# Combination of GRACE monthly gravity models on normal equation level

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## EGSIEM: Scientific combination service

The Scientific Combination Service of the European Gravity Service for Improved Emergency Management (EGSIEM) that is coordinated by the Astronomical Institute of the University of Bern (AIUB) aims at consistent, reliable and validated monthly gravity fields that are combined on Normal Equation (NEQ) level from standardized NEQs of all contributing Associated processing Centers (ACs). While the EGSIEM standards on reference frame and conventions guarantee consistency of the NEQs, the different ACs are free to use their specific approaches and parametrization and the a priori and background models of their choice. EGSIEM is open to all interested processing centers of GRACE-, GPS-, or SLR-based gravity fields.

## SINEX: Normal equation format

NEQs are exchanged in the Solution INdependent EXchange (SINEX) format, maintained by the International GNSS Service (IGS). Used are the SOLUTION/STATISTICS, /APRIORI, /ESTIMATE, /NORMAL_EQUATION_VECTOR, and /NORMAL_EQUATION_MATRIX blocks. Additional information on Earth Radius, GM and tide system is provided in the SOLUTION/COMMENT block in the gfc-format of the International Centre for Global Earth Models (ICGEM).

## Test of consistency

SINEX NEQs from GFZ and ITSG of GRACE gravity fields 2006/01 are transformed to the internal format used at AIUB, re-scaled to common a priori values, and combined and solved using the tools of the Celestial Mechanics Approach (CMA). As a first test of consistency the NEQs were solved individually and compared to the corresponding ESTIMATE-blocks of the SINEX files.

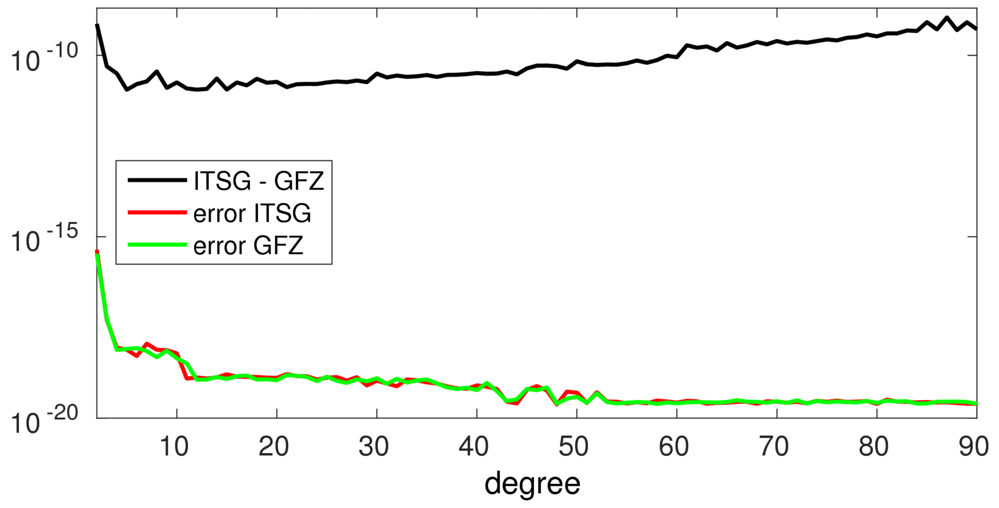


*Fig. 1: Difference degree amplitudes between reconstructed GFZ and ITSG solutions (black) and their respective reconstruction errors (ITSG: red, GFZ: green). The errors are at a numerically insignificant low level.*

## Temporal a priori variations and models

For consistency with existing monthly solutions the APRIORI-block includes the static a priori gravity model and its a priori temporal variations. It is under discussion to also include background models in the APRIORI-block to reconstruct and provide the full signal in the monthly fields (a variety of background models will be provided separately to the user).

