GNSS Satellite Orbit Modelling
Theory and Practice

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NGK Summer School
Introduction and Motivation

Overview on the GNSS Constellations

Effects Acting on Satellites and Related Models

Precise Orbit Determination for GNSS Satellites

GNSS Orbit Determination within the IGS
Errors in baseline components due to orbit errors following Bauršima, 1983:

$$\Delta X_l (m) \approx \frac{l}{d} \cdot \Delta X_{ORB} (m) \approx \frac{l (km)}{25'000 (km)} \cdot \Delta X_{ORB} (m)$$
Errors in baseline components due to orbit errors following Bauršima, 1983:

\[ \Delta X_l (m) \approx \frac{l}{d} \cdot \Delta X_{ORB} (m) \approx \frac{l (km)}{25'000 (km)} \cdot \Delta X_{ORB} (m) \]

<table>
<thead>
<tr>
<th>Orbit Error ( \Delta X_{ORB} )</th>
<th>Baseline Length ( l )</th>
<th>Baseline Error ( \frac{\delta X_{ORB}}{25'000 \text{ km}} )</th>
<th>Baseline Error ( \Delta X_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 m</td>
<td>1 km</td>
<td>0.1 ppm</td>
<td>1 mm</td>
</tr>
<tr>
<td>2.5 m</td>
<td>10 km</td>
<td>0.1 ppm</td>
<td>10 mm</td>
</tr>
<tr>
<td>2.5 m</td>
<td>100 km</td>
<td>0.1 ppm</td>
<td>100 mm</td>
</tr>
<tr>
<td>2.5 m</td>
<td>1000 km</td>
<td>0.1 ppm</td>
<td></td>
</tr>
<tr>
<td>0.05 m</td>
<td>1 km</td>
<td>0.002 ppm</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>0.05 m</td>
<td>10 km</td>
<td>0.002 ppm</td>
<td></td>
</tr>
<tr>
<td>0.05 m</td>
<td>100 km</td>
<td>0.002 ppm</td>
<td>2 mm</td>
</tr>
<tr>
<td>0.05 m</td>
<td>1000 km</td>
<td>0.002 ppm</td>
<td></td>
</tr>
</tbody>
</table>
Errors in baseline components due to orbit errors

Repeatability (north, east, up) when processing 90 days of GPS observations at Graz (Austria) and Onsala (Sweden) (1200 km baseline) with broadcast orbits (left) and with IGS orbits (right).
USA: GPS
Global Positioning System
USA: GPS
Global Positioning System

Russia: ГЛОНАСС
Глобальная навигационная спутниковая система
**GNSS: Global Navigation Satellite Systems**

**USA:** GPS  
Global Positioning System

**Russia:** GLONASS  
Global Satellite Navigation System
GNSS: Global Navigation Satellite Systems

- **USA**: GPS
  - Global Positioning System

- **Russia**: GLONASS
  - Global Satellite Navigation System

- **Europe**: Galileo
GNSS: Global Navigation Satellite Systems

**USA**: GPS
Global Positioning System

**Russia**: GLONASS
Global Satellite Navigation System

**Europe**: Galileo

**P.R. of China**: BeiDou
GPS Constellation

NAVSTAR GPS Block IIF Satellites

Approximate dimensions:
bus: $2 \times 2 \times 2.5\,\text{m}$
solar panels: $3 \times 2.5 \times 2\,\text{m}$
mass at launch: $\approx 1.6\,\text{t}$

Pictures from the manufacturer Boeing and www.gps.gov.
Fact sheet

Orbital elements for GPS satellites

- $a$: 26 560 km
- $e$: 0 (circular orbit)
- $i$: 55°

Distribution of orbital planes

- Number 6 separated by $\Omega_i = \Omega_0 + n \cdot 60°$
- Satellites 4 unequally distributed
- $= 24$ nominal constellation (today 32 active)
G06 for 10 days (from 09-May-2012 to 18-May-2012)
All GPS-satellites for 10 days (from 09-May-2012 to 18-May-2012)
• Revolution period $11^h\,58^m$
  (same constellation after 2 revolutions within 1 sidereal day)
- Revolution period $11^\text{h} 58^\text{m}$ (same constellation after 2 revolutions within 1 sidereal day)
- Repetition rates:
  - same geometry: 1 sidereal day
  - same constellation: 1 sidereal day

Elevation–Azimuth–Diagram for Zimmerwald
GPS Constellation

- Revolution period $11^h 58^m$ (same constellation after 2 revolutions within 1 sidereal day)
- Repetition rates:
  - same geometry: 1 sidereal day
  - same constellation: 1 sidereal day
- Signals:
  - Code: C1, P1, P2,
  - Phase: L1, L2
• Revolution period $11^h 58^m$ (same constellation after 2 revolutions within 1 sidereal day)

• Repetition rates:
  same geometry: 1 sidereal day
  same constellation: 1 sidereal day

