Simulation Study on Combination of GRACE Monthly Gravity Field Solutions

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1. Introduction

- **GRACE monthly gravity field solutions** from different processing centers using official data from GRACE satellites

- Combined by the project *EGSIEM* using different weighting schemes to produce more reliable solutions reducing systematic errors (Jean et al., 2015a,b)
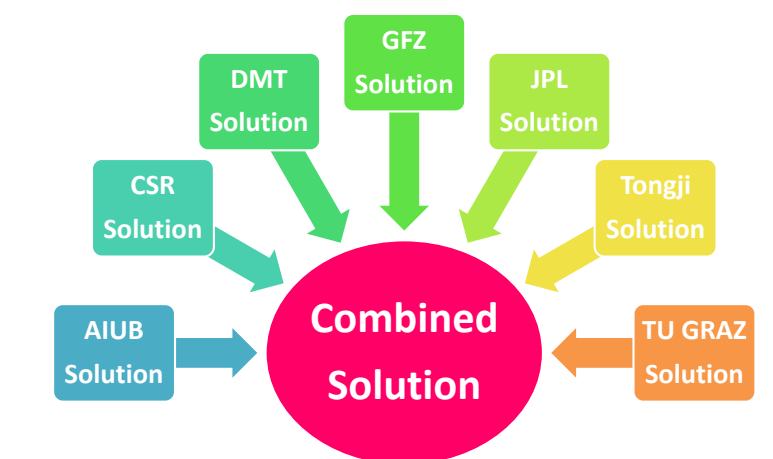
Figure 1: Combination of GRACE monthly gravity field solutions.

- Motivation:

The combined solutions' spherical harmonic coefficients

$0 <$ degree < 60 : The combined solutions work well.

$60 <$ degree < 90 : *An individual solution has smaller anomalies than the combined solutions in certain months.*
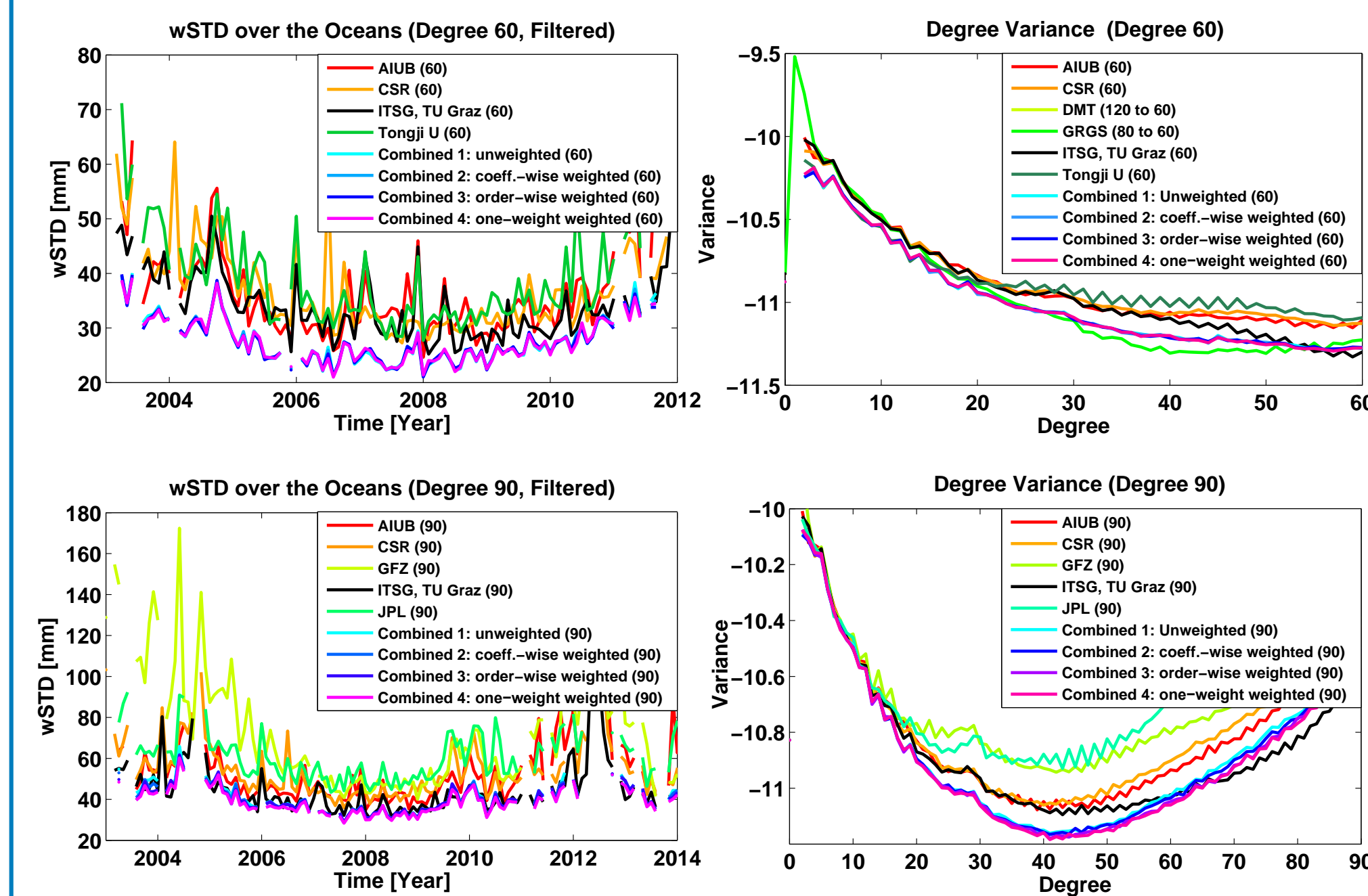
Figure 2: Weighted standard deviation (wSTD) over the oceans (Left Column) and degree variance of anomalies with respect to a model consisting of offset, trend, annual and semi-annual signals (Right Column) from/of different GRACE monthly individual and combined gravity field solutions up to degree 60 (Top Row) and 90 (Bottom Row).

