Combination of GRACE monthly gravity models on normal equation level

**Input-NEQ: AIUB-0601**

Observations are the kinematic satellite positions (POS), determined by a precise point positioning from phase observations (GPS), and the inter-satellite range-rates (KRR), observed by the K-Band link. The number of observations and their chosen a priori uncertainties s are:

\[ n_{\text{POS}} = 31 \times 17280 \text{ obs.}, n_{\text{GPS}} = 3 \times 76630, n_{\text{KRR}} = 3 \times 76630 \text{ s/m} \]

The kinematic positions are transformed back to GPS phases (s = 0.02 m) by their epoch wise covariance information. The kinematic positions (or GPS phases) are down-weighted by an empirical factor of F = 1.9. Additional pseudo-observations are set up to constrain the stochastic accelerations typical for the CMA. The following weighting schemes were tested (using either GPS or POS), all leading to comparable results.

**Input-NEQ: GFZ-0601**

The NEQ finally used for the EGSIEM-combination is the normalized one. All orbit- and instrument-parameters were pre-eliminated.

\[ n_{\text{GPS}} = 4 \times 10816763 \text{ (POS at 30 s, KRR at 5 s), with gaps} \]

\[ n_{\text{pos}} = 8277 \text{ (spherical harmonic coefficients of degrees l = 2 to 90)} \]

**Input-NEQ: ITSG-0601**

The NEQ is normalized, but all the gravity field parameters were pre-eliminated.

\[ n_{\text{GPS}} = 4 \times 2691802 \text{ (GPS at 30 s, KRR at 5 s)}, n_{\text{pos}} = 8277 \text{ (spherical harmonic coefficients of degrees l = 2 to 90)} \]

**Pair-wise comparisons**

The comparison of the three individual solutions reveals a high consistency between the AIUB and ITSG contributions (both relying upon strong stochastic elements, i.e., stochastic accelerations or a stochastic noise model). Relative weights for the final combination may be derived from the comparison of the gravity fields with their arithmetic mean or from the individual solution’s a posteriori RMS.

**Weights based on a posteriori RMS**

Degree Of Freedom: DoF = \( n_{\text{par}} - n_{\text{par}} + n_{\text{par}} \)

\[ \text{DoF} = \frac{n_{\text{par}} - n_{\text{par}} + n_{\text{par}}}{n_{\text{par}} - n_{\text{par}} + n_{\text{par}}} \]

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Weights may be derived on solution level by comparison of the spherical harmonic coefficients \( K_{lm} \). For the individual solutions of the 3 NEQs:

\[ w_{\text{AIUB}} = \frac{1}{1 + \chi^2 / \sigma_{\text{AIUB}}^2} \]

\[ w_{\text{ITSG}} = \frac{1}{1 + \chi^2 / \sigma_{\text{ITSG}}^2} \]

\[ w_{\text{GRFZ}} = \frac{1}{1 + \chi^2 / \sigma_{\text{GRFZ}}^2} \]

The resulting normalized weights for the example NEQs are

\[ w_{\text{AIUB}} = 0.44, w_{\text{ITSG}} = 0.18, w_{\text{GRFZ}} = 0.38 \]

These weights do not account for the statistical properties of the spherical harmonic coefficients. They can only be applied additionally to empirical weights that lead to a homogeneous contribution of the individual NEQs to a combined solution.

**Conclusion**

Technically the combination of the AC-specific NEQs works well, but the relative weights of the normalized NEQs determined by their a posteriori RMS lead to very inhomogeneous contributions of the individual NEQs to the combined solution. This is caused by the use of different observation types and sampling rates, leading to different number of observations, and by the diverse noise models, leading to very different formal errors of the individual solutions. The derivation of empirical weights seems to be indispensable to achieve a homogeneous contribution of all NEQs, because EGSIEM does not aim at the unification of the processing strategies. Weights derived by the combination of the individual solutions to their arithmetic mean may finally be multiplied to the empirical weights.

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