• Signals:
  Code: C1, C2 (since IIR–M), P1, P2,
  Phase: L1, L2 (L2C!!)
• Revolution period $11^h 58^m$ (same constellation after 2 revolutions within 1 sidereal day)
• Repetition rates:
  same geometry: 1 sidereal day
  same constellation: 1 sidereal day
• Signals:
  Code: C1, C2 (since IIR–M), P1, P2, C5 (since IIF)
  Phase: L1, L2 (L2C!!), L5
GLONASS Constellation

GLONASS-M Satellites

Approximate dimensions:
bus: cylinder $2.4 \times 3.7$ m
solar panels: width of $7.2$ m
mass at launch: $\approx 1.5$ t

GLONASS Constellation

Fact sheet

Orbital elements for GLONASS satellites

\begin{align*}
  a & : 25\,500 \text{ km} \\
  e & : 0 \quad \text{(circular orbit)} \\
  i & : 65^\circ
\end{align*}

Distribution of orbital planes

Number \quad 3 \quad \text{separated by } \Omega_i = \Omega_0 + n \cdot 120^\circ \\
Satellites \quad 8 \quad \text{equally distributed} \\
= 24 \text{ nominal constellation}
GLONASS Constellation

R04 for 1 day (09-May-2012)
GLONASS Constellation

R04 for 10 days (from 09-May-2012 to 18-May-2012)
GLONASS Constellation

R04 for 2 days (from 09-May-2012 to 10-May-2012)
GLONASS Constellation

R04 and R05 for 2 days (from 09-May-2012 to 10-May-2012)
GLONASS Constellation

R01 to R08 for 10 days (from 09-May-2012 to 18-May-2012)
All GLONASS satellites for 10 days (from 09-May-2012 to 18-May-2012)
• Revolution period $11^h 16^m$ (same constellation after 17 revolutions within 8 sidereal days)

Elevation–Azimuth–Diagram for Zimmerwald
GLONASS Constellation

- Revolution period $11^h\,16^m$ (same constellation after 17 revolutions within 8 sidereal days)
- Repetition rates:
  - same geometry:
    - same plane: 1 sidereal day
    - next plane: $\frac{1}{3}$ sidereal day
  - same constellation: 8 sidereal days

Elevation–Azimuth–Diagram for Zimmerwald
• Revolution period $11^h\ 16^m$ (same constellation after 17 revolutions within 8 sidereal days)

• Repetition rates:
  - same geometry:
    - same plane: 1 sidereal day
    - next plane: $\frac{1}{3}$ sidereal day
  - same constellation: 8 sidereal days

• Signals:
  Code: C1, C2, P1, P2
  Phase: L1, L2
Galileo Constellation

Galileo FOC Satellites

Approximate dimensions:
- bus: $2.5 \times 1.1 \times 1.2$ m; solar panels: width tip-to-tip 14.5 m
- Mass at launch: $\approx 733$ kg

Fact sheet

Orbital elements for Galileo satellites

- \( a \): 30 000 km
- \( e \): 0 (circular orbit)
- \( i \): 56°

Distribution of orbital planes

- Number: 3 separated by \( \Omega_i = \Omega_0 + n \cdot 120° \)
- Satellites: 9 equally distributed
  \[ = 27 \text{ nominal constellation} \]
Galileo Constellation

- Revolution period $13^h\ 45^m$ (same constellation after 17 revolutions within 10 sidereal days)
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• Repetition rates:
  same geometry/constellation: 10 sidereal days
• Revolution period 13\textsuperscript{h} 45\textsuperscript{m} (same constellation after 17 revolutions within 10 sidereal days)

• Repetition rates:
same geometry/constellation: 10 sidereal days
Fictive E04 for one day
Galileo Constellation

Fictive E04 for 10 days
Fictive E04 for two days
Fictive E04 and E05 for two days
Fictive E01 to E09 for two days
Galileo Constellation

Fictive E01 to E09 for 10 days
Fictive Galileo constellation for 10 days
BeiDou Constellation

Fact sheet (MEO)

Orbital elements for BeiDou satellites

\[ a: \ 28\ 000\ \text{km} \]
\[ e: \ 0 \quad \text{(circular orbit)} \]
\[ i: \ 55^\circ \]

Distribution of orbital planes

Number  3 \quad \text{separated by } \Omega_i = \Omega_0 + n \cdot 120^\circ
Satellites  9 \quad \text{equally distributed}

= 27 nominal constellation
BeiDou Constellation

Fact sheet (MEO)

Orbital elements for BeiDou satellites

- **a**: 28 000 km
- **e**: 0 (circular orbit)
- **i**: 55°

Distribution of orbital planes

- Number 3 separated by $\Omega_i = \Omega_0 + n \cdot 120°$
- Satellites 9 equally distributed
  
  $= 27$ nominal constellation

Repetition rates

- Revolution period 12 h 57 min
- Constellation after 17 revolutions within 7 sidereal days
BeiDou Constellation