Possible reasons "*why the combined solutions do not have the lowest noise in all cases*":

1) The individual solutions containing higher noise affected the noise of combined solutions in a negative way.

2) The individual solution containing the lowest noise is different from other solutions: e.g. *attenuated signals* as well as very low noise.

3) The weighting schemes based on difference from the arithmetic mean are not the best weighting schemes because arithmetic mean can be easily affected by outliers.

- Objectives:

Through simulations, we investigate

1) Effect of an individual solution with extreme noise level on the combination

2) Effect of an individual solution containing attenuated signal on the combination

3) An enhanced weighting scheme by an iterative process to minimize negative effects of deviated solutions on the combination

2. Simulated Gravity Field Solutions

- Reference solution:

The reference solution used in this study for simulation is generated from a model whose offset, slope, and annual signal are extracted from a weighted combined solution.

$$\hat{X}_{l_m}(t) = a_{0_{lm}} + a_{1_{lm}}\Delta t + a_{2_{lm}}\sin\omega\Delta t + b_{2_{lm}}\cos\omega\Delta t$$

- Simulated individual solutions:

$$X_{i_{lm}}(t) = \mathbf{k_0} a_{0_{lm}} + \mathbf{k_1} a_{1_{lm}}\Delta t + \mathbf{k_2}(a_{2_{lm}}\sin\omega\Delta t + b_{2_{lm}}\cos\omega\Delta t) + \mathbf{k_3}\epsilon$$

where

$\hat{X}_{lm}(t)$	model's spherical harmonic coefficient of degree l and order m in month t which is $t_0 + \Delta t$ where t_0 is reference epoch
$a_{0_{lm}}$	offset
$a_{1_{lm}}$	slope (trend)
$a_{2_{lm}}, b_{2_{lm}}$	amplitudes of annual signal
k_0, k_1, k_2, k_3	scaling factors to control the offset, slope, annual signal, and unmodeled random error ϵ

Tables 1 and 2 describe the scale factors and the simulated solutions used in this study.