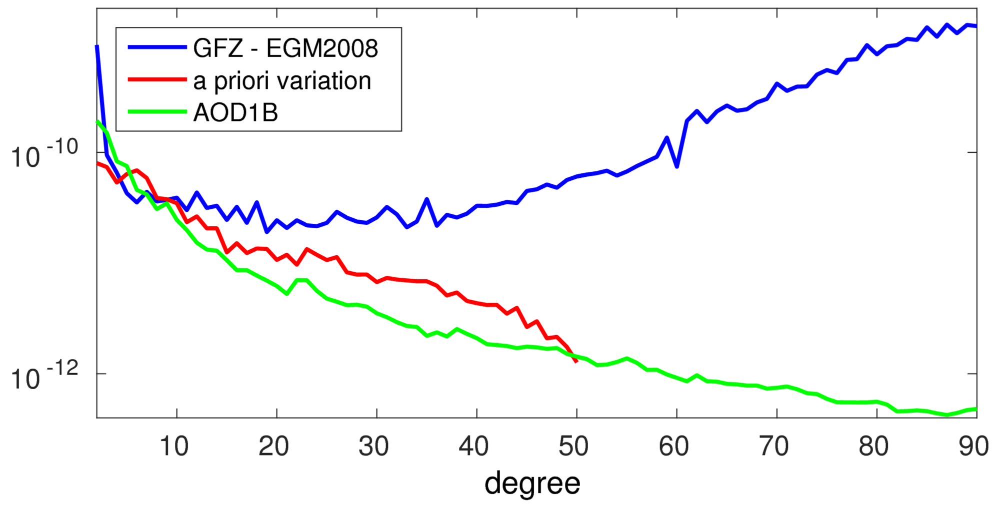


*Fig. 2: Difference degree amplitudes of GFZ monthly gravity field (excluding a priori trend, annual and semi-annual variations) with respect to EGM2008 (blue), a priori variations (red) and effect of dealiasing model (green).*

## Input-NEQ: AIUB-0601

Observations are the kinematic satellite positions (POS), determined by a precise point positioning from phase observations (GPS), and the inter-satellite range-rates (KRR), observed by the K-Band link. The number of observations and their chosen a priori uncertainties $s_0$ are:

$n_{KRR}$ = 31 d * 17280 obs. / d = 535680; $s_0$ = 3e-7 m/s
$n_{POS}$ = 31 d * 2 sat. * 2880 obs. / d / sat. * 3 = 535680; $s_0 \approx$ 0.02 m

The kinematic positions are transformed back to GPS phases ($s_0$ = 0.002 m) by their epoch wise covariance information. The kinematic positions (or GPS phases) are down-weighted by an empirical factor of $f^2=15^2$. Additional pseudo-observations are set up to constrain the stochastic accelerations typical for the CMA. The following weighting schemes were tested (using either GPS or POS), all leading to comparable results:

| f=15 | SO | $W_{KRR}$ | $W_{GPS}$ | $W_{POS}$ | $W_{norm}$ |
|---|---|---|---|---|---|
| KRR | $SO_{KRR}$ = 3e-7 m $s^{-1}$ | 1 | $SO_{GPS}^2 / SO_{KRR}^2$ = 1e10 | $SO_{POS}^2 / SO_{KRR}^2$ = 1e12 | $1 / SO_{KRR}^2$ = 1.11e13 |
| GPS | $SO_{GPS}$ = f*2e-3 m = 0.03 m | $SO_{KRR}^2 / SO_{GPS}^2$ = 1e-10 | 1 | | $1 / SO_{GPS}^2$ = 1111.11 |
| POS | $SO_{POS}$ = f*2e-2 m = 0.3 m | $SO_{KRR}^2 / SO_{POS}^2$ = 1e-12 | | 1 | $1 / SO_{POS}^2$ = 11.11 |
| STOCH. ACCEL. | $SO_{cons}$ = 3e-9 $s^{-2}$ | $SO_{KRR}^2 / SO_{cons}^2$ = 1e4 | $SO_{GPS}^2 / SO_{cons}^2$ = 1e14 | $SO_{POS}^2 / SO_{cons}^2$ = 1e16 | $1 / SO_{cons}^2$ = 1.11e17 |

The NEQ finally used for the EGSIEM-combination is the normalized one. All orbit- and instrument-parameters were pre-eliminated.
$n_{obs} = n_{POS} + n_{KRR}$ = 1016763 (POS at 30 s, KRR at 5 s, with gaps)
$n_{par} = n_{coef}$ = 8277 (spherical harmonic coefficients of degrees l = 2 to 90)