Fictive C04 for one day
BeiDou Constellation

Fictive C04 for 10 days
Fictive C01 to C09 for 10 days
Fictitive BeiDou constellation for 10 days
Beidou Constellation

Fictitious Beidou constellation for 10 days
BeiDou Constellation

Fact sheet (part 2)

Orbital elements for BeiDou satellites

- $a$: 42 000 km
- $e$: 0 (circular orbit)
- $i$: 55° (IGSO) 0° (GEO)

Distribution of orbital planes for IGSO–satellites

- Number 3 separated by $\Omega_i = \Omega_0 + n \cdot 120°$
- Satellites 1 distributed in a way that all satellites follow the same ground track

= 3 nominal constellation
BeiDou Constellation

Fact sheet (part 2)

Orbital elements for BeiDou satellites

- \(a\): 42 000 km
- \(e\): 0 (circular orbit)
- \(i\): 55° (IGSO) 0° (GEO)

Distribution of orbital planes for IGSO–satellites

- Number: 3
- Satellites: 1
  - separated by \(\Omega_i = \Omega_0 + n \cdot 120°\)
  - distributed in a way that all satellites follow the same ground track
  - = 3 nominal constellation

Repetition rates

- Revolution period: 23 h 56 min
- Constellation: after one revolutions within 1 sidereal day
Fact sheet

Orbital elements for QZSS satellites

- \( a \): 42 000 km
- \( e \): 0.075
- \( \omega \): 270°
- \( i \): 43°

Distribution of orbital planes

- Number 3 separated by \( \Omega_i = \Omega_0 + n \cdot 120° \)
- Satellites 1 distributed in a way that all satellites follow the same ground track

\( = 3 \) nominal constellation
QZSS Constellation

Fact sheet

Orbital elements for QZSS satellites

\[
\begin{align*}
    a & : 42\,000 \text{ km} \\
    e & : 0.075 \\
    \omega & : 270^\circ \\
    i & : 43^\circ
\end{align*}
\]

Distribution of orbital planes

Number 3 separated by \( \Omega_i = \Omega_0 + n \cdot 120^\circ \)
Satellites 1 distributed in a way that all satellites follow the same ground track

\[= 3 \text{ nominal constellation}\]

Repetition rates

Revolution period 23 h 56 min
Constellation after one revolutions within 1 sidereal day
Fictive QZSS constellation for 10 days
QZSS Constellation

QZSS Satellites

Approximate dimensions:
- **bus**: $3 \times 3 \times 6 \text{ m}$
- **solar panels**: $2.9 \times 3.1 \times 6.2 \text{ m}$
  - width tip-to-tip $25 \text{ m}$

Mass at launch: $\approx 4 \text{ t}$

Pictures from JAXA.
GNSS Constellation Summary

Global Navigation Systems

GPS
GLONASS
Galileo
BeiDou

Regional and Augmentation Systems

QZSS
NAVIC
SBAS
Effects Acting on Satellites and Related Models

Introduction and Motivation

Overview on the GNSS Constellations

Effects Acting on Satellites and Related Models
- Gravitational Forces
- Radiation Pressure Effects
- Emission Effects

Precise Orbit Determination for GNSS Satellites

GNSS Orbit Determination within the IGS
Gravitational Forces
Gravitational Forces
Gravitational Forces
Gravitational Forces
Gravitational Forces
Gravitational Forces

[Diagram of gravitational forces around a sphere]
Gravitational Forces
Gravitational Forces
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Gravitational Forces
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Acceleration due to centrifugal force:

\[ \ddot{\mathbf{r}} = \frac{\mathbf{r} \cdot \mathbf{r}}{|\mathbf{r}|^3} \]  

(1)

Acceleration due to gravitational force:

\[ \ddot{\mathbf{r}} = -GM_E \cdot \frac{\mathbf{r}}{|\mathbf{r}|^3} \]  

(2)

\( GM_E \) product of the constant of gravity and the mass of the Earth

\( \mathbf{r} \) geocentric vector to the satellite

\( \dot{\mathbf{r}} \) the related first time derivative (velocity vector)

\( \ddot{\mathbf{r}} \) the related second time derivative (acceleration vector)
Gravitational Forces

Velocities of selected GNSS satellites:

- Starting with the radius of the satellite orbit, the gravitational acceleration can be computed according to equation (2), with $GM_E = 398.6004415 \cdot 10^{12} \text{m}^3\text{s}^{-2}$.
- To compensate the gravitational acceleration a velocity of the satellite according to equation (1) is needed.

| Satellite          | $|\vec{r}|$ in km | $|\vec{v}|$ in $\text{m/s}^2$ | $|\vec{a}|$ in $\text{km/s}$ |
|--------------------|------------------|-----------------------------|-----------------------------|
| GLONASS            | 25 500           | 0.613                       | 3.95                        |
| GPS                | 26 560           | 0.565                       | 3.87                        |
| Galileo            | 30 000           | 0.443                       | 3.65                        |
| BeiDou, IGSO       | 42 000           | 0.226                       | 3.08                        |
Keplerian Orbit