Table 1: Scale factors used in this simulation study

Scale Factor	In Simulation
$k0, k1$	**Fixed** because the offset and slope do not affect amplitudes of signal and variability
$k2$	**Varied** to investigate effect of an individual solution containing attenuated signal
$k3$	**Varied** to investigate effect of an individual solution containing deviated level of noise

Table 2: A group of *simulated* solutions in each case (Cases 1–4)

Simulated Solution	Type
Individual solutions	1 deviated solution + 3 normal solutions
Combined solutions [1]	1) Unweighted 2) Weighted (by one-weight per month)
Combined solutions [2]	A series of combined solutions weighted by one-weight per month from an *iterative process**

* **Improved weighting scheme using *iteration*:**

The weights as well as the role of arithmetic mean in computation of the weights are updated in each iteration step using *Variance Component Estimation*.

5. Conclusions

- Relatively very large noise in an individual gravity field solution can make the combined solutions have larger noise than an involved individual solution containing the lowest noise.

- Attenuated signal in an individual gravity field solution make the combined solutions also have attenuated signal.

- The weighted combined solution can reduce the negative effects by an individual solution's large noise and also attenuated signal better than the unweighted combined solution.

- The weighted combined solutions through the iterative process performs significantly better than the weighted combined solution without the iterative process.