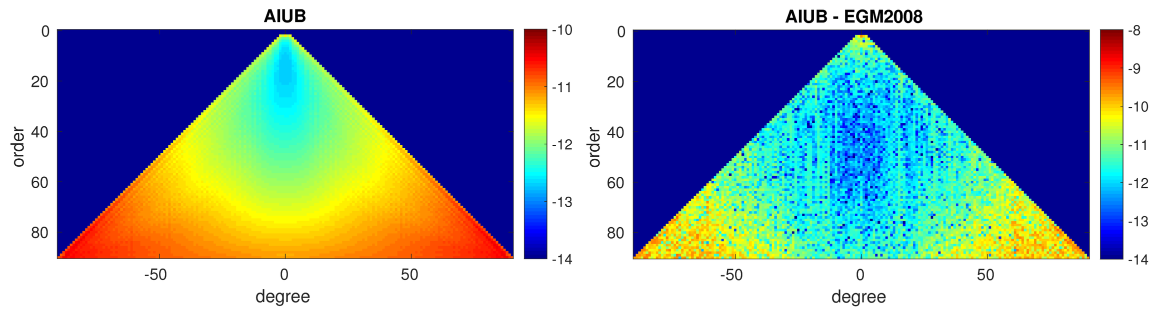


*Fig. 3: AIUB formal errors and differences to static field EGM2008.*

## Input-NEQ: GFZ-0601

The NEQ is normalized, all but the gravity field parameters were pre-eliminated.
$n_{obs} = n_{GPS} + n_{KRR}$ = 2691802 (GPS at 30 s, KRR at 5 s); $n_{par}$ = 8277

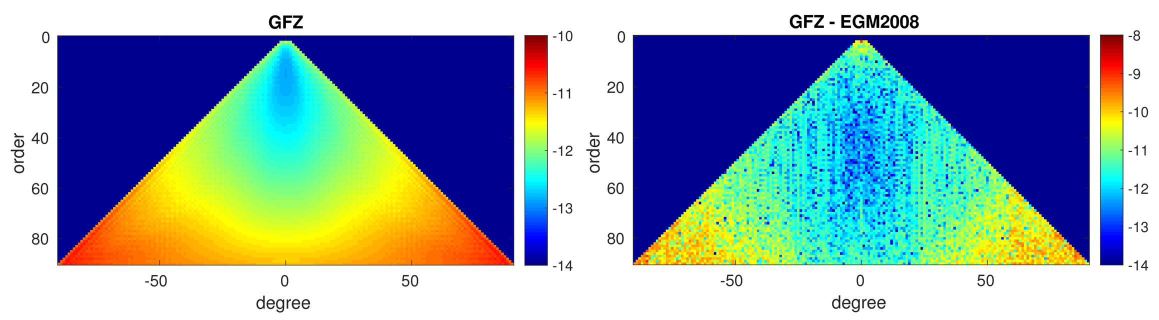


*Fig. 4: GFZ formal errors and differences to static field EGM2008.*

## Input-NEQ: ITSG-0601

The NEQ is normalized, all but the gravity field parameters were pre-eliminated. Empirical co-variances were applied to model the noise. The relative weighting of POS and KRR was defined by variance component estimation.
$n_{obs} = n_{POS} + n_{KRR}$ = 540481 (POS at 300 s, KRR at 5 s); $n_{par}$ = 8277

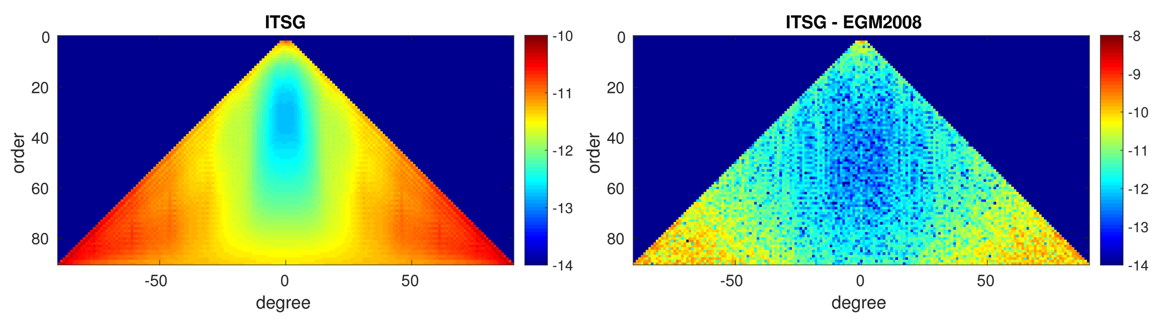


*Fig. 5: ITSG formal errors and differences to static field EGM2008. The formal errors differ from AIUB and GFZ due to the empirical co-variances.*

## Pair-wise comparisons

The comparison of the three individual solutions reveals a high consistency between the AIUB and ITSG contributions (both relying upon strong stochastic elements, i.e., stochastic accelerations or a stochastic noise model). Relative weights for the final combination may be derived from the comparison of the gravity fields with their arithmetic mean or from the individual solution's a posteriori RMS.

