixed trade costs on world trade volumes in percentage changes

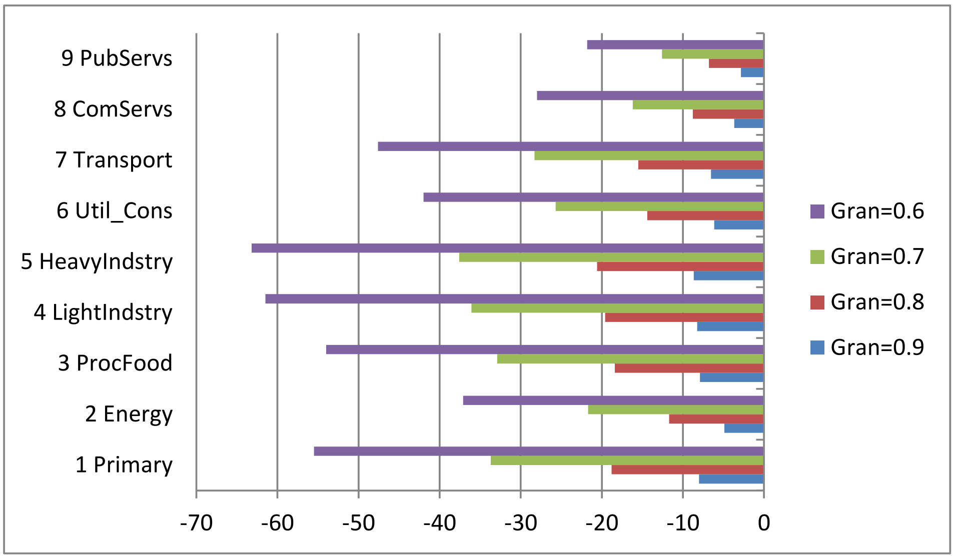

Figure 6: Effect of ten percent reduction in fixed trade costs on world trade prices in percentage changes

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Appendix A Ethier/Krugman Economy

The goal of this section is to derive the expressions for c_i and t_{ij} in the main text in equations (15)-(18). Before we go into the Ethier-Krugman model, we first rewrite the expressions for demand and the price index in the general model. The general setup-expressions for $q_{ij}e_j^s$ and P_j^{ag} implied by equations (3)-(9) are given by:

$$q_{ij}e_j^s = \left(\frac{p_{ij}}{e_j^s}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}} \tag{A.1}$$

$$P_j^{ag} = \left(\sum_{i=1}^{J} \left(\frac{p_{ij}ta_j^{s',ag}}{e_j^{s\prime}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{A.2}$$

With p_{ij} defined as follows:

$$p_{ij} = ta_{ij}t_{ij}c_i \left(te_{ij}b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) \tag{A.3}$$

With $s' = d$ if $i = j$ and $s' = m$ if $i \neq j$ and $ta_{ii} = t_{ii} = te_{ii} = 1$ and $\frac{p_{ii}^{tr}}{a_{ii}^{tr}} = 0$.

To show equivalence between the general model-representation and the normal representation of different models, we have to show that the expressions for demand in equation (A.1) and for the price index in equation (A.2) with the appropriate choices for c_i, t_{ij} and e_j^s in the general model-representation are identical to the demand and price index expressions in the normal representation of the different models.

In the Ethier-Krugman model agents of group $ag = \{s, p, f\}$ with g government, p private sector and f firms in country j have CES preferences over physical quantities $o(\omega)$ of varieties ω from different countries. The quantity and price index are defined in equations (13)-(14). Demand for a variety ω shipped from i to j and sold to group ag is equal to:

$$o_{ij}(\omega) = \sum_{ag=\{s,p,f\}} \left(ta_j^{ag}p_{ij}(\omega)\right)^{-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag} \tag{A.4}$$

Varieties are produced by identical firms with an increasing returns to scale technology with fixed cost a_i and marginal cost b_i, implying that each firm produces a unique variety. As firms are identical, ω can be dropped in the remainder.

Firms face iceberg trade costs τ_{ij}, bilateral export taxes te_{ij}, bilateral import tariffs ta_{ij}, and group specific import tariffs ta_j^{ag}. Moreover, there is a transport sector with firms having to spend a fixed quantity share of sales on transport services. Technically, the cif-quantity traded o_{ij}^{cif} is a Leontief function of the quantity in fob-terms o_{ij}^{fob} and transport services tr_{ij}:

$$o_{ij}^{cif} = \min\left(o_{ij}^{fob}, a_{ij}^{tr} tr_{ij}\right) \tag{A.5}$$

Profits are therefore given by:

$$\begin{aligned}\pi_{ij} &= ta_j^{ag} p_{ij}^o o_{ij} - \left(ta_j^{ag} - 1\right) p_{ij}^o o_{ij} - \frac{ta_{ij} - 1}{ta_{ij}} p_{ij} o_{ij} - \tau_{ij}\left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) o_{ij} \\ &= \frac{p_{ij} o_{ij}}{ta_{ij}} - \tau_{ij}\left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) o_{ij} \end{aligned} \tag{A.6}$$

This expression for profit implies the following markup pricing rule:

$$p_{ij}^o = \frac{\sigma}{\sigma - 1} ta_{ij} \tau_{ij} \left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) \tag{A.7}$$

p_{ij}^o is the cif price of physical output o_{ij} before the group-specific import tariff ta_j^{ag} is applied. Firms do not face destination specific fixed costs and can enter all markets upon paying the fixed costs a_i. Profits from sales to all markets are thus equal to:

$$\pi_i = \sum_{j=1}^{J} \frac{p_{ij}^o o_{ij}}{\sigma ta_{ij}} - a_i p_{Z_i} \tag{A.8}$$

As a next step, N_i is defined as the mass of varieties produced in country i. N_i is identical for all destinations by absence of destination specific fixed costs. It follows from the following labor market equilibrium:

$$\left(\sum_{j=1}^{J} \tau_{ij} o_{ij} + a_i\right) N_i = Z_i \tag{A.9}$$

To rewrite this expression, we first rewrite the expression for $\tau_{ij} o_{ij}$ using the markup equation (A.7):

$$\tau_{ij} o_{ij} = \frac{\sigma - 1}{\sigma} \frac{p_{ij}^o o_{ij}}{p_{Z_i} ta_{ij}} + \frac{\sigma - 1}{\sigma} \frac{p_{ij}^o o_{ij}}{p_{Z_i} ta_{ij}} \left(\frac{1}{te_{ij} b_i + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} - 1\right) \tag{A.10}$$

Using equations (A.8) and (A.10), we can solve for N_i from equation (A.9) as follows:

$$N_i = \frac{\widetilde{Z}_i}{\sigma a_i} = \frac{Z_i - \frac{\sigma-1}{\sigma}\left(\sum_{j=1}^{J}\frac{N_i\bar{r}_{ij}}{p_{Z_i}ta_{ij}\left(te_{ij}b_i + \frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}\right)} - \frac{N_i\bar{r}_{ij}}{p_{Z_i}ta_{ij}}\right)}{\sigma a_i} \tag{A.11}$$

With $\widetilde{Z}_i$ as defined in equation (17).

The price index in (14) can be written as equation (A.2) with $e_j^s = 1$ and p_{ij} defined as:

$$p_{ij}ta_j^{s,ag} = \left(\int\limits_{\omega\in\Omega_{ij}^{ag}} p^{ag,o}(\omega)^{1-\sigma}\, d\omega\right)^{\frac{1}{1-\sigma}} \tag{A.12}$$

Therefore, we only need to elaborate on $p_{ij}ta_j^{s,ag}$ to show equivalence of the price index. Given that all firms are identical and all varieties N_i are exported to all destinations, equation (A.12) can be rewritten as:

$$p_{ij}ta_j^{s,ag} = N_i^{\frac{1}{1-\sigma}} ta_j^{s,ag} p_{ij}^o = \left(\frac{\widetilde{Z}_i}{\sigma a_i}\right)^{\frac{1}{1-\sigma}} ta_j^{s,ag} p_{ij}^o \tag{A.13}$$

Substituting equation (A.7) for p_{ij}^o leads to:

$$p_{ij} = ta_j^{s,ag} ta_{ij}\tau_{ij}\left(\frac{\widetilde{Z}_i}{\sigma a_i}\right)^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\left(te_{ij}b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right) \tag{A.14}$$

Equation (A.14) shows that the externality is applied after expenditures on the transport sector have been incurred. t_{ij} is thus equal to 1 and we can write generalized marginal costs c_i thus as follows with $\widetilde{Z}_i$ as defined in equation (17):

$$c_i = \frac{\sigma}{\sigma-1}\left(\frac{\widetilde{Z}_i}{\sigma a_i}\right)^{\frac{1}{1-\sigma}}$$

Appendix B Melitz Economy

Appendix B.1 Demand and Production

Like in the Ethier/Krugman economy the goal of this section is to derive the expressions for generalized marginal costs c_i, generalized iceberg trade costs t_{ij} and the demand externality e_j in the Melitz economy in equations (21)-(24) and to derive the demand externality.

Agents of group ag in country j have the same CES preferences over varieties ω from different countries as in the Ethier/Krugman economy. The quantity and price index are thus given by equations (13)-(14) and demand for physical quantities $o_{ij}(\omega)$ of a variety ω by equation (A.4). In contrast to the Ethier/Krugman economy goods are produced by firms with heterogeneous productivity. Firms can sell both in domestic and foreign markets and have to pay fixed costs f_{ij} to sell in each market. The fixed costs are paid in wages of both countries with according to a Cobb Douglas specification a fraction μ paid in domestic input bundles. The fixed costs are destination-specific, but not agent-specific. So a firm pays the fixed costs ij only once for sales to all three groups of agents. Exporting firms also face iceberg trade costs τ_{ij}, bilateral tariffs ta_{ij}, agent-specific tariffs ta_j^g, export taxes te_{ij}. Moreover, there is a transport sector with firms having to spend a fixed quantity share of sales on transport services as in the Ethier-Krugman model with the cif-quantity traded o_{ij}^{cif} defined as in equation (A.5). Profits are therefore given by:

$$\begin{aligned}\pi_{ij} &= ta_j^{ag} p_{ij}^o o_{ij} - \left(ta_j^{ag} - 1\right) p_{ij}^o o_{ij} - \frac{ta_{ij} - 1}{ta_{ij}} p_{ij}^o o_{ij} - \tau_{ij}\left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\frac{o_{ij}}{\varphi} \\ &= \frac{p_{ij}^o o_{ij}}{ta_{ij}} - \tau_{ij} p_{Z_i}\left(te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}\right)\frac{o_{ij}}{\varphi} \end{aligned} \tag{B.1}$$

We assume that productivity φ operates both on the costs of production and on the transport sector.[6] Each firm produces a unique variety, so we can identify demand for variety ω by the productivity φ of the firm producing this variety. Demand $o_{ij}(\varphi)$ and revenues $r_{ij}(\varphi)$ of a firm with productivity φ producing in i and selling in j are equal to:

$$o_{ij}(\varphi) = \sum_{ag=\{s,p,f\}} \left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag} \tag{B.2}$$

$$r_{ij}(\varphi) = \sum_{ag=\{s,p,f\}} \left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag} \tag{B.3}$$

Maximizing profits implies the following markup pricing rule:

$$p_{ij}^o(\varphi) = \frac{\sigma}{\sigma-1} \frac{ta_{ij}\tau_{ij}\left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi} \tag{B.4}$$

Substituting equation (B.4) back into equation (B.1) shows that profits for sales to destination market j are equal to:

[6] In line with the GTAP model we define p_{ij}^o as the price before group specific import tariffs ta_j^{ag} are paid.

$$\pi_{ij}(\varphi) = \sum_{ag=\{s,p,f\}} \frac{\left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}\sigma} - f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \tag{B.5}$$

So we add up the revenues for sales to the three groups of agents to calculate profit. In the profit expression in equation (B.1) we have assumed that the bilateral tariffs ta_{ij} and the group-specific importer-specific tariffs ta_j^{ag} are paid both based on the marked-up price over marginal cost, respectively on the landed cif-price and on the landed cif-price inclusive of bilateral tariffs. Iceberg trade costs τ_{ij} and export taxes te_{ij} instead are paid based on the cost level, respectively the cif cost level (so inclusive of transport costs) and fob cost level. Both types of trade costs (based on marked-up landed prices and based on cost levels) affect the optimal markup price in equation (B.4) identically, but they affect the expression for profit as a function of revenues in equation (B.5) differently. Revenues are divided by import tariffs based on landed prices to calculate profit. Import tariffs are therefore revenue-shifting, whereas iceberg trade costs and export subsidies are cost-shifting. The distinction is relevant for the gravity equation in the Melitz model, since the revenue shifting tariffs affect the cutoff productivity and therefore display a different elasticity.

Appendix B.2 Entry and Exit

Entry and exit are like in Melitz (2003), i.e. firms can draw a productivity parameter φ from a distribution $G_i(\varphi)$ after paying a sunk entry cost en_i. The productivity of firms stays fixed and firms face a fixed death probability δ in each period. Firms either decide to start producing for at least one of the markets or leave the market immediately. In equilibrium there is a steady state of entry and exit with a steady number of entrants NE_i drawing a productivity parameter, implying that the productivity distribution of producing firms is constant. Denoting φ_{ij}^* as the cutoff productivity, only firms with a productivity $\varphi \geq \varphi_{ij}^*$ from country i sell in market j.

Appendix B.3 Free Entry and Zero Cutoff Profit Conditions

Equilibrium is defined with a zero cutoff profit condition (ZCP) and a free entry condition (FE). According to the zero cutoff profit condition firms from country i with cutoff productivity φ_{ij}^*

can just make zero profit from sales in country i:

$$\sum_{ag=\{p,g,f\}} \frac{\left(ta_j^{ag} p_{ij}^{o}\left(\varphi\right)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \tag{B.6}$$

Since the fixed costs are destination-specific and not group-specific there is only one ZCP for each source-destination pair and thus also only one cutoff productivity level φ_{ij}^{*}. Using equations (B.3)-(B.5) the ZCP can be written as follows:

$$\varphi_{ij}^{*} = \frac{\frac{\sigma}{\sigma-1} ta_{ij} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} ta_{ij}\right)^{\frac{1}{1-\sigma}}} \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}\right)^{\frac{1}{1-\sigma}} \tag{B.7}$$

The free entry condition (FE) equalizes the expected profits before entry with the sunk entry costs:

$$\sum_{ag=\{p,g,f\}} \sum_{j=1}^{J} \left(1 - G_i\left(\varphi_{ij}^{*}\right)\right) \pi_{ij}^{ag}\left(\widetilde{\varphi}_{ij}\right) = \delta en_i p_{Z_i} \tag{B.8}$$

$\widetilde{\varphi}_{ij}$ is a measure of average productivity and defined as:

$$\widetilde{\varphi}_{ij} = \left(\int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-1} \frac{g_i\left(\varphi\right)}{1 - G_i\left(\varphi_{ij}^{*}\right)} d\varphi\right)^{\frac{1}{\sigma-1}} \tag{B.9}$$

Using $\frac{r_{ij}^{ag}(\varphi_1)}{r_{ij}^{ag}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$ and the ZCP in equation (B.6), the FE in equation (B.8) can be written as:

$$\sum_{j=1}^{J} \left(1 - G_i\left(\varphi_{ij}^{*}\right)\right) p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} f_{ij} \left(\left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^{*}}\right)^{\sigma-1} - 1\right) = \delta en_i p_{Z_i} \tag{B.10}$$