The Equation of Motion:

$$\ddot{\mathbf{r}} = -G M_E \frac{\mathbf{r}}{|\mathbf{r}|^3}$$  \hspace{2cm} (3)

- describes the motion of a satellite around a spherically symmetric Earth.
Keplerian Orbit

The Equation of Motion:

\[ \ddot{\mathbf{r}} = -G M_E \frac{\mathbf{r}}{|\mathbf{r}|^3} \]  

(3)

- describes the motion of a satellite around a spherically symmetric Earth.
- is a differential equation with a solution describing either an ellipse, a parabola, or a hyperbola.
Keplerian Orbit

The Equation of Motion:

\[ \ddot{\vec{r}} = -GM_E \frac{\vec{r}}{|\vec{r}|^3} \]  

(3)

- describes the motion of a satellite around a spherically symmetric Earth.
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Keplerian Orbit

The Equation of Motion:

\[ \ddot{\mathbf{r}} = -GM_E \frac{\mathbf{r}}{|\mathbf{r}|^3} \]  

(3)

- describes the motion of a satellite around a spherically symmetric Earth.
- is a differential equation with a solution describing either an ellipse, a parabola, or a hyperbola.

It describes the trajectory of the satellite along a so called Keplerian orbit ellipse.
Quasi-Inertial Coordinate System

- Origin is located in the center of mass of the Earth.
• Z-axis corresponds to the mean rotation axis of the Earth.
• X-axis points to the vernal equinox (intersection with the ecliptic).
The coordinate system does not follow the rotation of the Earth but follows the motion of the Earth around the Sun.
Quasi-Inertial Coordinate System

- The coordinate system does not follow the rotation of the Earth but follows the motion of the Earth around the Sun.
Keplerian Orbit Ellipse
Keplerian Orbit Ellipse

$x$

$y$

$z$
Keplerian Orbit Ellipse
Keplerian Orbit Ellipse

Perigee

$\Omega$

$\omega$

$i$

$y$

$x$

$z$
Keplerian Orbit Ellipse

- $x$, $y$, $z$
- $\Omega$
- $i$
- Perigee

Satellite position:
- $v_0$
- $\omega$
- $\omega_0$
Keplerian Orbit Ellipse

Description of the orbit ellipse
- $a$ semimajor axis
- $e$ numerical eccentricity

Location of the orbit ellipse
- $i$ inclination of the orbital plane
- $\Omega$ right ascension of the ascending node
- $\omega$ argument of perigee

Location of the satellite within the orbit ellipse
- $u_0(t_0)$ argument of latitude of the satellite at $t_0$
- $v_0(t_0)$ true anomaly at epoch $t_0$
with $u_0(t_0) = \omega + v_0(t_0)$
Gravitational effect of other celestial bodies:
Advancing the Keplerian Orbit Theory

Gravitational effect of other celestial bodies:

\[ \vec{r}_{\text{sat}} \quad \vec{r}_{\text{Moon}} \]
Gravitational effect of other celestial bodies:
Advancing the Keplerian Orbit Theory

Gravitational effect of other celestial bodies:

\[ \vec{r}_{sat} \quad \vec{r}_i \quad \vec{a}_i \]
Advancing the Keplerian Orbit Theory

Gravitational effect of other celestial bodies:

\[
\ddot{\vec{r}}_{sat} = -GM_E \frac{\vec{r}_{sat}}{|\vec{r}_{sat}|^3} - G \sum_{i=1}^{n} M_i \frac{\vec{r}_i - \vec{r}_{sat}}{|\vec{r}_i - \vec{r}_{sat}|^3}
\]
Advancing the Keplerian Orbit Theory

Gravitational effect of other celestial bodies:

\[ \ddot{\mathbf{r}}_{\text{sat}} = -GM_E \frac{\mathbf{r}_{\text{sat}}}{|\mathbf{r}_{\text{sat}}|^3} - G \sum_{i=1}^{n} M_i \frac{\mathbf{r}_i - \mathbf{r}_{\text{sat}}}{|\mathbf{r}_i - \mathbf{r}_{\text{sat}}|^3} \]

Keplerian motion

relevant celestial bodies

• Further gravitational effects act on the satellite as well.

The resulting satellite motion is described by a perturbed Keplerian motion.
Advancing the Keplerian Orbit Theory

Gravitational effect of other celestial bodies:

\[
\ddot{\vec{r}}_{\text{sat}} = -G M_E \frac{\vec{r}_{\text{sat}}}{|\vec{r}_{\text{sat}}|^3} - G \sum_{i=1}^{n} M_i \frac{\vec{r}_i - \vec{r}_{\text{sat}}}{|\vec{r}_i - \vec{r}_{\text{sat}}|^3}
\]

- Further gravitational effects act on the satellite as well.