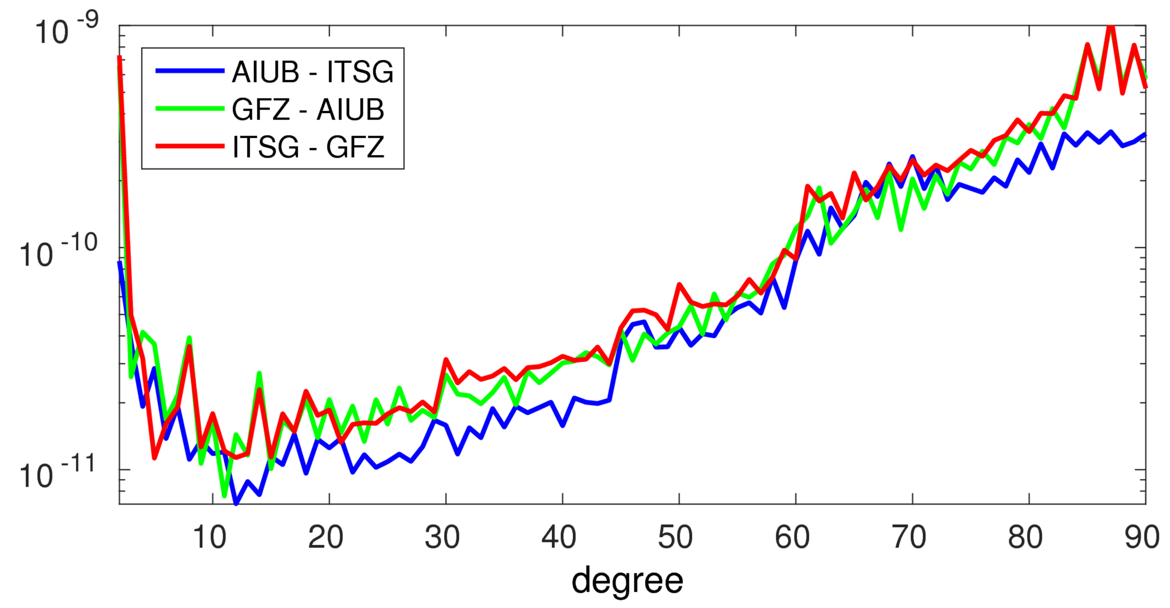


*Fig. 6: Difference degree amplitudes derived by pairwise comparisons of the three individual gravity field solutions.*