The distribution of initial productivities $G_i\left(\varphi\right)$ is Pareto:

$$G_i\left(\varphi\right) = 1 - \frac{\kappa_i^{\theta}}{\varphi^{\theta}} \tag{B.11}$$

with θ the shape parameter and κ_i the size parameter. We impose $\theta > \sigma - 1$ to guarantee that expected revenues are finite. With a Pareto distribution $\widetilde{\varphi}_{ij}$ is proportional to φ_{ij}^{*}:

$$\widetilde{\varphi}_{ij} = \left(\frac{\theta}{\theta - \sigma + 1}\right)^{\frac{1}{\sigma-1}} \varphi_{ij}^{*} \tag{B.12}$$

Substituting equations (B.11)-(B.12) into the fe, equation (B.10), gives:

$$\sum_{j=1}^{J}\left(\frac{\kappa_i}{\varphi_{ij}^{*}}\right)^{\theta} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} f_{ij}\frac{\sigma-1}{\theta-\sigma+1}=\delta en_i p_{Z_i} \tag{B.13}$$

Appendix B.4 Equivalence of The Price Index

To show equivalence of the price index in the general representation version of the Melitz model and the normal version, we write the price index in (14) as equation (A.2) with the representative price $\frac{p_{ij}ta_j^{s,ag}}{e_j^s}$ defined as:

$$\frac{p_{ij}ta_j^{s,ag}}{e_j^s}=\left(\int\limits_{\omega\in\Omega_{ij}^{ag}} p^{o}\left(\omega\right)^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}} \tag{B.14}$$

$\frac{p_{ij}ta_j^{s,ag}}{e_j^s}$ is the representative price including the demand externality. The representative price in equation (B.14) can be redefined as an integral over productivities of the producing firms as follows:

$$\frac{p_{ij}ta_j^{s,ag}}{e_j^s}=\left(\int\limits_{\varphi_{ij}^{*}}^{\infty} N_{ij}p_{ij}^{ag,o}\left(\varphi\right)^{1-\sigma}\frac{g_i\left(\varphi\right)}{1-G_i\left(\varphi_{ij}^{*}\right)}d\varphi\right)^{\frac{1}{1-\sigma}} \tag{B.15}$$

Using equations (B.4) and (B.9) the representative price in equation (B.15) can be rewritten as a function of average productivities:

$$\frac{p_{ij}ta_j^{s,ag}}{e_j^s}=\frac{\sigma}{\sigma-1}\left(N_{ij}\left(ta_{ij}ta_j^{s,ag}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}\widetilde{\varphi}_{ij}^{\sigma-1}\right)^{\frac{1}{1-\sigma}} \tag{B.16}$$

The mass of varieties sold from country i to country j, N_{ij} is related to the mass of entrants NE_i and the cutoff productivity φ_{ij}^{*} by the following steady state condition:

$$N_{ij}=\frac{\left(1-G_i\left(\varphi_{ij}^{*}\right)\right)NE_i}{\delta}=\left(\frac{\kappa_i}{\varphi_{ij}^{*}}\right)^{\theta}\frac{NE_i}{\delta} \tag{B.17}$$

The steady state of entry and exit implies that NE_i can be written as a function of the number of input bundles Z_i:

$$NE_i = \frac{\sigma-1}{\theta\sigma}\frac{\widetilde{Z}_i}{en_i} = \frac{\sigma-1}{\theta\sigma}\frac{Z_i - \sum_{j=1}^{J} N_{ij}\frac{\sigma-1}{\sigma}\left(\frac{1-te_{ij}+\frac{p^{tr}_{ij}}{p_{Z_i}a^{tr}_{ij}}}{te_{ij}+\frac{p^{tr}_{ij}}{p_{Z_i}a^{tr}_{ij}}}\right)\frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i}ta^{g}_{j}ta_{ij}}}{en_i} \tag{B.18}$$

Since $N_{ij}\overline{r}_{ij}\left(\widetilde{\varphi}_{ij}\right)$ is equal to the value of trade (inclusive of bilateral import tariffs ta_{ij}, but inclusive of group- and importer-specific tariffs ta^{ag}_{j}) and thus equal to $N_i\overline{r}_{ij}$ in the Ethier-Krugman model, we can use the same definition for $\widetilde{Z}_i$ in both models. Using equations (B.12), (B.17) and (B.18), the representative price in equation (B.16) can be written as:

$$\frac{p_{ij}ta^{s,ag}_{j}}{e^{s}_{j}} = \frac{\sigma}{\sigma-1}\left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)}\frac{\kappa^{\theta}_{i}\widetilde{Z}_i}{\delta en_i}\frac{\left(ta_{ij}ta^{s,ag}_{j}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}\right)\right)^{1-\sigma}}{\left(\varphi^{*}_{ij}\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}} \tag{B.19}$$

The final step is to substitute the ZCP solved for φ^{*}_{ij} in equation (B.7) into equation (B.19) generating the following expression:

$$\frac{p_{ij}ta^{s,ag}_{j}}{e^{s}_{j}} = \left(\frac{\gamma_m\kappa^{\theta}_{i}\widetilde{Z}_i\left(te_{ij}p_{Z_i}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}\right)^{-\theta}p_{Z_i}^{-\frac{\theta+\sigma-1}{\sigma-1}\mu}\left(ta_{ij}^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}\left(ta^{s,ag}_{j}\right)^{1-\sigma}}{\delta en_i}\right)^{\frac{1}{1-\sigma}}$$
$$* \left(\frac{\sum_{ag=\{p,g,f\}}\left(\frac{P^{ag,e}_{j}}{ta^{s,ag}_{j}}\right)^{\sigma-1}\frac{E^{ag}_{j}}{ta^{s,ag}_{j}}}{p_{Z_j}^{1-\mu}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{B.20}$$

γ_m is defined in equation (22) in the main text. From equation (B.20) we can easily determine the source-specific component, c_i, the bilateral component, $ta_{ij}t_{ij}$, and the destination specific component, e^{s}_{j}, in equation (A.3), the general setup-expression for the price in the Melitz model. The source specific component in equation (B.20) is equal to:

$$c_i = \left(\frac{\gamma_m\kappa^{\theta}_{i}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} p_{Z_i}^{\mu\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{B.21}$$

The pairwise component in equation (B.20) is given by:

$$t_{ij}ta_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)=\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta}{\sigma-1}}\left(ta_{ij}\tau_{ij}\right)^{\frac{\theta}{\sigma-1}}\left(ta_{ij}f_{ij}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{B.22}$$

Rearranging leads to the expression for t_{ij} in the main text, equation (23):

$$t_{ij}=\left(\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta-\sigma+1}{\sigma-1}}\tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}}ta_{ij}^{\frac{\sigma(\theta-\sigma+1)}{(\sigma-1)^2}}f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right)\tau_{ij} \tag{B.23}$$

Finally, the destination specific terms in equation (B.20) represent the demand externality, giving:

$$e_j^s=\left(\frac{\sum_{ag=\{p,g,f\}}\left(\frac{P_j^{ag,e}}{ta_j^{s,ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{s,ag}}}{p_{Z_j}^{1-\mu}}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tag{B.24}$$

So we have shown that the general setup-expression for the price index in equation (A.2) employing expressions for c_i in equation (21), t_{ij} in equation (23) and e_j^s in equation (24) follows from a Melitz structure and is thus equivalent to a Melitz structure.

Appendix B.5 Equivalence of Quantity Index

To prove equivalence between the general setup and the Melitz setup, we also show that the general setup-expression for demand in equation (A.1) is equivalent to the expression for demand following from the Melitz structure. Substituting the expressions for t_{ij}, c_i and e_j^s into the expression for $q_{ij}e_j^s$ in equation (A.1) leads to:

$$\begin{aligned} q_{ij}e_j^s=&\left(\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta}{\sigma-1}}\tau_{ij}^{\frac{\theta}{\sigma-1}}ta_{ij}^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}}\left(\frac{\gamma_m\kappa_i^\theta\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}}\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right)^{-\sigma}\\ &*\left(\sum_{ag\in\{p,g,f\}}\left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{s,ag}}\right)^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}} \end{aligned} \tag{B.25}$$

Next we show that the expression for quantity $q_{ij}e_j^s$ inclusive of the demand-side externality starting from the Melitz-setup is identical to the expression in equation (B.25). We can write the quantity starting from the Melitz-setup as follows:

$$q_{ij}e_j^s = \left(\int\limits_{\omega \in \Omega_{ij}} o(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{B.26}$$

Redefining quantity in equation (B.26) as an integral over the productivity of producing firms gives:

$$q_{ij}e_j^s = \left(N_{ij} \int\limits_{\varphi_{ij}^*}^{\infty} o_{ij}(\varphi)^{\frac{\sigma-1}{\sigma}} \frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{\sigma}{\sigma-1}} \tag{B.27}$$

Substituting the expression for $q_{ij}(\varphi)$ in equation (B.2), representative quantity in equation (B.27) can be written as a function of average productivity:

$$q_{ij}e_j^s = N_{ij}^{\frac{\sigma}{\sigma-1}} o_{ij}(\widetilde{\varphi}_{ij}) \tag{B.28}$$

The next step is to use $\frac{o_{ij}(\varphi_1)}{o_{ij}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}$ and equation (B.12) to write $o_{ij}(\widetilde{\varphi}_{ij})$ as a function of cutoff quantity $o_{ij}\left(\varphi_{ij}^*\right)$:

$$q_{ij}e_j^s = N_{ij}^{\frac{\sigma}{\sigma-1}} o_{ij}\left(\varphi_{ij}^*\right) \left(\frac{\theta}{\theta-\sigma+1} \right)^{\frac{\sigma}{\sigma-1}} \tag{B.29}$$

The ZCP in equation (B.6) can be employed to express cutoff quantity $o_{ij}^{ag}\left(\varphi_{ij}^*\right)$ as follows:

$$o_{ij}\left(\varphi_{ij}^*\right) = (\sigma-1) \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij}\left(p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)} \varphi_{ij}^* \tag{B.30}$$

Substituting equation (B.30) and also the expressions for N_{ij} and NE_i in equations (B.17)-(B.18) into equation (B.29) leads to:

$$q_{ij}e_j^s = \frac{\left(\frac{\sigma-1}{\sigma(\theta-\sigma+1)}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) \left(\frac{\kappa_i^{\theta} \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij}\left(p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}}{\left(\varphi_{ij}^*\right)^{\frac{\theta\sigma-\sigma+1}{\sigma-1}}} \tag{B.31}$$

Finally, the ZCP solved for φ_{ij}^* in equation (B.7) can be substituted into equation (B.31) and after several rearrangings, we get the same expression as the general setup-expression in equation (B.25).

Appendix B.6 Checking the Code in GAMS

As a check on the correctness of the expressions, we show in GAMS that a solution of the model in a setting with 10 countries generates the same solution using the initial equilibrium conditions of the Melitz firm heterogeneity model as using the single equilibrium condition. We work with a version of the model without intermediate linkages. The input bundle Z_i and its price p_{Z_i} will be equal to respectively factor input bundIes L_i and its price w_i. Imposing the general equilibrium condition that output $w_i L_i$ is equal to the value of exports to all destination countries j, leads to:

$$w_i L_i = \sum_{j=1}^{J} \frac{\alpha_{ij}^{\sigma} \left(t_{ij} c_i w_i\right)^{1-\sigma}}{\sum\limits_{k=1}^{J} \alpha_{kj}^{\sigma} \left(t_{kj} c_k w_k\right)^{1-\sigma}} w_j L_j \tag{B.32}$$

We have used in equation (B.32) that the absence of tariffs and trade imbalances implies that demand E_j is equal to $w_j L_j$.

Substituting the expressions for t_{ij} and c_i in the Melitz economy in equations (23)-(21) and abstracting from transport services and export taxes gives:

$$w_i L_i = \sum_{j=1}^{J} \frac{\frac{\kappa_i^{\theta}}{\delta en_i} L_i w_i^{-\left(\theta + \mu \frac{\theta - \sigma + 1}{\sigma - 1}\right)} \alpha_{ij}^{\frac{\sigma\theta}{(\sigma-1)}} \tau_{ij}^{-\theta} f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}}{\sum\limits_{k=1}^{J} \frac{\kappa_k^{\theta}}{\delta en_k} L_k w_k^{-\left(\theta + \mu \frac{\theta - \sigma + 1}{\sigma - 1}\right)} \alpha_{kj}^{\frac{\sigma\theta}{(\sigma-1)}} \tau_{kj}^{-\theta} f_{kj}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}} w_j L_j \tag{B.33}$$

With J equations (B.33) the model can be solved for J unknown w_i. We use population for the number of workers and fitted trade costs from the gravity regressions on distance for the biggest 10 countries in terms of population from the sample, the countries Bangladesh, Brazil, China, Indonesia, India, Nigeria, Pakistan, Russia and USA.

For the model with the full set of equations we use the following conditions: the expression for the price index following from equation (B.19); the expression for the number of varieties following from equations (B.17) and (B.18); a demand equation; an expression for cutoff revenues following from equation (B.3); a markup pricing expression in equation (B.4); and a zero cutoff profit condition in equation (B.6). The free entry condition is substituted in both the expression for the number of varieties and the demand equation. This gives the following set of equations.

$$(P_i)^{1-\sigma} = \sum_{j=1}^{J} N_{ji} \frac{\theta}{\theta - \sigma + 1} p_{ji} \left(\varphi_{ji}^*\right)^{1-\sigma} \tag{B.34}$$

$$N_{ij} = \left(\frac{\kappa_i}{\varphi_{ij}^*}\right)^{\theta_i} \frac{\sigma - 1}{\sigma\theta_i} \frac{Z_i}{\delta en_i} \tag{B.35}$$

$$p_{Z_i} Z_i = \sum_{j=1}^{J} N_{ij} \frac{\theta}{\theta - \sigma + 1} r_{ij} \left(\varphi_{ij}^*\right) \tag{B.36}$$

$$r_{ij} \left(\varphi_{ij}^*\right) = p_{ij} \left(\varphi_{ij}^*\right)^{1-\sigma} \left(P_i^e\right)^{\sigma-1} E_j \tag{B.37}$$

$$p_{ij} \left(\varphi_{ij}^*\right) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} p_{Z_i}}{\varphi_{ij}^*} \tag{B.38}$$

$$r_{ij} \left(\varphi_{ij}^*\right) = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{\mu} \tag{B.39}$$

GAMS code available upon request shows that both representations of the model generate exactly identical outcomes for the price of input bundles when identical parameters and data for population and trade costs are used. As parameter values we used $\sigma = 3.8$, $\theta = 3.4$ and $\mu = 0.5$. The single equation code solves the baseline in 13 iterations in GAMS, whereas the code with all equations requires 398 iterations. With 10 countries this is still a relatively fast process, but with more than 100 countries it is likely to encounter problems in solving the model.

Appendix C Eaton and Kortum Economy

The main structure of the Eaton and Kortum economy is described in the main text. Given the Frechet distribution of productivities φ in equation (Frechet) the price p of a good sold from country i to j is als Frechet distributed:

$$G_{ij}(p) = 1 - \exp\left(\frac{T_i}{\left(\left(1 + ta_{ij}\right) \tau_{ij} p_{Z_i}\right)^{\rho}} p^{\rho}\right) \tag{C.1}$$

The realised price of variety ω in country j is the minimum price of all potential suppliers:

$$p_j(\omega) = \min\left\{p_{1j}(\omega), .., p_{Jj}(\omega)\right\} \tag{C.2}$$

Therefore, the distribution of prices in country j is given by:[7]

$$G_j(p) = 1 - \prod_{i=1}^{J} \left(1 - G_{ij}(p)\right) = 1 - \exp\left(-\Phi_j p^{\rho}\right) \tag{C.3}$$

With Φ_j defined as:

$$\Phi_j = \sum_{i=1}^{J} T_i \left(ta_j^{ag} ta_{ij} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) \right)^{-\rho} \tag{C.4}$$

The probability that country i delivers a good to country j for group ag is equal to:

$$\pi_{ij}^{ag} = \frac{T_i(Z_i) \left(ta_j^{ag} ta_{ij} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) \right)^{-\rho}}{\sum_{k=1}^{J} T_k(Z_k) \left(ta_j^{ag} ta_{kj} \tau_{kj} \left(te_{kj} p_{Z_k} + \frac{p_{kj}^{tr}}{a_{kj}^{tr}} \right) \right)^{-\rho}} \tag{C.5}$$

Since ta_j^{ag} is both in numerator and denominator, equation (C.5) is equivalent to the expression for π_{ij} in the main text in equation (27).