The resulting satellite motion is described by a perturbed Keplerian motion.

- The elements of the orbit ellipse change continuously due to the perturbing forces – “osculating elements”.

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The Earth is not Spherically Symmetric

The dominate structure is oblateness of the Earth
The Earth is not Spherically Symmetric

In the next order it has a shape of a pear
The Earth is not Spherically Symmetric

There are also relevant longitude-dependent structures
The Earth is not Spherically Symmetric

Many more details of the gravity field are well known today...
The Earth is not Spherically Symmetric

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Considering the gravity field for GNSS orbit determination

Many more details of the gravity field are well known today...
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

\[
\ddot{\mathbf{r}}_{\text{sat}} = -GM_E \int_{V_E} q'(\mathbf{r}_P) \frac{\mathbf{r}_{\text{sat}} - \mathbf{r}_P}{|\mathbf{r}_{\text{sat}} - \mathbf{r}_P|^3} dV_E - G \sum_{i=1}^{n} M_i \frac{\mathbf{r}_i - \mathbf{r}_{\text{sat}}}{|\mathbf{r}_i - \mathbf{r}_{\text{sat}}|^3}
\]
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

\[ \ddot{\vec{r}}_{\text{sat}} = -G M_E \int_{V_E} \varrho'(\vec{r}_P) \frac{\vec{r}_{\text{sat}} - \vec{r}_P}{|\vec{r}_{\text{sat}} - \vec{r}_P|^3} dV_E - G \sum_{i=1}^{n} M_i \frac{\vec{r}_i - \vec{r}_{\text{sat}}}{|\vec{r}_i - \vec{r}_{\text{sat}}|^3} \]

- The first term represents the gravitational attraction by the Earth, where \( \varrho'(\vec{r}_P) \) is the density at \( \vec{r}_P \) in the Earth’s interior.
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

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\]

- The first term represents the gravitational attraction by the Earth, where \( \varrho'(\vec{r}_P) \) is the density at \( \vec{r}_P \) in the Earth’s interior.

- The density function of the Earth is given in an Earth-fixed system.

\[-GM_E \int_{V_E} \varrho'(\vec{r}_P) \ldots dV_E \quad \Rightarrow \quad -GM_E \mathbf{T} \int_{V_E} \varrho(\vec{r}_P) \ldots dV_E\]

where \( \mathbf{T} \) is the transformation matrix from the Earth-fixed into the quasi-inertial frame.
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

- The related gravity field of the Earth is considered as a conservative vector field

\[ GM_E \nabla V(\vec{r}) = GM_E \left( \nabla \int_{V_E} \frac{\rho(\vec{r}_P)}{|\vec{r}_{sat} - \vec{r}_P|} dV_E \right) \]
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

- The related gravity field of the Earth is considered as a conservative vector field that gradients may be represented by a spherical harmonic expansion of the potential:

\[
GM_E \nabla V(\vec{r}) = GM_E \left( \nabla \int_{V_E} \frac{\varrho(\vec{r}_P)}{|\vec{r}_{sat} - \vec{r}_P|} dV_E \right)
\]

\[
= \frac{GM}{|\vec{r}_{sat}|} \sum_{i=0}^{\infty} \left( \frac{a_e}{|\vec{r}_{sat}|} \right)^i \cdot \sum_{k=0}^{i} P_i^k(\sin \phi)\left\{ C_{ik} \cos k\lambda + S_{ik} \sin k\lambda \right\}
\]

with
- \( \phi, \lambda \) the spherical latitude and longitude of the satellite,
- \( P_i^k(\sin \phi) \) the associated Legendre functions of degree \( i \) and order \( k \),
- \( C_{ik}, S_{ik} \) the coefficients of the expansion of the potential into spherical harmonic functions.
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

- The related gravity field of the Earth is considered as a conservative vector field that gradients may be represented by a spherical harmonic expansion of the potential:

\[
G M_E \nabla V(\vec{r}) = G M_E \left( \nabla \int_{V_E} \frac{\varrho(\vec{r}_P)}{|\vec{r}_{sat} - \vec{r}_P|} dV_E \right) \\
= \frac{G M}{|\vec{r}_{sat}|} \sum_{i=0}^{\infty} \left( \frac{a_e}{|\vec{r}_{sat}|} \right)^i \sum_{k=0}^{i} P_i^k (\sin \phi) \{ C_{ik} \cos k \lambda + S_{ik} \sin k \lambda \}
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GM_E \nabla V(\vec{r}) = GM_E \left( \nabla \int_{V_E} \frac{\varrho(\vec{r}_P)}{\left| \vec{r}_{sat} - \vec{r}_P \right|} dV_E \right)
\]

\[
= \frac{GM}{|\vec{r}_{sat}|} \sum_{i=0}^{\infty} \left( \frac{ae}{|\vec{r}_{sat}|} \right)^i \cdot \sum_{k=0}^{i} P_i^k(\sin \phi) \{C_{ik} \cos k\lambda + S_{ik} \sin k\lambda\}
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with