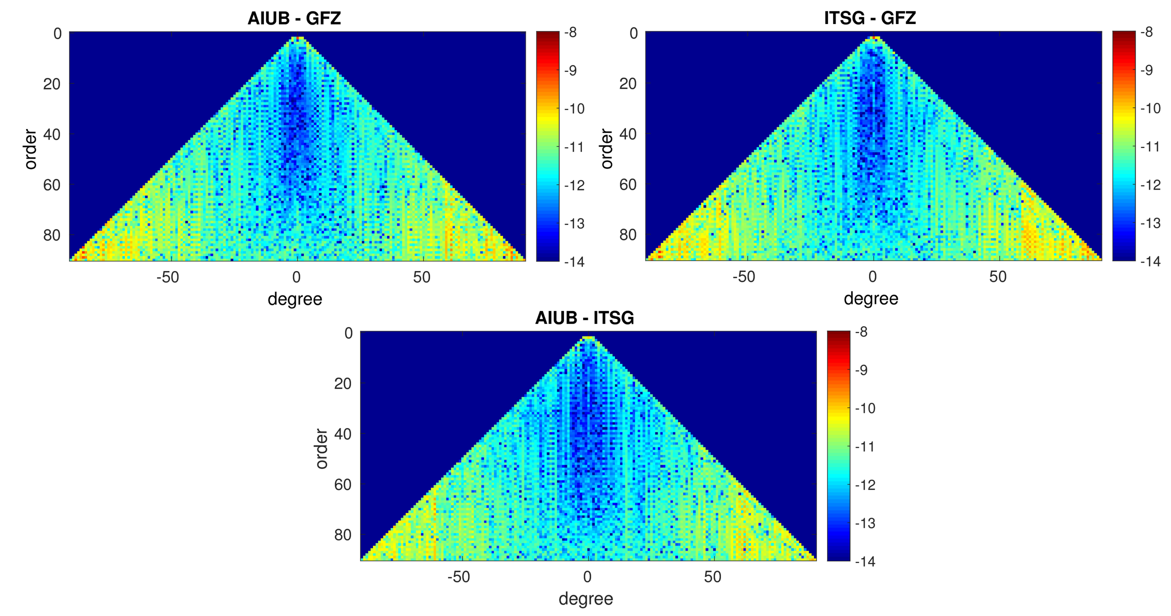


*Fig. 7: Coefficient wise differences between the three individual gravity field solutions.*

## Formal errors

The consistency of orbit models and observations is reflected by the formal errors of the estimated parameters. If systematic errors of the observations and background models are not taken into account properly, the formal errors tend to be optimistic. They may be scaled to realistic values by a noise model (e.g., by empirical co-variances) or calibrated a posteriori with external data. The formal errors of the three contributing solutions differ substantially, posing a problem for the combination because the variance factors of the NEQs are based on their individual a posteriori RMS.

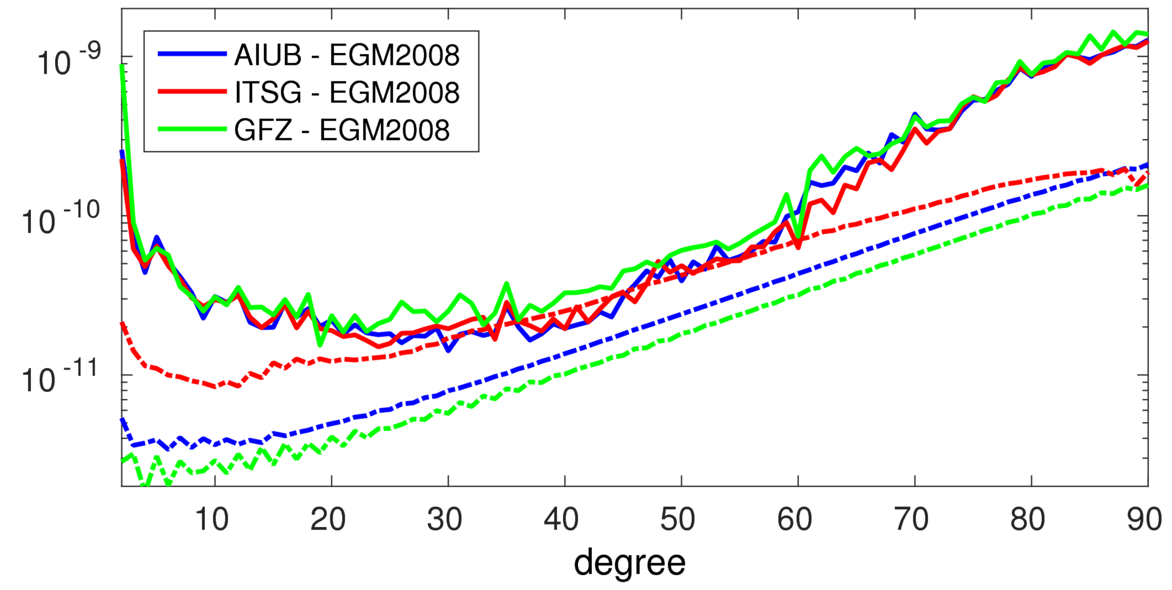


*Fig. 8: Difference degree amplitudes of the individual contributions with respect to EGM2008 and degree variances of their formal errors.*

## Weights based on a posteriori RMS

Degree Of Freedom: DOF = $n_{obs} - n_{par}$
($n_{obs}$ has to be reduced by the number of pre-eliminated parameters)

$v^TPv = l^TPl - dx^Tb$ with v = residuals, P = weights, l = observations, dx = ESTIMATE - APRIORI, b = NORMAL_EQUATION_VECTOR

$RMS^2 = v^TPv / DOF$

$w = S_0^2 / RMS^2 = 1 / RMS^2$

$n_{obs}$, $n_{par}$ and $l^TPl$ are given in the STATISTICS block. The derived weights strongly depend on the observation type, reflected in $n_{obs}$, and in the noise model, reflected in P. In the final combination they result in an a posteriori RMS = 1.