To show equivalence of the Eaton and Kortum quantity and price index equations (28)-(29) and the general representation equations (A.1)-(A.3) with c_i, t_{ij} and e_j^s as in equations (31)-(33), we substitute the expressions for c_i, t_{ij} and e_j^s into the general representation equations, imposing $\sigma = \rho$ in the quantity expression and $\sigma = \rho + 1$ in the price index expression. We start with the expression for the price index in equation (A.3), in turn replacing $\sigma - 1$ by ρ, substituting the expression for p_{ij} and the expressions for c_i, t_{ij} and e_j^s:

$$\begin{aligned}
P_j^{ag} &= \left(\sum_{i=1}^{J} \left(\frac{p_{ij} ta_j^{s,ag}}{e_j^s} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
&= \left(\sum_{i=1}^{J} \left(ta_{ij} \gamma_{eako} \left(T_i(Z_i)\right)^{-\frac{1}{\rho}} \tau_{ij} \left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) ta_j^{s,ag} \right)^{-\rho} \right)^{-\frac{1}{\rho}} \\
&= \gamma_{eako} \left(\sum_{i=1}^{J} T_i(Z_i) \left(ta_{ij} \tau_{ij} \left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) ta_j^{s,ag} \right)^{-\rho} \right)^{-\frac{1}{\rho}}
\end{aligned}$$

The expression for quantity in equation can be written as follows by using $E_j^{ag} = P_j^{ag} q_j^{ag}$, $\sigma = \rho$ and $e_j^s = 1$:

$$q_{ij} = p_{ij}^{-\rho} \sum_{ag \in \{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}} \right)^{\rho} q_j^{ag}$$

[7] The probability that a price in country j is smaller than p is equal to 1 minus the probability that none of the suppliers has a price smaller than p.

Substituting the expression for P_j^{ag} just derived and the definition of p_{ij} employing the expressions for c_i and t_{ij} in equations (31)-(32) leads to the expression for q_{ij} in the Eaton and Kortum model in equation (28) in the main text:

$$
\begin{aligned}
q_{ij} &= \left(ta_{ij}t_{ij}c_i\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{-\rho} \\
&* \sum_{ag\in\{p,g,f\}}\left(\frac{\left(\sum_{k=1}^{J}\left(ta_{kj}\gamma_{eako}\left(T_k\left(Z_k\right)\right)^{-\frac{1}{\rho}}\left(te_{kj}b_kp_{Z_k}+\frac{p_{kj}^{tr}}{a_{kj}^{tr}}\right)ta_j^{s,ag}\right)^{-\rho}\right)^{-\frac{1}{\rho}}}{ta_j^{s,ag}}\right)^{\rho} q_j^{ag} \\
&= \frac{T_i\left(Z_i\right)\left(ta_{ij}\tau_{ij}\left(te_{ij}b_ip_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{-\rho}}{\sum_{k=1}^{J}T_k\left(Z_k\right)\left(ta_{kj}\tau_{kj}\left(te_{kj}b_kp_{Z_k}+\frac{p_{kj}^{tr}}{a_{kj}^{tr}}\right)\right)^{-\rho}}\sum_{ag\in\{p,g,f\}}q_j^{ag}
\end{aligned}
$$

Appendix D Implementation in GTAP GEMPACK

We implement the Melitz structure with demand and supply side externalities and generalized iceberg trade costs in the GTAP model programmed in GEMPACK. We outline for each of the three topics first the blocks added to the GEMPACK code and then how the existing code is adjusted. Then we discuss parameterization in GEMPACK to continue this section with a discussion of how to move between the different models employing closure swaps. We finish this section with a discussion of the margin decomposition in GEMPACK. In the implementation we assume that all fixed exporting costs are paid in the source country, i.e. $\mu = 1$.

Appendix D.1 Supply-Side Externality

The supply-side externality in the Ethier-Krugman and Melitz model can be gathered by log differentiating respectively equations (15) and (21):

$$c_i = \gamma_{eako}\left(T_i\left(Z_i\right)\right)^{-\frac{1}{\rho}}$$

$$\widehat{c_i} = -\frac{1}{\sigma-1}\widehat{N_i} \tag{D.1}$$

$$\widehat{c_i} = -\frac{1}{\rho}\widehat{T_i\left(Z_i\right)} = -\frac{1}{\rho}\widehat{Z_i} \tag{D.2}$$

$$\widehat{c_i} = -\frac{1}{\sigma-1}\widehat{NE_i} + \frac{\theta-\sigma+1}{\left(\sigma-1\right)^2}\widehat{p_{Z_i}} \tag{D.3}$$

In GEMPACK we model respectively the Ethier-Krugman, Eaton-Kortum and Melitz supply-side externality as follows:

$$oscaleek(i,r) = ekscale(i,r) - [1/(\sigma-1)] * nne(i,r) \tag{D.4}$$

$$oscaleeako(i,r) = eakoscale(i,r) - \frac{1}{\sigma} * qo(i,r) \tag{D.5}$$

$$\begin{aligned} oscalem(i,r) &= mscale(i,r) - [1/(\sigma-1)] * nne(i,r) \\ &\quad + \frac{\theta-\sigma+1}{(\sigma-1)^2} * [ps(i,r) - pfactwld] \end{aligned} \tag{D.6}$$

In equation (D.5) we have used that in the Eaton-Kortum model $\sigma = \rho$.

We deflate the price change term $ps\,(i,r)$ in the calculation of the Melitz-externality in equation (D.6) by the numeraire $pfactwld$, such that a change in all prices does not change the size of the externality and is neutral. To move between the different supply-side externalities we add the following additional equation:

$$oscaleekm(i,r) = ekscale(i,r) + eakoscale\,(i,r) + emscale(i,r) - sext(i,r) \tag{D.7}$$

We use the same variable for the relative change in the number of firms in the Ethier-Krugman model and in the number of entrants in the Melitz model, $nne\,(i,r)$, since the two are identical. This becomes clear by log differentiating equation (A.11) or equivalently equation (B.18). In GEMPACK notation we get:

$$\begin{aligned} nneh(i,r) = {} & \frac{VOM\,(i,r)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum_{t=1}^{J}(VXMD\,(i,r,t) - VIWS\,(i,r,t))} qo\,(i,r) \\ & - \sum_{s=1}^{J} \frac{\frac{\sigma-1}{\sigma} VXMD\,(i,r,s)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum_{t=1}^{J}(VXMD\,(i,r,t) - VIWS\,(i,r,t))} (pcif\,(i,r,s) + qxs\,(i,r,s) \\ & - \frac{VXWD\,(i,r,s)}{VIWS\,(i,r,s)} (ps\,(i,r) + ao\,(i,r) - tx\,(i,r) - tx\,(i,r,s)) \\ & - \frac{VIWS\,(i,r,s) - VXWD\,(i,r,s)}{VIWS\,(i,r,s)} ptrans\,(i,r,s)) \\ & + \sum_{s=1}^{J} \frac{\frac{\sigma-1}{\sigma} VIWS\,(i,r,s)}{VOM\,(i,r) - \frac{\sigma-1}{\sigma}\sum_{t=1}^{J}(VXMD\,(i,r,t) - VIWS\,(i,r,t))} \\ & * (pcif\,(i,r,s) + qxs\,(i,r,s) - (ps\,(i,r) + ao\,(i,r))) - nne\,(i,r) \end{aligned} \tag{D.8}$$

So the expression for the number of varieties contains additional terms, reflecting the size of transport services and export subsidies to all destination partners. Moreover, we have to take into account that the variety scaling term has to be applied to the cif-price, so inclusive of transport costs, for the international price and quantity. Therefore, we have to write the iceberg trade costs technology shifter $ams\,(i,r,s)$ as a function of the supply-side externality. We cannot include the supply-side externality before the transport sector is added, since we would have to multiply all terms by $1/FOBSHR\,(i,r,s)$ which would be destination specific. Since the domestically sold goods do not feature transport costs, but do benefit from variety scaling, the variety scaling term also affects domestic prices and quantities, i.e. ppd, pgd and pfd and qpd, qgd and qfd.

Appendix D.2 Demand-Side Externality

To model the demand-side externality, we add a block to the model calculating the demand-side externality and we adjust the price and quantity expressions for domestic and imported goods for the three groups of agents, private households, governments and firms.

First, we discuss the additional block for the demand-side externality. Log differentiating the theoretical expression for the externality in equation (24) gives:

$$\widehat{e_j^s} = \sum_{ag=\{s,p,f\}} \frac{\left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}}{\sum_{ag'=\{s,p,f\}} \left(\frac{P_j^{ag'}}{ta_j^{s,ag'}}\right)^{\sigma-1} \frac{E_j^{ag'}}{ta_j^{s,ag'}}} \left(\frac{\theta-\sigma+1}{\sigma-1}\left(\widehat{P_j^{ag}} - \widehat{ta_j^{s,ag}}\right) + \frac{\theta-\sigma+1}{(\sigma-1)^2}\left(\widehat{E_j^{ag}} - \widehat{ta_j^{s,ag}}\right)\right) \tag{D.9}$$

Multiplying the numerator and denominator of the coefficient by $\left(p_j^s\right)^{1-\sigma}$, we can rewrite equation (D.9) as follows:

$$\widehat{e_j^s} = \sum_{ag=\{s,p,f\}} \frac{p_j^s q_j^{s,ag}}{\sum_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}} \left(\frac{\theta-\sigma+1}{\sigma-1}\left(\widehat{P_j^{ag}} - \widehat{ta_j^{s,ag}}\right) + \frac{\theta-\sigma+1}{(\sigma-1)^2}\left(\widehat{E_j^{ag}} - \widehat{ta_j^{s,ag}}\right)\right) \tag{D.10}$$

To find the equivalent expression in GTAP notation, we observe that $p_j^s q_j^{s,ag}$ represents the expenditures of group $ag = f, p, g$ on source $s = d, m, V, S, AG, M$. So, equation (D.10) can be written in GEMPACK notation as follows with $s = m, d$:

$$\begin{aligned} dscale1s\,(i,r) = {} & \frac{\theta-\sigma+1}{\sigma-1}\,(priceDs\,(i,r) - pfactwld) + \frac{\theta-\sigma+1}{(\sigma-1)^2}\,(valueDs\,(i,r) - pfactwld) \\ & - \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)^2} tariffDs\,(i,r) \end{aligned} \tag{D.11}$$

With $priceDs\,(i,r)$ the price index term of the externality in sector i in country r for source $s = d, m$, $valueDs\,(i,r)$ the value term and $tariffDs\,(i,r)$ the tariff term and defined for $s = m$ as (the expressions for $s = d$ are similar):

$$\begin{aligned} priceDm(i,r) = {} & SHRIPM * [pp(i,r)] + SHRIGM * [pg(i,r)] \\ & + sum(j, PROD\_COMM, SHRIFM(i,j,r)) * [pf(i,j,r)]) \end{aligned} \tag{D.12}$$

And:

$$\begin{aligned} valueDm(i,r) = {} & SHRIPM * [pp(i,r) + qp\,(i,r)] \\ & + SHRIGM * [pg(i,r) + qg\,(i,r)] \\ & + sum(j, PROD\_COMM, SHRIFM(i,j,r)) * [pf(i,j,r) + qf\,(i,j,r)]) \end{aligned} \tag{D.13}$$

And:

$$\begin{aligned} tariffDm(i,r) = {} & SHRIPM * tpm(i,r) + SHRIGM * tgm(i,r) \\ & + sum(j, PROD\_COMM, SHRIFM(i,j,r)) * tfm(i,j,r)) \end{aligned} \tag{D.14}$$

pp, pg, and pf are the relative price changes for private households, government and firms and qp, qg, and qf the quantity equivalents. $SHRIPM\,(i,r)$ is defined as:

$$SHRIPM\,(i,r) = \frac{VIPM\,(i,r)}{VIM\,(i,r)} \tag{D.15}$$

With $VIM\,(i,r)$ the sum of import demand at market prices:

$$VIM(i,r) = VIPM(i,r) + VIGM(i,r) + sum(j, PROD\_COMM, VIFM(i,j,r)) \tag{D.16}$$

$SHRIGM\,(i,r)$ and $SHRIFM\,(i,j,r)$ are defined similarly. As for the supply-side externality, we deflate the price and value changes (based on price changes) in the calculation of the externality by the numeraire, such that a change in all prices does not change the externality.

To determine how the expressions for domestic and importer demand and price for the three groups of agents in the GTAP model change, we define the domestic and importer price, inclusive of the externality and the agent-specific tax, $\widetilde{p}_j^{s,ag}$, as follows:

$$\widetilde{p}_j^{s,ag} = \frac{ta_j^{s,ag} p_j^{s,ag}}{e_j^s} \tag{D.17}$$

Log differentiating both equation (D.17) and the rewritten expression for demand in equation (3) gives:

$$\widehat{q_j^{s,ag}} = \sigma \left(\widehat{P_j^{ag,e}} - \widehat{\widetilde{p}_j^{s,ag}} \right) + \widehat{q_j^{ag,e}} - \widehat{e_j^s} \tag{D.18}$$

$$\widehat{\widetilde{p}_j^{s,ag}} = \widehat{ta_j^{s,ag}} + \widehat{p_j^{s,ag}} - \widehat{e_j^s} \tag{D.19}$$

The equivalent expressions in GTAP for domestic government goods is given by:

$$qgd(i,s) = ESUBD(i) * [pg(i,s) - pgd(i,s)] + qg(i,s) - Dextd(i,s) \tag{D.20}$$

$$pgd(i,s) = tgd(i,s) + pm(i,s) - Dextd(i,s) \tag{D.21}$$

$$pgm(i,s) = tgm(i,s) + pim(i,s) - Dextm(i,s) \tag{D.22}$$

with qgd and qg the domestic and total government demand; pgd, pgm and pg, the domestic, imported and overall price of government consumption; tgd and tgm the tax on domestic and imported government consumption; pm and pim the domestic and import price of goods; and $Dextd$ the domestic demand externality. So we model the demand externality as a technology shifter to domestic and imported demand.

Appendix D.3 Generalized Iceberg Trade Costs

The generalized iceberg trade costs are equal to the normal iceberg trade costs in the Armington, Ethier-Krugman and Eaton-Kortum model. Only in the Melitz model the two are distinct and generalized iceberg trade costs are defined in equation (23). Log differentiating this equation gives:

$$\widehat{t_{ij}} = \frac{\theta-\sigma+1}{\sigma-1}\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}} + \left(1+\frac{\theta-\sigma+1}{\sigma-1}\right)\widehat{\tau_{ij}}$$
$$+ \frac{\sigma(\theta-\sigma+1)}{(\sigma-1)^2}\widehat{ta_{ij}} + \frac{\theta-\sigma+1}{(\sigma-1)^2}\widehat{f_{ij}} \tag{D.23}$$

In the GTAP model (with all variables expressed in relative change terms) bilateral ad-valorem tariffs $\widehat{ta_{ij}}$ consist of import tariffs tm and tms and the iceberg trade costs $\widehat{\tau_{ij}}$ consist of an iceberg-trade-costs-like technology shifter ams. Tariffs are paid based on the marked-up prices, whereas iceberg trade costs and the transport margin operate on the physical quantities and are thus based on costs. As a result, the coefficient on tariffs in generalized trade costs is different.