- \(\phi, \lambda\) the spherical latitude and longitude of the satellite,
- \(P_i^k(\sin \phi)\) the associated Legendre functions of degree \(i\) and order \(k\),
- \(C_{ik}, S_{ik}\) the coefficients of the expansion of the potential into spherical harmonic functions.
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

\[ k = 0 \quad l = 0 \]

\[ k = 1 \quad l = 0, 1 \]

\[ k = 2 \quad l = 0, \ldots, 2 \]

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The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

\[ k = 0 \quad l = 0 \]

The first term \( C_{00} \) is a constant.

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The terms \( C_{10}, C_{11}, \) and \( S_{11} \) are related to the center of mass of the Earth.
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

- $k = 0 \quad l = 0$
  - The $C_{20}$ term represents the flattening of the Earth.
- $k = 1 \quad l = 0, 1$
- $k = 2 \quad l = 0, \ldots, 2$
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The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

The terms with $k = 0$ are called zonal terms (latitude depending terms).
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

The terms with \( k = i \) are called sectorial terms (longitude depending terms).
The Earth is not Spherically Symmetric

Considering the mass distribution of the Earth:

The other components are named tesseral terms.
The Earth is not Spherically Symmetric

To which extent the gravity field is relevant for orbit determination of GNSS satellites?
The Earth is not Spherically Symmetric

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3D-RMS of the orbit differences w.r.t. an orbit based on a gravity field expanded up to degree and order 20.
The Earth is not Spherically Symmetric

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3D-RMS of the orbit differences w.r.t. an orbit based on a gravity field expanded up to degree and order 20.

- for MEO satellites the gravity field needs to be considered up to degree and order 7,
- whereas for satellites in the higher IGSO or GEO a expansion up to degree and order 5 is sufficient.
The Earth is not Spherically Symmetric

Resolution of the Earth gravity field relevant for modelling the orbits of GNSS satellites in MEO orbits.
Resolution of the Earth gravity field relevant for modelling the orbits of GNSS satellites in IGSO/GEO orbits.
Gravitational Forces

Summary:

- All relevant masses acting on a GNSS satellite must be known (including their location).
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- Regarding the body Earth even a more detailed distribution of the masses need to be considered.
Gravitational Forces

The most relevant gravitational effects for GNSS orbit modelling:

- **Oblateness of the Earth**
  - GPS: $\approx 40$ km
  - Galileo: $\approx 27$ km
  - QZSS: $\approx 15$ km

Maximal influence of the effect on the orbit after one day of orbit integration.
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  - GPS: $\approx 500$ m
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- **Gravitational effect due to ocean tides**
  
  GPS: \(< 1 \text{ cm} \)  
  Galileo: \(< 5 \text{ mm} \)  
  QZSS: \( \approx 1 \text{ mm} \)

Maximal influence of the effect on the orbit after one day of orbit integration.
Effects Acting on Satellites and Related Models

- Gravitational Forces
- Radiation Pressure Effects
- Emission Effects
According to quantum mechanics, each photon of frequency $\nu$ and wavelength $\lambda = \frac{c}{\nu}$ carries the energy

$$E = h \cdot \nu$$
Radiation Pressure

According to quantum mechanics, each photon of frequency $\nu$ and wavelength $\lambda = \frac{c}{\nu}$ carries the energy

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and linear momentum

$$\vec{p} = \frac{h \cdot \nu}{c},$$

where

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An interaction of radiation with a surface causes an exchange of momentum and therefore a force.
For the orbit modelling we need the resulting acceleration:

\[
\vec{a}_{SRP} = \vec{C} \cdot \frac{(1 \text{ AU})^2}{|\vec{r}_{sat} - \vec{r}_{Sun}|^2} \cdot \frac{\Phi}{c} \cdot \frac{A_{sat}}{m_{sat}}
\]

where

- \( \vec{C} \) is the vectorial radiation pressure coefficient (on the optical properties of the surface),
- \( A_{sat} \) is the area of the surface,
- \( m_{sat} \) is the mass of the satellite,
- \( \Phi \approx 1367 \frac{\text{W}}{\text{m}^2} \) is the solar flux (the energy passing through a unit area in a unit time) at the distance of 1 AU, and
- \( \frac{(1 \text{ AU})^2}{|\vec{r}_{sat} - \vec{r}_{Sun}|^2} \) accounts for changes in the solar flux due to the eccentricity of the Earth’s orbits around the Sun.
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The direction of the resulting acceleration depends on the kind of interaction of the radiation with the surface.
Specular reflection:
As the photon is specularly reflected from the surface,
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**Specular reflection:**
As the photon is specularly reflected from the surface, only a normal force is produced:

$$\vec{C}_s = -2 \cdot \cos^2 \alpha \cdot \vec{n}$$
Interaction of a Photon with a Surface