3. Effect of Deviated Solutions on Combination

To investigate how a deviated individual gravity field solution affects the combined solutions, we generated three groups of simulated individual gravity field solutions and their combined solutions as described in Table 2 for three cases shown in Figure 3.
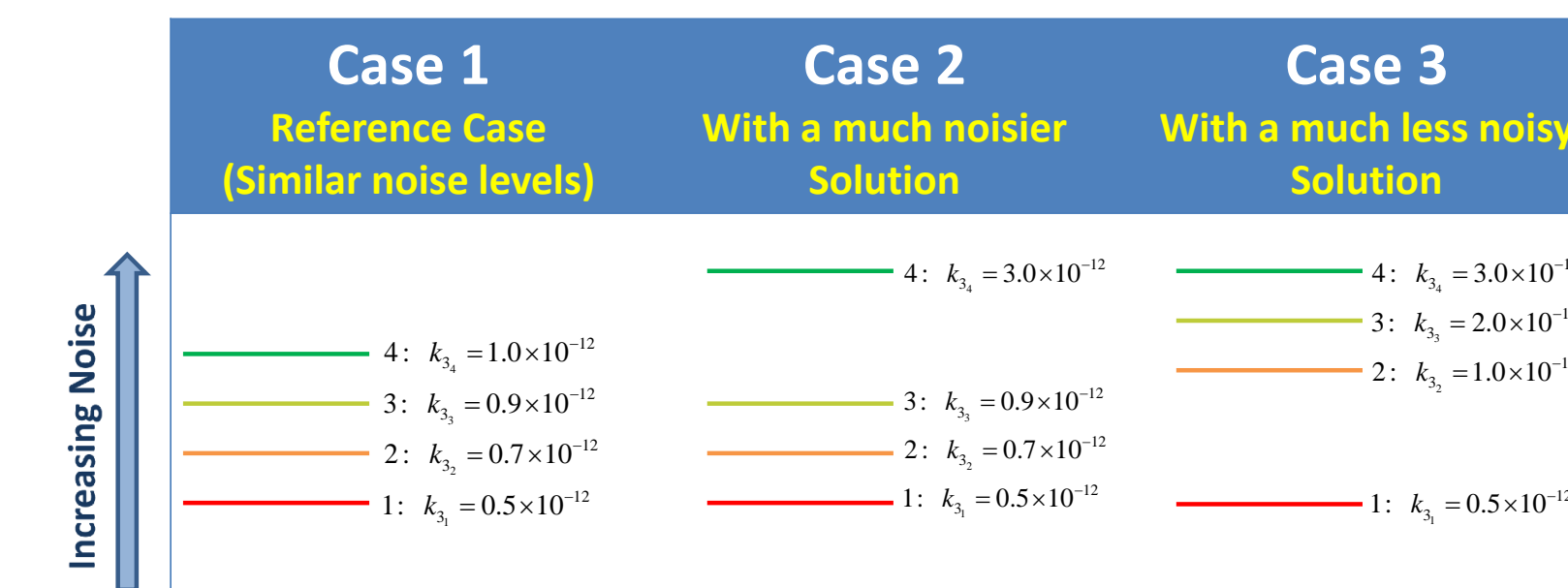
Figure 3: Three cases to investigate three different situations including four simulated individual solutions per case with different relative levels of noise by varing the scale factor k_3.
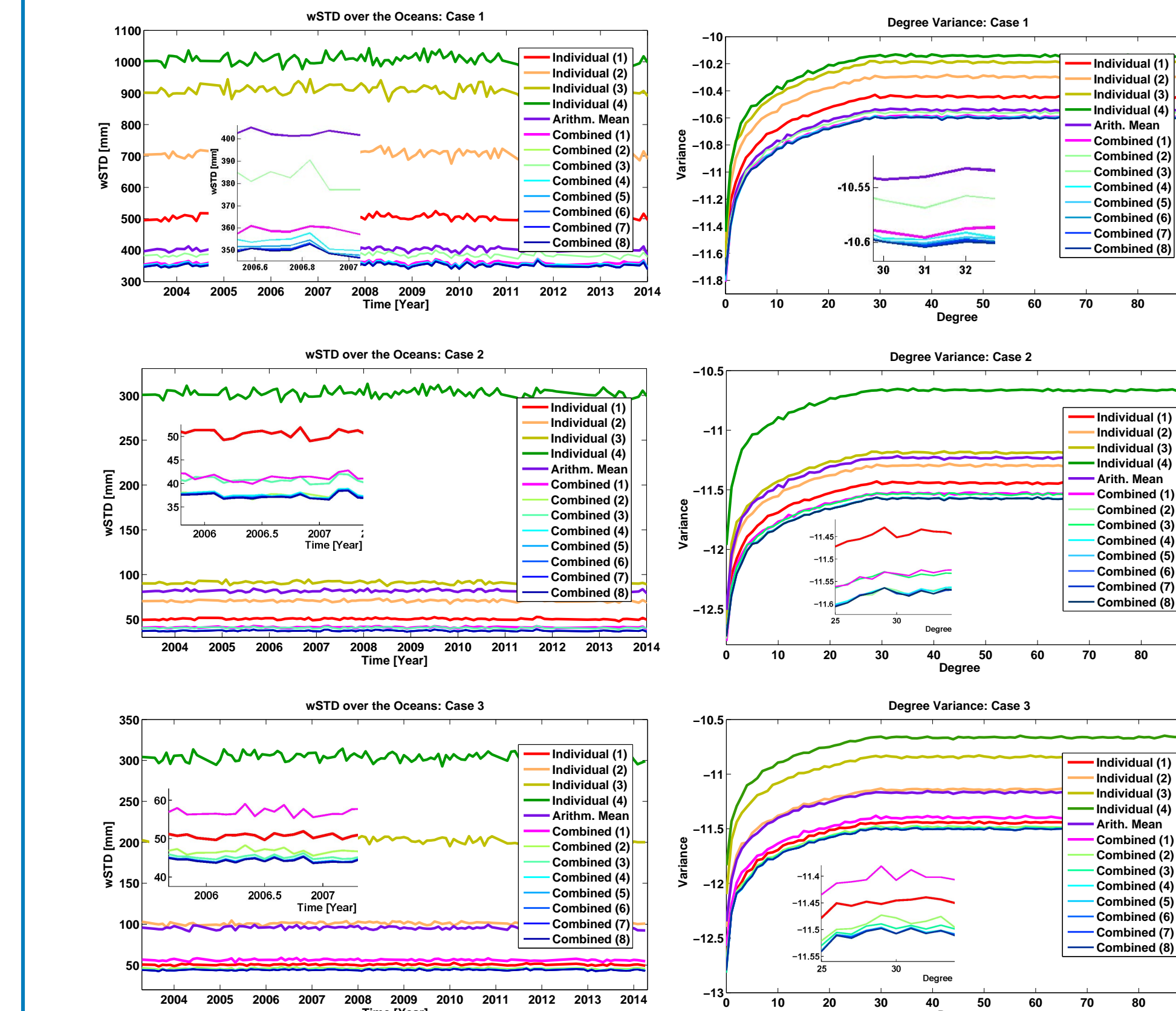
- Result and Discussion:

Figure 4: wSTD over the oceans (Left Column) and degree variance of anomalies with respect to a model consisting of offset, trend, and annual signal (Right Column) from/of simulated individual and combined gravity field solutions for case 1 (Top Row), case 2 (Middle Row), and case 3 (Bottom Row). (Zoomed-in subfigures are also shown on corresponding original figures; The numbers after 'Combined' in the legends indicate iteration step numbers.)