Since both the generalized iceberg trade costs t_{ij} and the generalized marginal costs c_i are applied on the cif-price, we endogenize the iceberg-trade-cost-like technology shifter $ams\,(i,r,s)$ as a function of the supply-side externality $sext\,(i,r)$ and generalized iceberg trade costs. In GEMPACK notation we get in the Ethier-Krugman/Eaton-Kortum and Melitz model respectively:

$$genitcekh(i,r,s) = -sext(i,r) + itc(i,r,s) - genitcek(i,r,s) \tag{D.24}$$

$$genitcmh(i,r,s) = -sext(i,r) + \frac{\sigma(\theta-\sigma+1)}{(\sigma-1)^2}(tm(i,s)+tms(i,r,s)) + \left(1+\frac{\theta-\sigma+1}{\sigma-1}\right)itc(i,r,s)$$
$$+ \frac{\theta-\sigma+1}{(\sigma-1)^2}fex\,(i,r,s) + \frac{\theta-\sigma+1}{\sigma-1}pcif\,(i,r,s) - genitcm\,(i,r,s) \tag{D.25}$$

We shift between the Ethier-Krugman/Eaton-Kortum and Melitz model with the following equation:

$$genitcekm(i,r,s) = genitcek(i,r,s) + genitcm(i,r,s) + ams(i,r,s) \tag{D.26}$$

We add the variable itc to the model, which represents normal iceberg trade cost in the Ethier-Krugman and Melitz specification of the model. Since $ams\,(i,r,s)$ is a technology-shifter and a positive shock to ams represents a reduction in iceberg trade costs in the standard model, we add ams in the above equation instead of subtracting it. The existing code of the model does not have to be adjusted to account for Melitz-generalized trade costs and only requires a closure swap. Since $sext\,(i,r)$ can be either Ethier-Krugman, Eaton-Kortum or Melitz depending on the swap chosen in equation (D.7) and since the generalized trade cost is given by iceberg trade costs τ_{ij} (itc in GTAP relative changes) in both Ethier-Krugman and Eaton-Kortum, we can use one equation, equation (D.24), for both models.

Appendix D.4 Parameterization

We need values for the parameters σ in the Armington, Ethier-Krugman and Melitz model, θ in the Melitz model and ρ and η in the Eaton-Kortum model. From the empirics we have estimates for the tariff elasticity $\widetilde{e}$ and the degree of granularity g. By varying the parameters *etil* and *gran*, based on the estimated $\widetilde{e}$ and g, we switch between the parameterizations of the different models.

Starting with the Melitz model, we have:

$$\widetilde{e} = \theta + 1 + \frac{\theta - \sigma + 1}{\sigma - 1} \tag{D.27}$$

$$g = \frac{\sigma - 1}{\theta} \tag{D.28}$$

We can thus express θ and σ as a function of the estimated $\widetilde{e}$ and g as follow:

$$\sigma = g * \widetilde{e} \tag{D.29}$$

$$\theta = \widetilde{e} - \frac{1}{g} \tag{D.30}$$

Granularity g approaching 1 means that the model is approaching so-called "full granularity" with $\theta = \sigma - 1$.

In the Armington and Ethier-Krugman model we only need a value for σ, which is equal to $\widetilde{e}$. In the Eaton and Kortum model we need a value for the dispersion parameter ρ, which is equal to the tariff elasticity minus one, $\widetilde{e} - 1$. In the implementation in GTAP we do not replace the substitution elasticity $\sigma = esubd$ in the code by $\rho = rho$, but keep working with *esubd* and recognize that we get the Eaton-Kortum equations if we impose $esubd = rho = \widetilde{e} - 1$ and adjust the parameter values accordingly.[8] To work with *esubd* set equal to $\widetilde{e} - 1$, we introduce the parameter *etil* in the parameter file based on the estimated tariff elasticity and set it at $\widetilde{e} - 1$ in the Eaton-Kortum model.

We thus introduce the parameters *gran* as a measure for granularity g and *etil* as a measure

[8] In the quantity equations for qpd, qpm, qgd, qgm, qfd, qfm, and qxs, σ is equal to ρ, so we impose $\sigma = \rho$ in the quantity equations. In the price equations σ is equal $\rho + 1$, but in relative changes the parameter ρ does not play a role, so we do not have to allow for the different value of σ in the pricing equations.

Parameters	Armington	Ethier-Krugman	Melitz	Eaton-Kortum
etil	$\widetilde{e}$	$\widetilde{e}$	$\widetilde{e}$	$\widetilde{e}-1$
gran	1	1	γ	1
esubd	$\widetilde{e}$	$\widetilde{e}$	$\gamma * \widetilde{e}$	$\widetilde{e}-1$
theta	$-$	$-$	$\widetilde{e}-\frac{1}{\gamma}$	$-$

Table 1: Parameterization of the four models

for the trade elasticity $\widetilde{e}$ and employ the following equations in all four models:

$$esubd = gran * etil \tag{D.31}$$

$$theta = etil - \frac{1}{gran} \tag{D.32}$$

esubd is the substitution elasticity σ in the original GTAP model and *theta* is the dispersion parameter θ in the added Melitz-block of the model. By varying the values for *gran* and *etil*, we can then move between the different models. First, in the Ethier-Krugman and Armington model the substitution elasticity *esubd* is equal to the tariff elasticity $\widetilde{e}$, thus requiring $gran = 1$ and $etil = \widetilde{e}$. Second, in the Melitz model we have the expressions (D.29)-(D.30) for $esubd = \sigma$ and $theta = \theta$, thus requiring $etil = \widetilde{e}$ and $gran = g$. Third, by setting *gran* at 1 and *etil* at $\widetilde{e}-1$, we get the Eaton-Kortum parameterization with $esubd = rho = \widetilde{e}-1$. The parameterization is summarized in Table 1. The table shows the values required for the parameters *etil* and *gran* read from the parameter file and the implied values for *esubd* and *theta* based on the use of different parameter files.

Appendix D.5 Moving between Different Models with Closure Swaps

We move between the different models using closure swaps and employing different parameter files with different parameter values. First we discuss closure swaps. The baseline model with the additional blocks and without closure swaps implies the Armington model. We move from Armington to Ethier-Krugman by turning on the Ethier-Krugman supply-side externality and by endogenizing iceberg trade costs. We move from Armington to Melitz by turning on the Melitz supply-side and demand-side externalities and by endogenizing iceberg trade costs. We move from Armington to Eaton-Kortum by turning on the Eaton-Kortum supply-side externality and by endogenizing iceberg trade costs.

By swapping *oscaleekm* with *sext* in equation (D.7) and *nneh* with *nne* in equation (D.8) for the Ethier-Krugman and Melitz model and *tekh* with *tek* in the Eaton-Kortum model we turn on the supply-side externality. By swapping *oscaleek* with *ekscale*, *oscalem* with *mscale* or *eakoscale* with *eakoscale* in respectively equations (D.4)-(D.6) we turn respectively the Ethier-Krugman, Melitz and Eaton-Kortum supply-side externality on.

To turn on the Melitz demand-side externality, we swap *dscaled* with *Dextd* (*dscalem* with *Dextm*) in the following equation:

$$dscale2d(i,r) = dscale1d(i,r) - Dextd(i,r) \tag{D.33}$$

Finally, to model generalized trade costs in Ethier-Krugman, Eaton-Kortum or Melitz, $ams\,(i,r,s)$ is swapped with $genitcekm\,(i,r,s)$ in equation (D.26). By swapping *genitcekh* with *genitcek* or *genitcmh* with *genitcm* in respectively equations (D.24)-(D.25) we choose for respectively Ethier-Krugman/Eaton-Kortum or Melitz generalized iceberg trade costs.

To move between the different models, we also have to use different parameter values. We do this by employing different parameter files in the command file, with the parameter files differing in their values of *etil* and *gran* according to Table 1. The table makes clear that the values for *etil* and *gran* are identical for Armington and Ethier-Krugman. Hence, we use the same parameter file for these two models, whereas Melitz and Eaton-Kortum have their own parameter files.

Appendix D.6 Margin Decomposition

To calculate the three margins in GEMPACK, we rewrite equations (37)-(41) in GEMPACK notation as follows:

$$\begin{aligned} psistarh\,(i,r,s) = {} & \frac{1}{\sigma-1}[ps(i,r) + ao\,(i,r) - pfactwld] + \left(1 + \frac{1}{\sigma-1}\right)(tm\,(i,s) + tms\,(i,r,s)) \\ & + pcif\,(i,r,s) + itc\,(i,r,s) + \frac{1}{\sigma-1} fex\,(i,r,s) \\ & - priceDs(i,s) - \frac{1}{\sigma-1} valueDs\,(i,s) + \frac{\sigma}{\sigma-1} tariffDs\,(i,s) - psistar\,(i,r,s) \end{aligned}$$

The extensive margin is given by:

$$extm\,(i,r,s) = -\theta psistar(i,r,s) + nne(i,r)$$

And the intensive margin is defined by:

$$
\begin{aligned}
intm\left(i,r,s\right) = &-\left(\sigma-1\right)\left(itc\left(i,r,s\right)+tm\left(i,s\right)+tms\left(i,r,s\right)+pcif\left(i,r,s\right)\right) \\
&+\left(\sigma-1\right)priceDs(i,s)+valueDs\left(i,s\right)-\sigma tariffDs\left(i,s\right)
\end{aligned}
$$

The compositional margin can be expressed as:

$$
compm\left(i,r,s\right)=\left(\sigma-1\right)psistar\left(i,r,s\right)
$$

And finally the overall effect can be written as:

$$
\begin{aligned}
d\ln V_{ij} &= TM = EM + IM + CM \\
&= -\frac{\theta-\sigma-1}{\sigma-1}\left(ps\left(i,r\right)+ao\left(i,r\right)-pfactwld\right)+nne(i,r) \\
&-\left(\theta+\frac{\theta-\sigma-1}{\sigma-1}\right)\left(tm\left(i,s\right)+tms\left(i,r,s\right)\right) \\
&-\theta\left(itc\left(i,r,s\right)+pcif\left(i,r,s\right)\right)-\frac{\theta-\sigma-1}{\sigma-1}fex\left(i,r,s\right) \\
&+\theta priceDs(i,s)+\frac{\theta}{\sigma-1}valueDs\left(i,s\right)-\frac{\sigma\theta}{\sigma-1}tariffDs\left(i,s\right) \qquad \text{(D.34)}
\end{aligned}
$$

With $priceDs$, $valueDs$ and $tariffDs$ defined as in equations (D.12)-(D.14), except for the fact that values are expressed employing agents prices instead of market prices.

Supplementary Appendices of Derivations

Equation (34)

To convert Melitz into Ethier/Krugman the following should hold:

$$\gamma_m^{\frac{1}{\sigma-1}} = \gamma_{ek}$$

Substituting the expressions for γ_{ek} and γ_m in equation (22) leads to the following expression for ψ:

$$\begin{aligned}
\left(\psi \left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1}\right)^{\frac{1}{\sigma-1}} &= \frac{\sigma-1}{\sigma}\sigma^{\frac{1}{1-\sigma}} \\
\psi \left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1} &= \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{1}{\sigma} \\
\psi &= \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left(\frac{\sigma}{\sigma-1}\right)^{\theta+1} \frac{\theta-\sigma+1}{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}+1}} \\
\psi &= \left(\frac{\sigma}{\sigma-1}\right)^{\theta-\sigma+2} \frac{\theta-\sigma+1}{\sigma^{-\frac{\theta}{\sigma-1}}} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{\theta-\sigma+2} \sigma^{\frac{\theta}{\sigma-1}}\left(\theta-\sigma+1\right)
\end{aligned}$$

Equation (36)

Differentiating equation (35) on the RHS and LHS wrt to the endogenous variables gives:

$$dV_{ij} = dN_{ij}\widetilde{r}_{ij} + N_{ij}\frac{1}{1-G\left(\varphi_{ij}^*\right)}\int_{\varphi_{ij}^*}^{\infty} dr_{ij}\left(\varphi\right) g\left(\varphi\right) d\varphi - N_{ij}\frac{1}{1-G\left(\varphi_{ij}^*\right)} r_{ij}\left(\varphi_{ij}^*\right) g\left(\varphi_{ij}^*\right) d\varphi_{ij}^*$$

$$+ N_{ij}\frac{1}{1-G\left(\varphi_{ij}^*\right)}\int_{\varphi_{ij}^*}^{\infty} r_{ij}\left(\varphi\right) g\left(\varphi\right) d\varphi \frac{g\left(\varphi_{ij}^*\right)}{1-G\left(\varphi_{ij}^*\right)} d\varphi_{ij}^*$$

$$= dN_{ij}\widetilde{r}_{ij} + N_{ij}\frac{1}{1-G\left(\varphi_{ij}^*\right)}\int_{\varphi_{ij}^*}^{\infty} dr_{ij}\left(\varphi\right) g\left(\varphi\right) d\varphi - N_{ij}\frac{1}{1-G\left(\varphi_{ij}^*\right)} r_{ij}\left(\varphi_{ij}^*\right) g\left(\varphi_{ij}^*\right) d\varphi_{ij}^*$$

$$+ V_{ij}\frac{g\left(\varphi_{ij}^*\right)}{1-G\left(\varphi_{ij}^*\right)} d\varphi_{ij}^*$$

Writing in logs and using $g\left(\varphi_{ij}^{*}\right)=-\frac{\partial\left(1-G\left(\varphi_{ij}^{*}\right)\right)}{\partial\varphi_{ij}^{*}}$:

$$\begin{aligned}
d\ln V_{ij} &= d\ln N_{ij}\frac{N_{ij}}{V_{ij}}\widetilde{r}_{ij}+N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty}d\ln r_{ij}\left(\varphi\right)r_{ij}\left(\varphi\right)g\left(\varphi\right)d\varphi\frac{1}{V_{ij}} \\
&- N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}r_{ij}\left(\varphi_{ij}^{*}\right)g\left(\varphi_{ij}^{*}\right)d\ln\varphi_{ij}^{*}\varphi_{ij}^{*}\frac{1}{V_{ij}} \\
&- V_{ij}\frac{\frac{\partial\left(1-G\left(\varphi_{ij}^{*}\right)\right)}{\partial\varphi_{ij}^{*}}}{1-G\left(\varphi_{ij}^{*}\right)}d\ln\varphi_{ij}^{*}\varphi_{ij}^{*}\frac{1}{V_{ij}} \\
&= d\ln N_{ij}+\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty}d\ln r_{ij}\left(\varphi\right)\frac{r_{ij}\left(\varphi\right)}{\overline{r}_{ij}}g\left(\varphi\right)d\varphi \\
&+ N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}r_{ij}\left(\varphi_{ij}^{*}\right)\frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}\frac{1-G\left(\varphi_{ij}^{*}\right)}{\varphi_{ij}^{*}}d\ln\varphi_{ij}^{*}\frac{\varphi_{ij}^{*}}{V_{ij}} \\
&- \frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}\frac{1-G\left(\varphi_{ij}^{*}\right)}{\varphi_{ij}^{*}}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}d\ln\varphi_{ij}^{*}\varphi_{ij}^{*} \\
&= d\ln N_{ij}+N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty}d\ln r_{ij}\left(\varphi\right)\frac{r_{ij}\left(\varphi\right)}{\overline{r}_{ij}}g\left(\varphi\right)d\varphi \\
&+ \frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}\frac{\partial\ln\varphi_{ij}^{*}}{\partial\ln\tau_{ij}}\frac{r_{ij}\left(\varphi_{ij}^{*}\right)}{\overline{r}_{ij}}-\frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}d\ln\varphi_{ij}^{*} \\
&= d\ln N_{ij}+N_{ij}\frac{1}{1-G\left(\varphi_{ij}^{*}\right)}\int\limits_{\varphi_{ij}^{*}}^{\infty}d\ln r_{ij}\left(\varphi\right)\frac{r_{ij}\left(\alpha\right)}{\overline{r}_{ij}}g\left(\varphi\right)d\varphi \\
&+ \frac{\partial\ln 1-G\left(\varphi_{ij}^{*}\right)}{\partial\ln\varphi_{ij}^{*}}d\ln\varphi_{ij}^{*}\left(\frac{r_{ij}\left(\varphi_{ij}^{*}\right)}{\overline{r}_{ij}}-1\right)
\end{aligned}$$