| $S_0 = 1$ | DOF | $l^TPl$ | $v^TPv$ | RMS | $S_0^2/RMS^2$ |
|---|---|---|---|---|---|
| AIUB | 1008486 | 178615 | 161893 | 0.40 | 6.25 |
| GFZ | 2683525 | 2599539 | 2065152 | 0.88 | 1.30 |
| ITSG | 532204 | 517610 | 495045 | 0.96 | 1.08 |

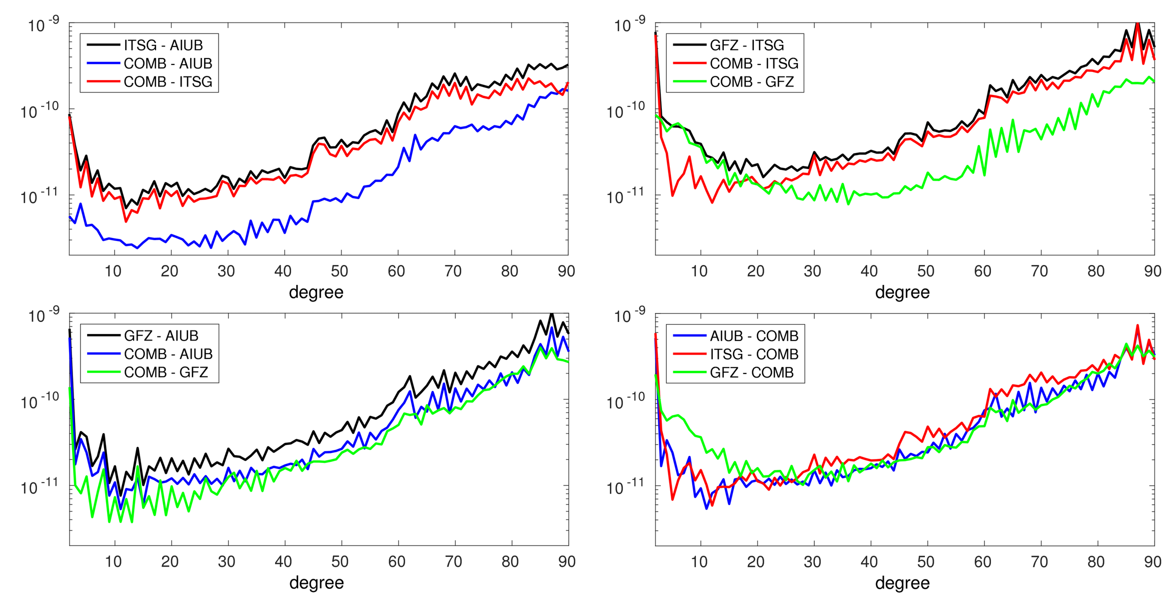


*Fig. 9: Combination based on a posteriori RMS leads to a down-weighting of the ITSG contribution (red) due to its empirical noise model and an up-weighting of the GFZ contribution (green) due to the higher number of GPS phase observations.*

## Weights based on comparison of solutions

Weights may be derived on solution level by comparison of the spherical harmonic coefficients $K^k_{lm}$ of the $n_{sol}$ = 3 individual solutions (AIUB, GFZ, ITSG) to their arithmetic mean:

$w_k = 1 / \sigma_k^2$; $\sigma_k^2 = \Sigma_{l,m}(K^k_{lm} - \overline{K}_{lm})^2 / (n_{coef} - 1)$; $\overline{K}_{lm} = \Sigma_k(K^k_{lm}) / n_{sol}$; $k = 1, \ldots, n_{sol}$

The resulting normalized weights for the example NEQs are

$w_{AIUB} = 0.44$; $w_{GFZ} = 0.18$; $w_{ITSG} = 0.38$

These weights do not account for the statistical properties of the individual NEQs and can only be applied additionally to empirical weights that lead to a homogeneous contribution of the individual NEQs to a combined solution.

## Conclusion

Technically the combination of the AC-specific NEQs works well, but the relative weights of the normalized NEQs determined by their a posteriori RMS lead to very inhomogeneous contributions of the individual NEQs to the combined solution. This is caused by the use of different observation types and sampling rates, leading to different numbers of observations, and by the diverse noise models, leading to very different formal errors of the individual solutions. The derivation of empirical weights seems to be indispensable to achieve a homogeneous contribution of all NEQs, because EGSIEM does not aim at the unification of the processing strategies. Weights derived by comparison of the individual solutions to their arithmetic mean may finally be multiplied to the empirical weights.

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