- The combined solutions including individual solutions with similar noise levels show the lowest noise even without iteration (Case 1).

- The combined solutions including an individual solution with deviated noise level have larger noise than the individual solution with the lowest noise level, similar to the real case shown in Figure 2 (Cases 2 and 3).

- The weighted combined solution can minimize the negative effect of an individual solution with deviated level of noise better than the simple arithmetic mean especially when the noise is white noise.

- The series of combined solutions from the iterative process, labeled Combined (2) to (8) in Figure 4, converge to the minimums of wSTD over the oceans and degree variance in all of the three cases regardless of relative noise levels of individual solutions.

- The weighted combined solutions through the iterative process can be the best combined solutions which can cancel out the negative effect by included individual solutions with very low or large noise.

4. Effect of Systematic Error on Combination

To investigate the effect of systematic errors on the combination of gravity field solutions, we also generated simulated solution whose signal is attenuated as described in Figure 5.
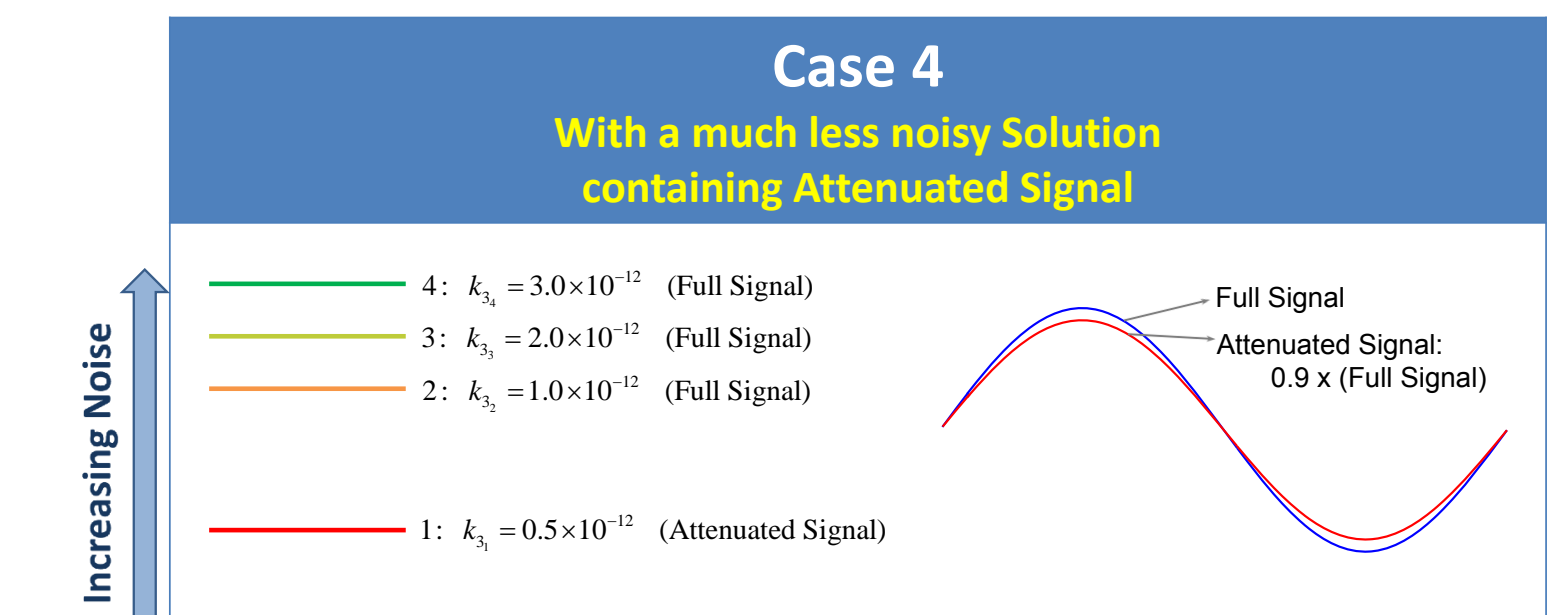
Figure 5: A case with simulated solutions containing full signal and attenuated signal (90% of full signal: $k_2 = 0.9$) with different relative noise levels identical to the noise levels in case 3.
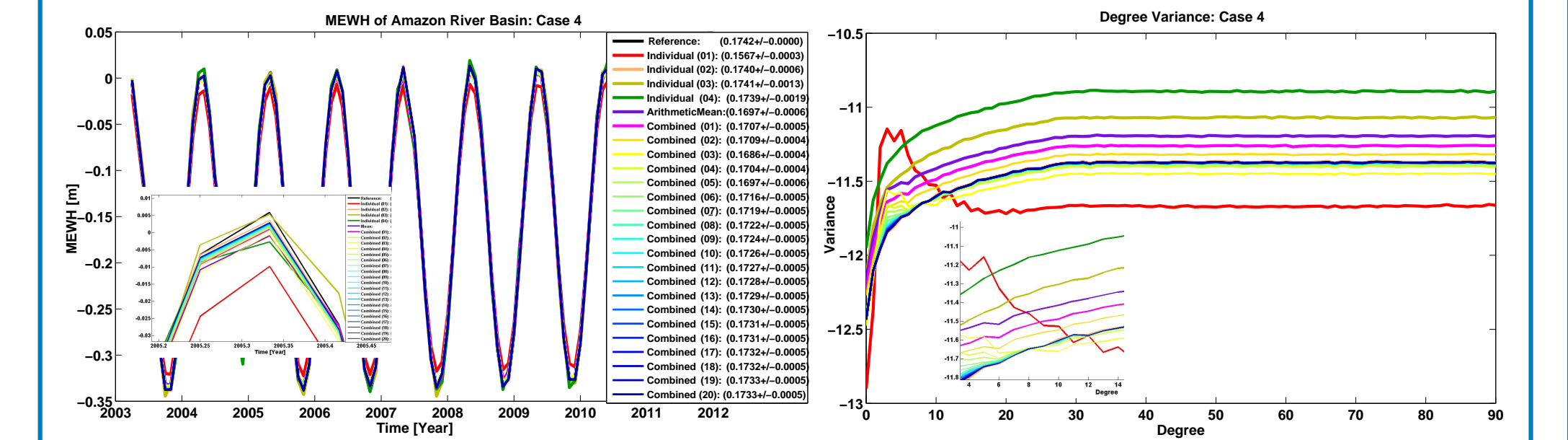
- Result and Discussion:

Figure 6: Mean equivalent water height of Amazon river basin (Left) and degree variance of anomalies with respect to a model (Right) from different simulated solutions including a solution with attenuated signal. (The values after the labels in the legends indicate amplitude of annual signal of each solution.)

- The attenuated signal in an individual solution results in the attenuated signal also in the combined solutions.

- When compared to the case 3 with the same relative noise levels, the individual solution with attenuated signal shows high anomalies in low degrees containing more signal and lower anomalies in higher degree. It can explain the cross point around degree 60 in the real situation shown in the degree variance graph in Figure 2.

- The weighted combined solution, labeled 'Combined (1)' in Figure 6, can reduce the negative effect due to the individual solutions containing attenuated signal better than the unweighted combined solution.

- The series of combined solutions from the iterative process, labeled Combined (2) to (20) in Figure 6, converge to the reference full signal of the reference solution.

- The weighted combined solutions through the iterative process can be the best combined solutions which can cancel out the negative effect by included individual solutions with attenuated signal.

References

Jean, Y., U. Meyer, and A. Jäggi. Combination of GRACE monthly gravity field solutions from different processing centers. EGU General Assembly, Vienna, Austria, April, 2015a.

Jean, Y., U. Meyer, and A. Jäggi. Combination of GRACE monthly gravity field solutions with different weighting schemes. Geodätische Woche, Stuttgart, Germany, September, 2015b.

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