Equation (41)

Adding up the three margins in equations (38)-(40), we get:

$$
\begin{aligned}
d\ln V_{ij} &= TM = EM + IM + CM \\
&= -\frac{\mu\theta}{\sigma-1}\widehat{p_{Z_i}} - \frac{\theta}{\sigma-1}\left(1-\mu\right)\widehat{p_{Z_j}} - \theta\left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} \\
&- \theta\widehat{\tau_{ij}} - \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) - \frac{\theta}{\sigma-1}\widehat{f_{ij}} + \widehat{NE_i} \\
&+ \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left(\left(\sigma-1\right)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right) \\
&- \left(\sigma-1\right)\left(\widehat{\tau_{ij}} + \widehat{ta_{ij}} + \left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right)\right) \\
&+ \sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left(\left(\sigma-1\right)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right) \\
&+ \mu\widehat{p_{Z_i}} + \left(1-\mu\right)\widehat{p_{Z_j}} + \sigma\widehat{ta_{ij}} + \left(\sigma-1\right)\widehat{\tau_{ij}} + \left(\sigma-1\right)\left(\widehat{te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}\right) + \widehat{f_{ij}} \\
&- \sum_{ag=\{s,p,f\}}\frac{p_j^s q_j^{s,ag}}{\sum\limits_{ag'=\{s,p,f\}} p_j^s q_j^{s,ag'}}\left(\left(\sigma-1\right)\widehat{P_j^{ag}} + \widehat{E_j^{ag}} - \sigma\widehat{ta_j^{s,ag}}\right)
\end{aligned}
$$

Elaborating and merging terms, we get:

$$\begin{aligned}
TM &= -\frac{\theta\mu}{\sigma-1}\widehat{p_{Z_i}} + \mu\widehat{p_{Z_i}} + \widehat{NE}_i \\
&- \frac{\theta}{\sigma-1}(1-\mu)\widehat{p_{Z_j}} + (1-\mu)\widehat{p_{Z_j}} \\
&- \theta\left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} - (\sigma-1)\widehat{ta_{ij}} + \sigma\widehat{ta_{ij}} \\
&- \theta\widehat{\tau_{ij}} - (\sigma-1)\widehat{\tau_{ij}} + (\sigma-1)\widehat{\tau_{ij}} \\
&- \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\right) - (\sigma-1)\left(\widehat{te_{ij}p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\right) + (\sigma-1)\left(\widehat{te_{ij}p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\right) \\
&- \frac{\theta}{\sigma-1}\widehat{f_{ij}} + \widehat{f_{ij}} \\
&+ \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p^s_j q^{s,ag}_j}{\sum\limits_{ag'=\{s,p,f\}} p^s_j q^{s,ag'}_j}\left((\sigma-1)\widehat{P^{ag}_j} + \widehat{E^{ag}_j} - \sigma\widehat{ta^{s,ag}_j}\right) \\
&= -\mu\frac{\theta-\sigma+1}{\sigma-1}\widehat{p_{Z_i}} - (1-\mu)\frac{\theta-\sigma-1}{\sigma-1}\widehat{p_{Z_j}} \\
&- \left(\theta\left(1+\frac{1}{\sigma-1}\right)-1\right)\widehat{ta_{ij}} - \theta\widehat{\tau_{ij}} - \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\right) - \frac{\theta-\sigma-1}{\sigma-1}\widehat{f_{ij}} \\
&+ \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p^s_j q^{s,ag}_j}{\sum\limits_{ag'=\{s,p,f\}} p^s_j q^{s,ag'}_j}\left((\sigma-1)\widehat{P^{ag}_j} + \widehat{E^{ag}_j} - \sigma\widehat{ta^{s,ag}_j}\right)
\end{aligned}$$

So we have:

$$\begin{aligned}
TM &= -\mu\frac{\theta-\sigma+1}{\sigma-1}\widehat{p_{Z_i}} - (1-\mu)\frac{\theta-\sigma-1}{\sigma-1}\widehat{p_{Z_j}} - \left(\theta+\frac{\theta-\sigma-1}{\sigma-1}\right)\widehat{ta_{ij}} - \theta\widehat{\tau_{ij}} \\
&- \theta\left(\widehat{te_{ij}p_{Z_i} + \frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\right) - \frac{\theta-\sigma-1}{\sigma-1}\widehat{f_{ij}} + \frac{\theta}{\sigma-1}\sum_{ag=\{s,p,f\}}\frac{p^s_j q^{s,ag}_j}{\sum\limits_{ag'=\{s,p,f\}} p^s_j q^{s,ag'}_j}\left((\sigma-1)\widehat{P^{ag}_j} + \widehat{E^{ag}_j} - \sigma\widehat{ta^{s,ag}_j}\right)
\end{aligned}$$

Equation (37)

Log differentiating the expression for φ_{ij}^{*} in equation (B.7) gives:

$$\begin{aligned}
\widehat{\varphi_{ij}^{*}} &= \left(1+\frac{\mu}{\sigma-1}\right)\widehat{p_{Z_i}} + \frac{1-\mu}{\sigma-1}\widehat{p_{Z_j}} + \left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} + \widehat{\tau_{ij}} \\
&+ \frac{1}{1-\sigma}\sum_{ag=\{s,p,f\}} \widehat{\left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{ag}}} \\
&= \left(1+\frac{\mu}{\sigma-1}\right)\widehat{p_{Z_i}} + \frac{1-\mu}{\sigma-1}\widehat{p_{Z_j}} + \left(1+\frac{1}{\sigma-1}\right)\widehat{ta_{ij}} + \widehat{\tau_{ij}} \\
&- \frac{1}{\sigma-1}\sum_{ag=\{s,p,f\}} \frac{\frac{E_j^{ag}}{ta_j^{ag}}}{\sum_{ag'=\{s,p,f\}}\frac{E_j^{ag'}}{ta_j^{ag'}}}\left((\sigma-1)\left(\widehat{P_j^{ag}} - \widehat{ta_j^{ag}} - \widehat{\tau_j^{ag}}\right) + \widehat{E_j^{ag}} - \widehat{ta_j^{ag}}\right)
\end{aligned}$$

Equation (43)

Substituting equation (23) into equation (42) gives:

$$\begin{aligned}
v_{ij} &= ta_{ij}^{-\sigma}\left(\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta-\sigma+1}{\sigma-1}} \tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}} ta_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}+\frac{\theta-\sigma+1}{(\sigma-1)^2}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \tau_{ij}c_i\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\sigma}\left(\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta}{\sigma-1}} \tau_{ij}^{\frac{\theta}{\sigma-1}} ta_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}\frac{\sigma}{\sigma-1}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} c_i\right)^{1-\sigma}\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\left(1+\frac{\theta-\sigma+1}{\sigma-1}\right)\sigma}\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\frac{\theta\sigma}{\sigma-1}}\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\frac{\theta\sigma+\theta(\sigma-1)-\theta(\sigma-1)}{\sigma-1}}\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\left(\theta+\frac{\theta\sigma-\theta\sigma+\theta}{\sigma-1}\right)}\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\left(\theta+\frac{\theta}{\sigma-1}\right)}\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m \\
&= ta_{ij}^{-\left(\theta+1+\frac{\theta-\sigma+1}{\sigma-1}\right)}\left(te_{ij}b_ip_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\tau_{ij}^{-\theta}f_{ij}^{-\frac{\theta-\sigma+1}{(\sigma-1)}}c_i\left(p_j^m\right)^{-\sigma}q_j^m
\end{aligned}$$

Equation (A.1)

Substituting equations (6)-(8) into equations (2)-(5) gives for q_{ij}:

$$
\begin{aligned}
q_{ij} &= \left(\frac{p_{ij}}{p_j^m}\right)^{-\sigma} q_j^m = \left(\frac{p_{ij}}{p_j^m}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} q_j^{m,ag} \\
&= \left(\frac{p_{ij}}{p_j^m}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} \left(e_j^m\right)^{\sigma-1} \left(\frac{ta_j^{m,ag} p_j^m}{P_j^{ag}}\right)^{-\sigma} q_j^{ag} \\
&= p_{ij}^{-\sigma} \left(e_j^m\right)^{\sigma-1} \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{m,ag}}\right)^{\sigma} q_j^{ag} \qquad \text{(S.1)}
\end{aligned}
$$

Substituting equation (9) and rearranging gives:

$$
q_{ij} e_j^m = \left(\frac{ta_{ij} t_{ij} c_i \left(te_{ij} b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{e_j^m}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{m,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{m,ag}} \qquad \text{(S.2)}
$$

To derive the expression for $q_j^{d,ag}$ we substitute equation (7) into equations (2)-(3):

$$
q_j^d e_j^d = \left(\frac{c_j b_j p_{Z_j}}{e_j^d}\right)^{-\sigma} \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{d,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{d,ag}} \qquad \text{(S.3)}
$$

Together equations (S.2)-(S.3) imply the general expression for $q_j^{s,ag}$ in equation (A.1).

Equation (D.8)

Log differentiating equation (A.11) gives:

$$
\begin{aligned}
\widehat{N_i} = & \frac{Z_i}{Z_i - \sum\limits_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}\left(te_{ij}+\frac{p^{tr}_{ij}}{p_{Z_i}a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}}\right)}\widehat{Z_i} \\
& - \sum_{j=1}^{J} \frac{\frac{\sigma-1}{\sigma}\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(p_{z_i}te_{ij}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}\right)}}{Z_i - \sum\limits_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{p_{z_i} ta_{ij}\left(te_{ij}+\frac{p^{tr}_{ij}}{p_{z_i}a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}}\right)}(\widehat{\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}} \\
& - \frac{p_{z_i}te_{ij}}{p_{z_i}te_{ij}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\left(\widehat{p_{Z_i}}+\widehat{te_{ij}}\right) - \frac{\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}{p_{z_i}te_{ij}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\widehat{p^{tr}_{ij}}) \\
& + \sum_{j=1}^{J} \frac{\frac{\sigma-1}{\sigma}\frac{N_{ij}\overline{r}_{ij}}{p_{Z_i}ta_{ij}}}{Z_i - \sum\limits_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{p_{z_i} ta_{ij}\left(te_{ij}+\frac{p^{tr}_{ij}}{p_{z_i}a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{p_{Z_i} ta_{ij}}\right)}\left(\widehat{\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}} - \widehat{p_{Z_i}}\right) \\
= & \frac{p_{Z_i}Z_i}{p_{Z_i}Z_i - \sum\limits_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(te_{ij}+\frac{p^{tr}_{ij}}{p_{Z_i}a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}\right)}\widehat{Z_i} \\
& - \sum_{j=1}^{J} \frac{\frac{\sigma-1}{\sigma}\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(te_{ij}+\frac{p^{tr}_{ij}}{p_{Z_i}a^{tr}_{ij}}\right)}}{p_{z_i}Z_i - \sum\limits_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(te_{ij}+\frac{p^{tr}_{ij}}{p_{Z_i}a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}\right)}(\widehat{\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}} \\
& - \frac{p_{z_i}te_{ij}}{p_{z_i}te_{ij}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\left(\widehat{p_{Z_i}}+\widehat{te_{ij}}\right) - \frac{\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}{p_{z_i}te_{ij}+\frac{p^{tr}_{ij}}{a^{tr}_{ij}}}\widehat{p^{tr}_{ij}}) \\
& + \sum_{j=1}^{J} \frac{\frac{\sigma-1}{\sigma}\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}}{p_{z_i}Z_i - \sum\limits_{j=1}^{J} \frac{\sigma-1}{\sigma}\left(\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}\left(te_{ij}+\frac{p^{tr}_{ij}}{p_{Z_i}a^{tr}_{ij}}\right)} - \frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}\right)}\left(\widehat{\frac{N_{ij}\overline{r}_{ij}}{ta_{ij}}} - \widehat{p_{Z_i}}\right)
\end{aligned}
$$

In GEMPACK notation we get:

$$
\begin{aligned}
oscale(i,r) = {} & nne(i,r) - \frac{VOM\left(i,r\right)}{VOM\left(i,r\right) - \frac{\sigma-1}{\sigma}\sum_{t=1}^{J}\left(VXMD\left(i,r,t\right) - VIWS\left(i,r,t\right)\right)} qo\left(i,r\right) \\
& + \sum_{s=1}^{J} \frac{\frac{\sigma-1}{\sigma} VXMD\left(i,r,s\right)}{VOM\left(i,r\right) - \frac{\sigma-1}{\sigma}\sum_{t=1}^{J}\left(VXMD\left(i,r,t\right) - VIWS\left(i,r,t\right)\right)} (pcif\left(i,r,s\right) + qxs\left(i,r,s\right) \\
& - \frac{VXWD\left(i,r,s\right)}{VIWS\left(i,r,s\right)}\left(ps\left(i,r\right) + ao\left(i,r\right) - tx\left(i,r\right) - tx\left(i,r,s\right)\right) \\
& - \frac{VIWS\left(i,r,s\right) - VXWD\left(i,r,s\right)}{VIWS\left(i,r,s\right)} ptrans\left(i,r,s\right)) \\
& - \sum_{s=1}^{J} \frac{\frac{\sigma-1}{\sigma} VIWS\left(i,r,s\right)}{VOM\left(i,r\right) - \frac{\sigma-1}{\sigma}\sum_{t=1}^{J}\left(VXMD\left(i,r,t\right) - VIWS\left(i,r,t\right)\right)} \\
& * \left(pcif\left(i,r,s\right) + qxs\left(i,r,s\right) - \left(ps\left(i,r\right) + ao\left(i,r\right)\right)\right)
\end{aligned}
$$

Equation (A.7)

Taking the FOC wrt p^o_{ij} in equation (A.6) gives:

$$
\begin{aligned}
0 = {} & (1-\sigma)\frac{\left(\sum_{ag=\{s,p,f\}}\left(ta_j^{ag}p_{ij}\right)^{-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}\right)}{ta_j^{ag}ta_{ij}te_{ij}} \\
& + \sigma\tau_{ij}\left(c_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\left(\sum_{ag=\{s,p,f\}}\left(ta_j^{ag}p_{ij}\right)^{-(\sigma+1)}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}\right) \\
0 = {} & (1-\sigma)\frac{1}{ta_j^{ag}ta_{ij}te_{ij}} + \sigma\tau_{ij}\left(te_{ij}c_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\frac{1}{ta_j^{ag}p_{ij}}
\end{aligned}
$$

$$
p_{ij} = \frac{\sigma}{\sigma-1}ta_{ij}te_{ij}\tau_{ij}\left(te_{ij}c_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)
$$

Equation (A.8)

Substituting equation (A.7) back into equation (A.6) gives:

$$
\begin{aligned}
\pi_{ij} = {} & \frac{ta_j^{ag}p_{ij}\left(\sum_{ag=\{s,p,f\}}\left(ta_j^{ag}p_{ij}^{o}\right)^{-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}\right)}{ta_j^{ag}ta_{ij}} - \frac{p_{ij}^{o}}{ta_{ij}}\frac{\sigma-1}{\sigma}\left(\sum_{ag=\{s,p,f\}}\left(ta_j^{ag}p_{ij}\right)^{-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}\right) \\
= {} & \frac{p_{ij}^{o}o_{ij}}{\sigma ta_{ij}}
\end{aligned}
$$

Equation (A.11)