Diffuse reflection:
Interaction of a Photon with a Surface

Diffuse reflection:

normal vector to the surface

\[ \vec{n} \]

[\[ \alpha \] angle]
Interaction of a Photon with a Surface

Diffuse reflection:

- normal vector to the surface

\[ \vec{n} \]
Diffuse reflection:
This kind of reflection produces both normal and tangential forces:

$$\vec{C}_d = \left( \frac{\vec{r}_{sat} - \vec{r}_{Sun}}{|\vec{r}_{sat} - \vec{r}_{Sun}|} - \frac{2}{3} \vec{n} \right) \cdot \cos \alpha$$

(assuming diffuse reflection according to Lambert's cosine law)
Absorption:
The photon is fully absorbed by the surface.
Interaction of a Photon with a Surface

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\[ \vec{C}_a = \left( \frac{\vec{r}_{sat} - \vec{r}_{Sun}}{|\vec{r}_{sat} - \vec{r}_{Sun}|} \right) \cdot \cos \alpha \]
Interaction of a Photon with a Surface

In general, realistic satellite surfaces show a mixture of the three optical properties. If

- $p_s$ is the portion of specularly reflected photons,
- $p_d$ is the portion of diffusely reflected photons, and
- $(1 - p_d - p_s)$ is the portion of absorbed photons,

the resulting radiation coefficient is

$$\vec{C}_r = p_s \cdot \vec{C}_s + p_d \cdot \vec{C}_d + (1 - p_d - p_s) \cdot \vec{C}_a.$$
Thermal Re-radiation Effect

The heat generated by the absorption (or any other thermal emission of the satellite) produces an additional force as a Lambert diffuser:

\[ d\vec{F}_{\text{therm}} = -\frac{2}{3} \cdot \frac{\epsilon \sigma T_A^4}{c} \, dA \cdot \vec{e}_A \]

with

- \( \epsilon \) is the emissivity,
- \( \sigma \) the Stephan-Boltzmann constant,
- \( c \) the speed of light,
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Radiation Effects in the Orbit Determination

We need to know which amount of photons arrives at the satellite. According to the surface properties the resulting force can be derived.
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- a detailed decomposition of the satellite into the **geometrical elements**,

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With a ray tracing the resulting acceleration can be computed but this needs a big computational effort.
Semi-Analytical Modelling
To reduce the computational effort, the satellite is typically represented by a **box-wing model**.
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Semi-Analytical Modelling

Elevation of the Sun above the orbital plane

$\beta$
Semi-Analytical Modelling

\[ \beta \]  

Elevation of the Sun above the orbital plane

\[ \Delta u \]  

Difference \( u_{sat} - u_{Sun} \)
Semi-Analytical Modelling

- $\beta$: Elevation of the Sun above the orbital plane
- $\gamma$: Elongation angle
- $\Delta u$: Difference $u_{sat} - u_{Sun}$
Semi-Analytical Modelling

$\vec{Z}$ Direction satellite $\rightarrow$ Earth
(antenna pointing direction)

- $\beta$: Elevation of the Sun above the orbital plane
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Semi-Analytical Modelling

$\vec{Z}$  Direction satellite $\rightarrow$ Earth  
(antenna pointing direction)

$\vec{D}$  Direction satellite $\rightarrow$ Sun

$\beta$  Elevation of the Sun  
above the orbital plane

$\gamma$  Elongation angle

$\Delta u$  Difference $u_{sat} - u_{Sun}$
**Semi-Analytical Modelling**

- $\vec{Z}$: Direction satellite → Earth (antenna pointing direction)
- $\vec{D}$: Direction satellite → Sun
- $\vec{Y}$: Direction along the solar panel axis

**Parameters:**

- $\beta$: Elevation of the Sun above the orbital plane
- $\gamma$: Elongation angle
- $\Delta u$: Difference $u_{sat} - u_{Sun}$
Semi-Analytical Modelling

\[
\begin{align*}
\vec{Z} & \quad \text{Direction satellite } \to \text{ Earth} \\
& \quad \text{(antenna pointing direction)} \\
\vec{D} & \quad \text{Direction satellite } \to \text{ Sun} \\
\vec{Y} & \quad \text{Direction along the solar panel axis} \\
\vec{B} & \quad \text{Completes the system: } \vec{B} = \vec{D} \times \vec{Y}
\end{align*}
\]

- \( \beta \) Elevation of the Sun above the orbital plane
- \( \gamma \) Elongation angle
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Semi-Analytical Modelling

\[ \Delta u \]

\[ \vec{B} \]