Substituting equations (A.8) and (A.10) into equation (A.9) gives:

$$\left(\sum_{j=1}^{J}\frac{\sigma-1}{\sigma}\frac{p_{ij}^{o}o_{ij}}{p_{Z_i}ta_{ij}}+\frac{\sigma-1}{\sigma}\frac{p_{ij}^{o}o_{ij}}{p_{Z_i}ta_{ij}}\left(\frac{1}{te_{ij}b_i+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}-1\right)+a_i\right)N_i=Z_i$$

$$N_i\sigma a_i+\sum_{j=1}^{J}N_i\frac{\sigma-1}{\sigma}\frac{p_{ij}^{o}o_{ij}}{p_{Z_i}ta_{ij}}\left(\frac{1}{te_{ij}b_i+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}-1\right)=Z_i$$

$$N_i=\frac{Z_i-\frac{\sigma-1}{\sigma}\sum_{j=1}^{J}N_i\frac{p_{ij}^{o}o_{ij}}{p_{Z_i}ta_{ij}}\left(\frac{1}{te_{ij}b_i+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}-1\right)}{\sigma a_i} \tag{S.4}$$

$$=\frac{Z_i-\frac{\sigma-1}{\sigma}\sum_{j=1}^{J}N_i\frac{\overline{r}_{ij}}{p_{Z_i}ta_{ij}}\left(\frac{1}{te_{ij}b_i+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}-1\right)}{\sigma a_i} \tag{S.5}$$

Equation (B.5)

With tariffs as revenues shifters, profits for sales from i to j can be written as:

$$\pi_{ij}=\sum_{ag=\{s,p,f\}}\left(\frac{ta_j^{ag}p_{ij}^{o}o_{ij}}{ta_j^{ag}ta_{ij}}-\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\frac{o_{ij}}{\varphi}\right)-f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}$$

$$=\sum_{ag=\{s,p,f\}}\left(\frac{ta_j^{ag}p_{ij}^{o}o_{ij}}{ta_j^{ag}ta_{ij}}-\frac{\sigma-1}{\sigma}\frac{ta_j^{ag}p_{ij}o_{ij}^{ag}}{ta_j^{ag}ta_{ij}}\right)-f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}$$

$$=\sum_{ag=\{s,p,f\}}\frac{ta_j^{ag}p_{ij}^{o}o_{ij}}{\sigma ta_j^{ag}ta_{ij}}-f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}$$

$$=\sum_{ag=\{s,p,f\}}\frac{\left(ta_j^{ag}p_{ij}^{o}(\varphi)\right)^{1-\sigma}\left(P_j^{ag}\right)^{\sigma-1}E_j^{ag}}{ta_j^{ag}ta_{ij}\sigma}-f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}$$

Equation (B.7)

Using equations (B.3)-(B.5) the ZCP can be written as follows:

$$p_{ij}^{o}(\varphi)=\frac{\sigma}{\sigma-1}\frac{ta_{ij}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi}$$

$$\sum_{ag=\{s,p,f\}} \frac{\left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \tag{S.6}$$

$$\sum_{ag=\{s,p,f\}} \frac{\left(ta_j^{ag} p_{ij}^o(\varphi)\right)^{1-\sigma} \left(P_j^{ag}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}$$

$$\sum_{ag=\{s,p,f\}} \left(\frac{\sigma}{\sigma-1} \frac{ta_j^{ag} ta_{ij} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\varphi_{ij}^*}\right)^{1-\sigma} \frac{\left(P_j^{ag,e}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}$$

$$\sum_{ag=\{s,p,f\}} \left(\frac{\sigma}{\sigma-1} ta_j^{ag} ta_{ij} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma} \frac{\left(P_j^{ag,e}\right)^{\sigma-1} E_j^{ag}}{\sigma ta_j^{ag} ta_{ij} f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}} = \left(\varphi_{ij}^*\right)^{1-\sigma}$$

$$\frac{\left(\frac{\sigma}{\sigma-1} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}} \sum_{ag=\{s,p,f\}} \left(ta_j^{ag} ta_{ij}\right)^{1-\sigma} \frac{\left(P_j^{ag,e}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \left(\varphi_{ij}^*\right)^{1-\sigma}$$

$$\frac{\left(\frac{\sigma}{\sigma-1} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}} \sum_{ag=\{s,p,f\}} \frac{\left(\frac{P_j^{ag,e}}{ta_j^{ag} ta_{ij} \tau_{ij}}\right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \left(\varphi_{ij}^*\right)^{1-\sigma}$$

$$\begin{aligned}
\varphi_{ij}^* &= \frac{\frac{\sigma}{\sigma-1} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}\right)^{\frac{1}{1-\sigma}}} \left(\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag} ta_{ij} \tau_{ij}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag} ta_{ij}}\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\frac{\sigma}{\sigma-1} ta_{ij} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} ta_{ij}\right)^{\frac{1}{1-\sigma}}} \left(\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

Equation (B.10)

Writing expected profit $\pi_{ij}\left(\widetilde{\varphi}_{ij}\right)$ as a function of expected revenues $r_{ij}\left(\widetilde{\varphi}_{ij}\right)$ using equation (B.5) and expressing expected revenues $r_{ij}\left(\widetilde{\varphi}_{ij}\right)$ as a function of cutoff revenues $r_{ij}\left(\varphi_{ij}^*\right)$ using $\frac{r_{ij}(\varphi_1)}{r_{ij}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$ gives:

$$\pi_{ij}\left(\widetilde{\varphi}_{ij}\right) = \sum_{ag=\{s,p,f\}} \frac{r_{ij}^{ag}\left(\varphi_{ij}^*\right)}{ta_j^{ag} ta_{ij} \sigma} \left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^*}\right)^{\sigma-1} - f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}$$

Using the ZCP in equation (B.6) this can be rewritten as:

$$\begin{aligned}\pi_{ij}\left(\widetilde{\varphi}_{ij}\right) &= f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^{*}}\right)^{\sigma-1} - f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \\ &= f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \left(\left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^{*}}\right)^{\sigma-1} - 1\right)\end{aligned} \tag{S.7}$$

Substituting equation (S.7) into the FE, equation (B.8) leads to equation (B.10).

Equation (B.12)

Using the Pareto distribution in equation (B.11) average productivity $\widetilde{\varphi}_{ij}$ can be written as:

$$\begin{aligned}\widetilde{\varphi}_{ij}^{\sigma-1} &= \int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-1} \frac{g_i(\varphi)}{1-G_i\left(\varphi_{ij}^{*}\right)} d\varphi = \int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-1} \frac{\theta \frac{\kappa_i^{\theta}}{\varphi^{\theta+1}}}{\left(\frac{\kappa_i}{\varphi_{ij}^{*}}\right)^{\theta_i}} d\varphi \\ &= \int_{\varphi_{ij}^{*}}^{\infty} \theta \left(\frac{\kappa_i}{\varphi_{ij}^{*}}\right)^{-\theta} \varphi^{\sigma-1-\theta+1} \frac{\kappa_i^{\theta}}{\varphi^{\theta+1}} d\varphi = \int_{\varphi_{ij}^{*}}^{\infty} \theta \varphi_{ij}^{*\theta} \varphi^{\sigma-1-\theta-1} d\varphi \\ &= \theta \varphi_{ij}^{*\theta} \int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-\theta-2} d\varphi = \theta \varphi_{ij}^{*\theta} \int_{\varphi_{ij}^{*}}^{\infty} \varphi^{\sigma-\theta-2} d\varphi \\ &= \frac{\theta}{\sigma-\theta-1} \varphi_{ij}^{*-\theta} \varphi^{\sigma-\theta-1} \Big|_{\varphi_{ij}^{*}}^{\infty} = -\frac{\theta}{\sigma-\theta-1} \varphi_{ij}^{*\theta} \varphi_{ij}^{*\sigma-\theta-1} \\ &= \frac{\theta}{\theta-\sigma+1} \varphi_{ij}^{*\sigma-1}\end{aligned}$$

Equation (B.16)

Substituting equations (B.4) and (B.9) into equation (B.15) gives:

$$
\begin{aligned}
\frac{p_{ij}^{ag} ta_j^{s,ag}}{e_j} &= \left(\int_{\varphi_{ij}^*}^{\infty} N_{ij} \left(\frac{\sigma}{\sigma-1} \frac{ta_{ij} ta_j^{s,ag} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\varphi} \right)^{1-\sigma} \frac{g_i(\varphi)}{1-G_i\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1} \left(N_{ij} \left(ta_{ij} ta_j^{s,ag} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) \right)^{1-\sigma} \int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \frac{g_i(\varphi)}{1-G_i\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1} \left(N_{ij} \left(ta_{ij} ta_j^{s,ag} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right) \right)^{1-\sigma} \left(\int_{\varphi_{ij}^*}^{\infty} \varphi^{\sigma-1} \frac{g_i(\varphi)}{1-G_i\left(\varphi_{ij}^*\right)} d\varphi \right)^{\frac{\sigma-1}{\sigma-1}} \right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1} \left(\sum_{i=1}^{J} N_{ij} \left(\frac{ta_{ij} ta_j^{s,ag} \tau_{ij} \left(te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\widetilde{\varphi}_{ij}} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}
\end{aligned}
$$

Equation (B.18)

Equation (B.18) can be derived from labor market equilibrium. First, we write the expression for $q_{ij}(\varphi)$ as a function of revenues, using the rewritten markup equation $\frac{\tau_{ij}}{\varphi} = \frac{\sigma-1}{\sigma} \frac{p_{ij}^{o}(\varphi)}{\left(te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}} \right) ta_{ij} p_{Z_i}}$. This gives:

$$
\begin{aligned}
\frac{\tau_{ij} o_{ij}(\varphi)}{\varphi} &= \frac{\sigma-1}{\sigma} \frac{p_{ij}^{o}(\varphi)}{\left(te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}} \right) ta_{ij} p_{Z_i}} o_{ij} = \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{\left(te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}} \right) p_{Z_i} ta_{ij}} \\
&= \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{p_{Z_i} ta_{ij}} + \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{p_{Z_i} ta_{ij}} \left(\frac{1 - te_{ij} p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \qquad \text{(S.8)}
\end{aligned}
$$

Input bundle demand consists of demand for labor bundles in production, fixed costs and sunk entry costs. This gives the following equilibrium condition:

$$
Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{\tau_{ij} o_{ij}(\varphi)}{\varphi} \frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)} d\varphi + \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}}
$$

Substituting equation (S.8) and elaborating the expression using $\frac{r_{ij}(\varphi_1)}{r_{ij}(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}$ gives:

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{p_{Z_i} ta_{ij}} \frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)} d\varphi$$

$$+ \sum_{j=1}^{J} N_{ij} \int_{\varphi_{ij}^*}^{\infty} \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\varphi)}{p_{Z_i} ta_{ij}} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)} d\varphi$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{ta_{ij}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_j^g ta_{ij}}$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \frac{\theta}{\theta-\sigma+1} \frac{r_{ij}\left(\varphi_{ij}^*\right)}{p_{Z_i} ta_j^g ta_{ij}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{r_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_j^g ta_{ij}}$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

Substituting equation (B.12) for the ratio of productivities and the ZCP in equation (B.6) gives:

$$Z_i = NE_i en_i + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \frac{\theta}{\theta-\sigma+1} \sigma f_{ij} \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{r_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

$$= NE_i en_i + \sum_{j=1}^{J} \frac{\theta(\sigma-1)}{\theta-\sigma+1} N_{ij} f_{ij} \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}$$

$$+ \sum_{j=1}^{J} N_{ij} f_{ij} \frac{\mu p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

$$= NE_i en_i + \sum_{j=1}^{J} N_{ij} \frac{\theta(\sigma-1) + \mu(\theta-\sigma+1)}{\theta-\sigma+1} f_{ij} \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i} ta_{ij}}$$

$$+ \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1-\mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}$$

Next, equation (B.17) is used to express N_{ij} as a function of NE_i and φ_{ij}^*:

$$
\begin{aligned}
Z_i = {} & NE_i en_i + \frac{NE_i}{\delta} \sum_{j=1}^{J} \left(\frac{\kappa_i}{\varphi_{ij}^*} \right)^{\theta} \frac{\theta (\sigma - 1) + \mu (\theta - \sigma + 1)}{\theta_i - \sigma + 1} f_{ij} \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} \\
& + \sum_{j=1}^{J} N_{ij} \frac{\sigma - 1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij} \left(\widetilde{\varphi}_{ij} \right)}{p_{Z_i} ta_{ij}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1 - \mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}}
\end{aligned}
$$

The next step is to substitute the FE from equation (B.13):

$$
\begin{aligned}
Z_i = {} & NE_i en_i + NE_i \sum_{j=1}^{J} \left(\frac{\kappa_i}{\varphi_{ij}^*} \right)^{\theta_i} \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}} f_{ij} \frac{\sigma - 1}{\theta_i - \sigma + 1} \frac{1}{\delta} \frac{\theta_i (\sigma - 1) + \mu (\theta_i - \sigma + 1)}{\sigma - 1} \\
& + \sum_{j=1}^{J} N_{ij} \frac{\sigma - 1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij} \left(\widetilde{\varphi}_{ij} \right)}{p_{Z_i} ta_{ij}} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1 - \mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}} \\
= {} & NE_i en_i + NE_i en_i \frac{\theta (\sigma - 1) + \mu (\theta - \sigma + 1)}{\sigma - 1} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1 - \mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}} \\
= {} & NE_i en_i \frac{(\theta + 1)(\sigma - 1) + \mu (\theta - \sigma + 1)}{\sigma - 1} + \sum_{k=1}^{J} N_{ki} f_{ki} \frac{(1 - \mu) p_{Z_k}^{\mu} p_{Z_i}^{1-\mu}}{p_{Z_i}} \\
& + \sum_{j=1}^{J} N_{ij} \frac{\sigma - 1}{\sigma} \left(\frac{1 - te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}}{te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij} \left(\widetilde{\varphi}_{ij} \right)}{p_{Z_i} ta_{ij}} \qquad \text{(S.9)}
\end{aligned}
$$

To rewrite the second term on the RHS of equation (S.9) we substitute the relation between $\widetilde{\varphi}_{ki}^{\sigma-1}$ and $\varphi_{ki}^{*\sigma-1}$ from equation (B.12) into the expression for the price index implied by equation (B.16) :

$$
\begin{aligned}
P_i^{1-\sigma} & = \sum_{k=1}^{J} N_{ki} \left(\frac{\sigma}{\sigma - 1} ta_{ki} \tau_{ki} \left(te_{ki} p_{Z_k} + \frac{p_{ki}^{tr}}{a_{ki}^{tr}} \right) \right)^{1-\sigma} \widetilde{\varphi}_{ki}^{\sigma-1} \\
& = \sum_{k=1}^{J} N_{ki} \left(\frac{\sigma}{\sigma - 1} ta_{ki} \tau_{ki} \left(te_{ki} p_{Z_k} + \frac{p_{ki}^{tr}}{a_{ki}^{tr}} \right) \right)^{1-\sigma} \varphi_{ki}^{*\sigma-1} \frac{\theta}{\theta - \sigma + 1}
\end{aligned}
$$

P_i is the group-uniform price index before the group-specific tariff is imposed. Substituting the rewritten ZCP from equation (B.7) gives:

$$
\begin{aligned}
P_i^{1-\sigma} &= \sum_{k=1}^{J}(N_{ki}\left(\frac{\sigma}{\sigma-1}ta_{ki}\tau_{ki}\left(te_{ki}p_{Z_k}+\frac{p_{ki}^{tr}}{a_{ki}^{tr}}\right)\right)^{1-\sigma}\left(\frac{\sigma}{\sigma-1}ta_{ki}\tau_{ki}\left(te_{ki}p_{Z_k}+\frac{p_{ki}^{tr}}{a_{ki}^{tr}}\right)\right)^{\sigma-1} \\
&\quad * \frac{\sigma f_{ki}ta_{ki}p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}}{\sum\limits_{ag=\{s,p,f\}}\left(\frac{P_i^{ag}}{ta_i^{ag}}\right)^{\sigma-1}\frac{E_i^{ag}}{ta_i^{ag}}})\frac{\theta}{\theta-\sigma+1} \\
&= \sum_{k=1}^{J}N_{ki}\frac{\sigma f_{ki}ta_{ki}p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}}{\sum\limits_{ag=\{s,p,f\}}P_i^{\sigma-1}\frac{E_i^{ag}}{ta_i^{ag}}}\frac{\theta}{\theta-\sigma+1} = P_i^{1-\sigma}\sum_{k=1}^{J}N_{ki}\frac{\sigma f_{ki}ta_{ki}p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}}{\sum\limits_{ag=\{s,p,f\}}\frac{E_i^{ag}}{ta_i^{ag}}}\frac{\theta}{\theta-\sigma+1}
\end{aligned}
$$

This expression can be written as:

$$
\sum_{k=1}^{J}N_{ki}f_{ki}w_k^{\mu}w_i^{1-\mu}+\sum_{k=1}^{J}N_{ki}f_{ki}\left(ta_{ki}-1\right)p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}=\sum_{ag=\{s,p,f\}}\frac{E_i^{ag}}{ta_i^{ag}}\frac{\theta-\sigma+1}{\theta\sigma} \tag{S.10}
$$

Next, tariff revenues can be written as:

$$
\begin{aligned}
\frac{ta_{ki}-1}{ta_{ki}}N_{ki}\bar{r}_{ki}\left(\widetilde{\varphi}_{ki}\right) &= \frac{ta_{ki}-1}{ta_{ki}}N_{ki}\bar{r}_{ki}\left(\widetilde{\varphi}_{ki}\right) \\
&= \frac{ta_{ki}-1}{ta_{ki}}N_{ki}\bar{r}_{ki}\left(\varphi_{ki}^{*}\right)\left(\frac{\widetilde{\varphi}_{ij}}{\varphi_{ij}^{*}}\right)^{\sigma-1} \\
&= \frac{ta_{ki}-1}{ta_{ki}}N_{ki}\sigma f_{ki}ta_{ki}p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}\frac{\theta}{\theta-\sigma+1} \\
&= \left(ta_{ki}-1\right)N_{ki}f_{ki}p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}\frac{\theta\sigma}{\theta-\sigma+1}
\end{aligned}
$$

Substituting this into equation (S.10) gives:

$$
\begin{aligned}
\sum_{k=1}^{J}N_{ki}f_{ki}p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}+\frac{ta_{ki}-1}{ta_{ki}}N_{ki}\bar{r}_{ki}\left(\widetilde{\varphi}_{ki}\right)\frac{\theta-\sigma+1}{\theta\sigma} &= \sum_{ag=\{s,p,f\}}\frac{E_i^{ag}}{ta_i^{ag}}\frac{\theta-\sigma+1}{\theta\sigma} \\
\sum_{k=1}^{J}N_{ki}f_{ki}p_{Z_k}^{\mu}p_{Z_i}^{1-\mu} &= \left(\sum_{ag=\{s,p,f\}}\frac{E_i^{ag}}{ta_i^{ag}}-\frac{ta_{ki}-1}{ta_{ki}}N_{ki}\bar{r}_{ki}\left(\widetilde{\varphi}_{ki}\right)\right)\frac{\theta-\sigma+1}{\theta\sigma}
\end{aligned}
$$

Using $p_{Z_i}Z_i+\sum\limits_{k=1}^{J}\frac{ta_{ki}-1}{ta_{ki}}N_{ki}\bar{r}_{ki}\left(\widetilde{\varphi}_{ki}\right)=\sum\limits_{ag=\{s,p,f\}}\frac{E_i^{ag}}{ta_i^{ag}}$ therefore leads to:

$$
\sum_{k=1}^{J}N_{ki}f_{ki}\frac{p_{Z_k}^{\mu}p_{Z_i}^{1-\mu}}{p_{Z_i}}=Z_i\frac{\theta-\sigma+1}{\theta\sigma} \tag{S.11}
$$

Substituting (S.11) into (S.9) then gives:

$$
\begin{aligned}
Z_i = {} & NE_i en_i \frac{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)}{\sigma-1} + (1-\mu) Z_i \frac{\theta-\sigma+1}{\theta\sigma} \\
& + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1-te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}{te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}\left(\widetilde{\varphi}_{ij}\right)}{p_{Z_i}ta_{ij}}
\end{aligned} \tag{S.12}
$$

Rearranging then leads to:

$$
\begin{aligned}
Z_i \left(\frac{\theta\sigma-(1-\mu)(\theta-\sigma+1)}{\theta\sigma} \right) = {} & NE_i en_i \frac{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)}{\sigma-1} \\
& + \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1-te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}{te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}\left(\widetilde{\varphi}_{ij}\right)}{p_{Z_i}ta_{ij}}
\end{aligned}
$$

And solving for NE_i:

$$
\begin{aligned}
NE_i = {} & \frac{\sigma-1}{\theta\sigma} \frac{\theta\sigma-(1-\mu)(\theta-\sigma+1)}{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)} \frac{Z_i}{en_i} \\
& - \frac{\sigma-1}{en_i} \frac{\sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1-te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}}{te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{ta_{ij}}}{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)} \\
= {} & \frac{\sigma-1}{\theta\sigma} \frac{\theta\sigma-\theta+\sigma-1+\mu(\theta-\sigma+1)}{\theta\sigma-\theta+\sigma-1+\mu(\theta-\sigma+1)} \frac{Z_i}{en_i} \\
& - \frac{\sigma-1}{en_i} \frac{\sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1-te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}{te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i}ta_{ij}}}{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)} \\
= {} & \frac{\sigma-1}{\theta\sigma} \frac{Z_i}{en_i} - \frac{\sigma-1}{en_i} \frac{\sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1-te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}{te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}(\widetilde{\varphi}_{ij})}{p_{Z_i}ta_{ij}}}{(\theta+1)(\sigma-1)+\mu(\theta-\sigma+1)}
\end{aligned}
$$

Imposing $\mu = 1$ gives:

$$
NE_i = \frac{\sigma-1}{\theta\sigma} \frac{1}{en_i} \left(Z_i - \sum_{j=1}^{J} N_{ij} \frac{\sigma-1}{\sigma} \left(\frac{1-te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}}{te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}} \right) \frac{\overline{r}_{ij}\left(\widetilde{\varphi}_{ij}\right)}{p_{Z_i}ta_{ij}} \right) \tag{S.13}
$$

Equation (B.19)

Substituting equations (B.12) and (B.17) into equation (B.16) gives:

$$\begin{aligned}
\frac{p_{ij}^{ag} ta_j^{s,ag}}{e_j} &= \frac{\sigma}{\sigma-1}\left(\left(\frac{\kappa_i}{\varphi_{ij}^*}\right)^{\theta}\frac{NE_i}{\delta}\left(\frac{ta_{ij} ta_j^{s,ag}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}{\left(\frac{\theta}{\theta-\sigma+1}\right)^{\frac{1}{\sigma-1}}\varphi_{ij}^*}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}\left(\frac{\theta\kappa_i^{\theta_i}}{\theta-\sigma+1}\frac{NE_i}{\delta}\frac{\left(ta_{ij} ta_j^{s,ag}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\varphi_{ij}^*\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

Substituting next equation (B.18) leads to:

$$\begin{aligned}
\frac{p_{ij} ta_j^{s,ag}}{e_j} &= \frac{\sigma}{\sigma-1}\left(\frac{\theta\kappa_i^{\theta}}{\theta-\sigma+1}\frac{\sigma-1}{\sigma\theta}\frac{\widetilde{Z}_i}{\delta en_i}\frac{\left(ta_{ij} ta_j^{s,ag}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\varphi_{ij}^*\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma}{\sigma-1}\left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)}\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\frac{\left(ta_{ij} ta_j^{s,ag}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\varphi_{ij}^*\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

Equation (B.20)

Substituting equation (B.7) into equation (B.19) gives:

$$
\begin{aligned}
\frac{p_{ij}^{ag} ta_j^{s,ag}}{e_j} &= \frac{\sigma}{\sigma-1}\left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)}\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\frac{\left(ta_{ij} ta_j^{s,ag}\tau_{ij}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1-\sigma}}{\left(\frac{\frac{\sigma}{\sigma-1}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)ta_{ij}\tau_{ij}}{\left(\sigma f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}ta_{ij}\right)^{\frac{1}{1-\sigma}}}\left(\sum_{ag=\{s,p,f\}}\left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{1}{1-\sigma}}\right)^{\theta-\sigma+1}}\right)^{\frac{1}{1-\sigma}} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{1+\frac{1}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\theta-\sigma+1}{\sigma-1}}\frac{\left(\theta-\sigma+1\right)^{\frac{1}{\sigma-1}}}{\sigma^{\frac{\theta-\sigma+1}{1-\sigma}\frac{1}{\sigma-1}}}\left(\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\left(ta_j^{ag}ta_{ij}\tau_{ij}\right)^{1-\sigma}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{1-\sigma-\theta+\sigma-1}\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}ta_{ij}\right)^{\frac{\theta-\sigma+1}{1-\sigma}}\left(ta_{ij}\tau_{ij}\right)^{-(\theta-\sigma+1)}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\sum_{ag=\{s,p,f\}}\left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{ag}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}} \\
&= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma+\theta-\sigma+1}{\sigma-1}}\frac{\left(\theta-\sigma+1\right)^{\frac{1}{\sigma-1}}}{\sigma^{\frac{\theta-\sigma+1}{1-\sigma}\frac{1}{\sigma-1}}}\left(\frac{\kappa_i^{\theta}\widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \\
&* \left(p_{Z_i}^{-\frac{\theta+\sigma-1}{\sigma-1}\mu}\left(te_{ij}p_{Z_i}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta}\left(\left(ta_j^{ag}\tau_j^{ag}\right)^{\frac{\sigma-1}{\theta}}\left(ta_{ij}\tau_{ij}\right)f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\left(ta_{ij}\right)^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta}p_{Z_j}^{-(1-\mu)\frac{\theta-\sigma+1}{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\sum_{ag=\{s,p,f\}}\left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1}\frac{E_j^{ag}}{ta_j^{ag}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma+\theta-\sigma+1}{\sigma-1}} \frac{(\theta-\sigma+1)^{\frac{1}{\sigma-1}}}{\sigma^{\frac{\theta-\sigma+1}{1-\sigma}\frac{1}{\sigma-1}}} \left(\frac{\kappa_i^\theta Z_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta} p_{Z_i}^{-\frac{\theta+\sigma-1}{\sigma-1}\mu} \left(ta_{ij}\tau_{ij}f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}} (ta_{ij})^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta} \left(ta_j^{ag}\tau_j^{ag}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}} \\
&= \left(\frac{\gamma_m \kappa_i^\theta \widetilde{Z}_i \left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{-\theta} p_{Z_i}^{-\frac{\theta+\sigma-1}{\sigma-1}\mu}}{\delta en_i} \left(ta_{ij}^{1+\frac{\theta-\sigma+1}{\theta(\sigma-1)}} \tau_{ij} f_{ij}^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\right)^{-\theta} \left(ta_j^{ag}\tau_j^{ag}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\
&* \left(\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}\tau_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{-\frac{\theta-\sigma+1}{(\sigma-1)^2}}
\end{aligned}
$$

With γ_m defined as:

$$
\gamma_m = \psi \left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1}
$$

Equations (D.29)-(D.30)

From equation (D.27) we can write θ as:

$$
\theta = \widetilde{e} - \frac{1}{d}
$$

We can rewrite the expression for $\widetilde{e}$ in equation (D.27) as follows:

$$
\begin{aligned}
\widetilde{e} &= \frac{(\theta+1)(\sigma-1)}{\sigma-1} + \frac{\theta-\sigma+1}{\sigma-1} \\
&= \frac{\theta\sigma+\sigma-\theta-1+\theta-\sigma+1}{\sigma-1} \\
&= \frac{\theta\sigma}{\sigma-1}
\end{aligned}
$$

Therefore we can write σ as:

$$
\widetilde{e} = \frac{\sigma}{d}
$$

$$
\sigma = d\widetilde{e}
$$

Equation (B.25)

Substituting the expressions for t_{ij}, c_i and e_j^s into equation (A.1) gives:

$$q_{ij}e_j^s = \left(\frac{ta_{ij} \left(\left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)^{\frac{\theta-\sigma+1}{\sigma-1}} \tau_{ij}^{\frac{\theta-\sigma+1}{\sigma-1}} ta_{ij}^{\frac{\sigma(\theta-\sigma+1)}{(\sigma-1)^2}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \right) \tau_{ij} \left(\frac{\gamma_m \kappa_i^\theta \widetilde{Z}_i}{\delta en_i} \right)^{\frac{1}{1-\sigma}} p_{Z_i}^{\mu\frac{\theta-\sigma+1}{(\sigma-1)^2}} \left(te_{ij}b_i p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\left(\frac{\sum_{ag=\{s,p,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{s,ag}} \right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}}{p_{Z_j}^{1-\mu}} \right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}} \right)^{-\sigma}$$

$$* \sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}} \right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}$$

$$= \left(\left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)^{\frac{\theta}{\sigma-1}} \tau_{ij}^{\frac{\theta}{\sigma-1}} ta_{ij}^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}} f_{ij}^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \left(\frac{\gamma_m \kappa_i^\theta \widetilde{Z}_i}{\delta en_i} \right)^{\frac{1}{1-\sigma}} \left(f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu} \right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} \right)^{-\sigma}$$

$$* \left(\sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}} \right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}} \right)^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}}$$

Equation (B.30)

Elaborating on equation (B.6) gives:

$$\sum_{ag=\{s,p,f\}} \frac{\left(ta_j^{ag} p_{ij}^{o} \left(\varphi_{ij}^* \right) \right)^{1-\sigma} \left(P_j^{ag} \right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}$$

$$\sum_{ag=\{s,p,f\}} \frac{\frac{ta_j^{ag} \frac{\sigma}{\sigma-1} ta_{ij}\tau_{ij} \left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\varphi_{ij}^*} \left(\frac{ta_j^{ag} \frac{\sigma}{\sigma-1} ta_{ij}\tau_{ij} \left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\varphi_{ij}^*} \right)^{-\sigma} \left(P_j^{ag} \right)^{\sigma-1} E_j^{ag}}{ta_j^{ag} ta_{ij}} = \sigma f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}$$

Rearranging:

$$\sum_{ag=\{s,p,f\}} \left(\frac{\sigma}{\sigma-1} \frac{ta_j^{ag} ta_{ij}\tau_{ij} \left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}{\varphi_{ij}^*} \right)^{-\sigma} \left(P_j^{ag} \right)^{\sigma-1} E_j^{ag} = (\sigma-1) \frac{f_{ij}}{\tau_{ij} \left(te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}} \right)} \varphi_{ij}^* \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}}$$

$$o_{ij}\left(\varphi_{ij}^* \right) = (\sigma-1) \frac{f_{ij}}{\tau_{ij} \left(te_{ij} + \frac{p_{ij}^{tr}}{p_{Z_i} a_{ij}^{tr}} \right)} \varphi_{ij}^* \frac{p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{p_{Z_i}}$$

Equation (B.31)

Substituting equation (B.30) and also the expressions for N_{ij} and NE_i in equations (B.17)-

(B.18) into equation (B.29) leads to:

$$
\begin{aligned}
q_{ij}^{ag} e_j^s &= N_{ij}^{\frac{\sigma}{\sigma-1}} (\sigma-1) \varphi_{ij}^* \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij} \left(p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)} \left(\frac{\theta}{\theta - \sigma + 1} \right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\left(\frac{\kappa_i}{\varphi_{ij}^*} \right)^{\theta} \frac{\sigma-1}{\sigma\theta} \frac{\widetilde{Z}_i}{\delta en_i} \right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) \varphi_{ij}^* \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij} \left(p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)} \left(\frac{\theta}{\theta - \sigma + 1} \right)^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{\sigma-1}{\sigma(\theta-\sigma+1)} \right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) \left(\frac{\kappa_i^{\theta} \widetilde{Z}_i}{\delta en_i} \right)^{\frac{\sigma}{\sigma-1}} \frac{1}{\left(\varphi_{ij}^* \right)^{\frac{\theta\sigma-\sigma+1}{\sigma-1}}} \frac{f_{ij} p_{Z_i}^{\mu} p_{Z_j}^{1-\mu}}{\tau_{ij} \left(p_{Z_i} te_{ij} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}} \right)}
\end{aligned}
$$

Equivalence Equation (B.25)

Substituting the expression for φ_{ij}^* in equation (B.7) into equation (B.31) gives:

$$q_{ij}e_j^s = \left(\frac{\sigma-1}{\sigma\left(\theta-\sigma+1\right)}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{1}{\left(\varphi_{ij}^*\right)^{\frac{\theta\sigma-\sigma+1}{\sigma-1}}} \frac{f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}}{p_{Z_i}\tau_{ij}\left(p_{Z_i}te_{ij}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}$$

$$= \frac{\left(\frac{\sigma-1}{\sigma(\theta-\sigma+1)}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}}{\tau_{ij}\left(p_{Z_i}te_{ij}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)}}{\left(\frac{\frac{\sigma}{\sigma-1}ta_{ij}\tau_{ij}p_{Z_i}\left(te_{ij}+\frac{p_{ij}^{tr}}{p_{Z_i}a_{ij}^{tr}}\right)}{\left(\sigma f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}ta_{ij}\right)^{\frac{1}{1-\sigma}}} \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{1}{1-\sigma}}\right)^{\frac{\theta\sigma-\sigma+1}{\sigma-1}}}$$

$$= \frac{(\theta-\sigma+1)^{-\frac{\sigma}{\sigma-1}}(\sigma-1)}{\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\sigma}{\sigma-1}+\frac{\theta\sigma-\sigma+1}{\sigma-1}} \sigma^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}} \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{1-\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}}{\left(\tau_{ij}\left(p_{Z_i}te_{ij}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{1+\frac{\theta\sigma-\sigma+1}{\sigma-1}}} ta_{ij}^{-\sigma\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$* \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$= \frac{(\theta-\sigma+1)^{-\frac{\sigma}{\sigma-1}}}{\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\theta\sigma+1}{\sigma-1}} \sigma^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}-1}\frac{\sigma}{\sigma-1}} \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\sigma^2-2\sigma+1-(\theta\sigma-\sigma+1)}{(\sigma-1)^2}}}{\left(\tau_{ij}\left(p_{Z_i}te_{ij}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{\frac{\theta\sigma}{\sigma-1}}} ta_{ij}^{-\sigma\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$* \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$= \frac{(\theta-\sigma+1)^{-\frac{\sigma}{\sigma-1}}}{\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\theta\sigma+\sigma}{\sigma-1}} \sigma^{\frac{\theta\sigma-\sigma+1-(\sigma^2-2\sigma+1)}{(\sigma-1)^2}}} \left(\frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{\sigma}{\sigma-1}} \frac{\left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\sigma^2-\sigma-\theta\sigma}{(\sigma-1)^2}}}{\left(\tau_{ij}\left(p_{Z_i}te_{ij}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{\frac{\theta\sigma}{\sigma-1}}} ta_{ij}^{-\sigma\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$* \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

$$= \left(\left(\frac{\left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)}\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1} \frac{\kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \left(\tau_{ij}\left(p_{Z_i}te_{ij}+\frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)\right)^{\frac{\theta}{\sigma-1}} \left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}} ta_{ij}^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}\right)^{-\sigma}$$

$$* \left(\sum_{ag=\{p,g,f\}} \left(\frac{P_j^{ag,e}}{ta_j^{ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{ag}}\right)^{\frac{\theta\sigma-\sigma+1}{(\sigma-1)^2}}$$

Using the definition for γ_m in equation (22) this expression is identical to the expression in equation $(B.25)$:

$$q_{ij}e_j^s = \left(\left(\left(te_{ij}p_{Z_i} + \frac{p_{ij}^{tr}}{a_{ij}^{tr}}\right)^{\frac{\theta}{\sigma-1}} \tau_{ij}^{\frac{\theta}{\sigma-1}} ta_{ij}^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}} \left(\frac{\gamma_m \kappa_i^\theta \widetilde{Z}_i}{\delta en_i}\right)^{\frac{1}{1-\sigma}} \left(f_{ij}p_{Z_i}^{\mu}p_{Z_j}^{1-\mu}\right)^{\frac{\theta-\sigma+1}{(\sigma-1)^2}}\right)^{-\sigma}$$
$$* \left(\sum_{ag\in\{p,g,f\}} \left(\frac{P_j^{ag}}{ta_j^{s,ag}}\right)^{\sigma-1} \frac{E_j^{ag}}{ta_j^{s,ag}}\right)^{\frac{\sigma\theta-\sigma+1}{(\sigma-1)^2}}$$

With:

$$\gamma_m = \psi\left(\frac{\sigma}{\sigma-1}\right)^{-(\theta+1)} \frac{\sigma^{-\frac{\theta-\sigma+1}{\sigma-1}}}{\theta-\sigma+1}$$

Equality of total trade flows in GEMPACK from model and from margin decomposition

We can check the correctness of the margin decomposition expressions by comparing the total margin TM in equation (D.34) with the change in trade flows following from the main model. We do that employing GEMPACK notation. The change in the quantity of trade in the main model is given by:

$$qxs(i,r,s) = -ams(i,r,s) + qim(i,s) - \sigma[pms(i,r,s) - ams(i,r,s) - pim(i,s)]$$

In value terms the change in trade flows is given by:

$$\begin{aligned}
pms(i,r,s) + qxs(i,r,s) &= qim(i,s) + pim\,(i,s) \\
&- (\sigma-1)\,[pms(i,r,s) - ams(i,r,s) - pim(i,s)] \\
&= qim(i,s) + pim\,(i,s) - (\sigma-1)\,pms(i,r,s) \\
&+ (\sigma-1)\,ams\,(i,r,s) + (\sigma-1)\,pim(i,s) \\
&= qim(i,s) + pim\,(i,s) \\
&- (\sigma-1)\,(tm(i,s) + tms(i,r,s)) - (\sigma-1)\,pcif\,(i,r,s) \\
&+ (\sigma-1)\,sext\,(i,r) \\
&- \frac{\sigma\,(\theta-\sigma+1)}{\sigma-1}\,(tm\,(i,s) + tms\,(i,r,s)) - \theta itc\,(i,r,s) \\
&- \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s) - (\theta-\sigma+1)\,pcif\,(i,r,s) \\
&+ (\sigma-1)\,pim(i,s)
\end{aligned}$$

Rearranging gives:

$$\begin{aligned}
pms(i,r,s) + qxs(i,r,s) = {} & qim(i,s) + pim\,(i,s) + (\sigma-1)\,pim(i,s) \\
& - \left((\sigma-1) + \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)} \right) (tm\,(i,s) + tms\,(i,r,s)) - \theta pcif\,(i,r,s) \\
& + (\sigma-1)\,sext\,(i,r) - \theta itc\,(i,r,s) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s)
\end{aligned}$$

And further rearranging we get:

$$\begin{aligned}
pms(i,r,s) + qxs(i,r,s) &= qim(i,s) + pim\,(i,s) + (\sigma-1)\,pim(i,s) \\
& - \left(\frac{\sigma^2 - 2\sigma + 1 + \sigma\theta - \sigma^2 + \sigma}{(\sigma-1)} \right) (tm\,(i,s) + tms\,(i,r,s)) - \theta pcif\,(i,r,s) \\
& + (\sigma-1)\,sext\,(i,r) - \theta itc\,(i,r,s) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s) \\
&= qim(i,s) + pim\,(i,s) + (\sigma-1)\,pim(i,s) \\
& - \left(\frac{\theta\sigma - \theta + \theta + 1 - \sigma}{(\sigma-1)} \right) (tm\,(i,s) + tms\,(i,r,s)) - \theta pcif\,(i,r,s) \\
& + (\sigma-1)\,sext\,(i,r) - \theta itc\,(i,r,s) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s) \\
&= qim(i,s) + pim\,(i,s) + (\sigma-1)\,pim(i,s) \\
& - \left(\theta + \frac{\theta-\sigma+1}{\sigma-1} \right) (tm\,(i,s) + tms\,(i,r,s)) - \theta pcif\,(i,r,s) \\
& + (\sigma-1)\,sext\,(i,r) - \theta itc\,(i,r,s) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s)
\end{aligned}$$

We have employed both the expression for ams:

$$\begin{aligned}
ams\,(i,r,s) = {} & sext\,(i,r) - \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)^2} (tm\,(i,s) + tms\,(i,r,s)) \\
& - \left(1 + \frac{\theta-\sigma+1}{\sigma-1} \right) itc\,(i,r,s) - \frac{\theta-\sigma+1}{(\sigma-1)^2} fex\,(i,r,s) \\
& - \frac{\theta-\sigma+1}{\sigma-1} pcif\,(i,r,s)
\end{aligned} \tag{S.14}$$

And for pms:

$$pms(i,r,s) = tm(i,s) + tms(i,r,s) + pcif(i,r,s)$$

Next, we elaborate on $qim(i,s) + pim\,(i,s) + (\sigma - 1)\,pim(i,s)$. For $qim + pim$ we have:

$$
\begin{aligned}
\sigma pim\,(i,r) + qim(i,r) &= pim\,(i,r) + qim(i,r) + (\sigma-1)\,pim\,(i,r) \\
&= pim\,(i,r) + sum(j, PROD\_COMM, SHRIFM(i,j,r) * qfm(i,j,r)) \\
&+ SHRIPM(i,r) * qpm(i,r) + SHRIGM(i,r) * qgm(i,r) + (\sigma-1)\,pim\,(i,r) \\
&= pim\,(i,r) \\
&+ sum(j, PROD\_COMM, SHRIFM(i,j,r)\,(qf(i,j,r) - \sigma * [pfm(i,j,r) - pf(i,j,r)])) \\
&+ SHRIPM(i,r)\,(qp(i,r) - \sigma[ppm(i,r) - pp(i,r)]) \\
&+ SHRIGM(i,r)\,(qg(i,r) - \sigma[pgm(i,r) - pg(i,r)]) - Dextm\,(i,r) + (\sigma-1)\,pim\,(i,r) \\
&= -\,(\sigma-1)\,pim\,(i,r) + (\sigma-1)\,Dextm(i,r) + (\sigma-1)\,pim\,(i,r) \\
&+ sum(j, PROD\_COMM, SHRIFM(i,j,r)(qf(i,j,r) + pf(i,j,r) \\
&- \sigma tfm(i,j,r) + (\sigma-1)\,pf(i,j,r)])) \\
&+ SHRIPM(i,r)\,(qp(i,r) + pp(i,r) - \sigma tpm(i,r) + (\sigma-1)\,pp(i,r)]) \\
&+ SHRIGM(i,r)\,(qg(i,r) + pg(i,r) - \sigma tgm(i,r) + (\sigma-1)\,pg(i,r)])
\end{aligned}
$$

$$
\begin{aligned}
\sigma pim\,(i,r) + qim(i,r) &= (\sigma-1)\,Dextm(i,r) \\
&+ sum(j, PROD\_COMM, SHRIFM(i,j,r)(qf(i,j,r) + pf(i,j,r) \\
&- \sigma tfm(i,j,r) + (\sigma-1)\,pf(i,j,r)])) \\
&+ SHRIPM(i,r)\,(qp(i,r) + pp(i,r) - \sigma tpm(i,r) + (\sigma-1)\,pp(i,r)]) \\
&+ SHRIGM(i,r)\,(qg(i,r) + pg(i,r) - \sigma tgm(i,r) + (\sigma-1)\,pg(i,r)]) \\
&= (\sigma-1)\,Dextm(i,r) + valueD(i,r) + (\sigma-1)\,priceDm(i,r) - \sigma tariffDm(i,r)
\end{aligned}
$$

Using:

$$
\begin{aligned}
pfm\,(i,j,r) &= tfm(i,j,r) + pim(i,r) - Dextm(i,r) \\
pgm\,(i,r) &= tgm(i,r) + pim(i,r) - Dextm(i,r) \\
ppm\,(i,r) &= tpm(i,r) + pim(i,r) - Dextm(i,r)
\end{aligned}
$$

Elaborating on $Dextm\,(i,r)$ gives:

$$\begin{aligned}
Dextm\,(i,r) &= [g(i) * [\sigma-1]/\sigma] * [priceDm(i,r) - pfactwld] \\
&\quad + [g(i)/\sigma] * (valueDm(i,r) - pfactwld) \\
&\quad + g(i) * tariffDm(i,r) \\
&= \frac{\theta-\sigma+1}{\sigma-1}[priceDm(i,r) - pfactwld] + \frac{\theta-\sigma+1}{(\sigma-1)^2}(valueDm(i,r) - pfactwld) \\
&\quad + \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)^2} tariffDm(i,r)
\end{aligned}$$

Substituting in gives then:

$$\begin{aligned}
\sigma pim\,(i,r) + qim(i,r) &= (\sigma-1)\,(\frac{\theta-\sigma+1}{\sigma-1}[priceDm(i,r) - pfactwld] \\
&\quad + \frac{\theta-\sigma+1}{(\sigma-1)^2}(valueDm(i,r) - pfactwld) + \frac{\sigma\,(\theta-\sigma+1)}{(\sigma-1)^2} tariffDm(i,r)) \\
&\quad + valueDm(i,r) + (\sigma-1)\,priceDm(i,r) - \sigma tariffDm(i,r) \\
&= \theta priceDm(i,r) + \frac{\theta}{\sigma-1} valueDm(i,r) - \frac{\theta}{\sigma-1}\sigma tariffDm(i,r)
\end{aligned}$$

So, the overall effect becomes:

$$\begin{aligned}
pms(i,r,s) + qxs(i,r,s) &= (\sigma-1)\,sext\,(i,r) - \left(\theta + \frac{\theta-\sigma+1}{\sigma-1}\right)(tm\,(i,s) + tms\,(i,r,s)) \\
&\quad - \theta\,(itc\,(i,r,s) + pcif\,(i,r,s)) - \frac{\theta-\sigma+1}{\sigma-1} fex\,(i,r,s) \\
&\quad + \theta priceDm(i,r) + \frac{\theta}{\sigma-1} valueDm(i,r) - \frac{\sigma\theta}{\sigma-1} tariffDm(i,r)
\end{aligned}$$

And from the decomposition in equation (D.34) we had:

$$\begin{aligned}
d\ln V_{ij} &= TM = EM + IM + CM \\
&= (\sigma-1)\,sext\,(i,r) - \left(\theta + \frac{\theta-\sigma-1}{\sigma-1}\right)(tm\,(i,s) + tms\,(i,r,s)) \\
&\quad - \theta\,(itc\,(i,r,s) + pcif\,(i,r,s)) - \frac{\theta-\sigma-1}{\sigma-1} fex\,(i,r,s) \\
&\quad + \theta priceD(i,s) + \frac{\theta}{\sigma-1} valueD\,(i,s) - \frac{\sigma\theta}{\sigma-1} tariffDs\,(i,s)
\end{aligned}$$

So, the two approaches generate identical expressions, which is confirmed by calculating the change in trade flows in GEMPACK in the two alternative ways.