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\[ \vec{Z} \]

\[ \vec{B} \]

\[ \vec{Y} \]
Semi-Analytical Modelling
Semi-Analytical Modelling

\[ \Delta u \]

\[ \beta \]

\[ \vec{B} \]

\[ \vec{Y} \]

\[ \vec{Z} \]

\[ \vec{D} \]
Semi-Analytical Modelling

\[ \Delta u \]

\[ \vec{B}, \vec{Z}, \vec{Y} \]

\[ \beta \]

\[ \text{Bullet} \]

\[ \text{Bullet} \]
Semi-Analytical Modelling
Semi-Analytical Modelling

Accelerations derived for GPS (Block IIA) satellites from a boxwing\(^1\) and Rock-S\(^2\) model

Computed for \(\beta = 10^\circ\) \(\beta = 45^\circ\) \(\beta = 78^\circ\)

\(^1\)as proposed by Carlos Rodriguez–Solano based on Fliegel et al. (1992)
\(^2\)Fliegel et al. (1992)
If the radiation pressure effects cannot fully be described by analytical models one has to adjust empirical solar radiation pressure parameters.
Semi-Analytical Modelling

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**Adjustable box-wing model:**
C. Rodriguez-Solano has proposed to directly adjust the effect acting on the solar panels and the body of the satellite in the parameter adjustment.
These parameters are highly correlated and need a sophisticated system of constraints to become solvable.
Semi-Analytical Modelling

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Adjustable box-wing model:
C. Rodriguez-Solano has proposed to directly adjust the effect acting on the solar panels and the body of the satellite in the parameter adjustment. These parameters are highly correlated and need a sophisticated system of constraints to become solvable.

Box-wing a priori model:
A (more or less detailed) radiation pressure model is introduced in the orbit modelling process. Empirical parameters are estimated during the parameter adjustment process as well.
Empirical Modelling

Accelerations derived for GPS (Block IIA) satellites from a boxwing\(^1\) and Rock-S\(^2\) model

**D component**

![Graph showing D component accelerations](image)

**B component**

![Graph showing B component accelerations](image)

Computed for \(\beta = 10^\circ\), \(\beta = 45^\circ\), \(\beta = 78^\circ\)

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Observing the satellite from the Sun
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Conclusions

- A Sun–fixed argument for the periodic terms is helpful to obtain interpretable series of these parameters:

\[ \Delta u = u_{sat} - u_{Sun} \]
Conclusions

- A Sun–fixed argument for the periodic terms is helpful to obtain interpretable series of these parameters:

\[ \Delta u = u_{\text{sat}} - u_{\text{Sun}} \]

- Solar radiation pressure for satellites flying according to the previously mentioned models can be represented by:

\[ D = D_0 + D_2 \cos(2\Delta u) + D_4 \cos(4\Delta u) + \ldots \]
\[ Y = Y_0 \]
\[ B = B_1 \cos(1\Delta u) + B_3 \cos(3\Delta u) + \ldots \]

\[ Y_0 \neq 0 \] if the satellite is flying “misaligned” with a \( Y \–\)bias (e.g., GPS, except for Block IIF).
**Component:** $D_0$

**GPS Block IIA**

**GPS Block IIR**

**GPS Block IIF**

**GLONASS–M**

Estimated Solar Radiation Pressure

Acceleration in nm/s$^2$
Estimated Solar Radiation Pressure

Component: \( Y_0 \) (small scale)

GPS Block IIA

GPS Block IIR

GPS Block IIF

GLONASS–M

Elevation of the Sun above the orbital plane

Acceleration in \( \text{nm/s}^2 \)
Estimated Solar Radiation Pressure

Component: $B_1 \cdot \cos(1\Delta u)$
Estimated Solar Radiation Pressure

Component: $D_2 \cdot \cos(2\Delta u)$

GPS Block IIA

GPS Block IIR

GPS Block IIF

GLONASS–M
**Component:** $B_1 \cdot \sin(1\Delta u)$

---

**GPS Block IIA**

![Graph showing acceleration vs. elevation of the Sun above the orbital plane for GPS Block IIA]

**GPS Block IIR**

![Graph showing acceleration vs. elevation of the Sun above the orbital plane for GPS Block IIR]

**GPS Block IIF**

![Graph showing acceleration vs. elevation of the Sun above the orbital plane for GPS Block IIF]

**GLONASS–M**

![Graph showing acceleration vs. elevation of the Sun above the orbital plane for GLONASS–M]
Estimated Solar Radiation Pressure

**Component:** \( D_2 \cdot \sin(2\Delta u) \)

---

**GPS Block IIA**

<table>
<thead>
<tr>
<th>Acceleration in nm/s**2</th>
<th>Elevation of the Sun above the orbital plane</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-90 -60 -30 0 30 60 90</td>
</tr>
<tr>
<td></td>
<td>-20 -10 0 10 20</td>
</tr>
</tbody>
</table>

**GPS Block IIR**

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</tr>
</thead>
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</tr>
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<td></td>
<td>-20 -10 0 10 20</td>
</tr>
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</table>

**GPS Block IIF**

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</tr>
</thead>
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<td>-90 -60 -30 0 30 60 90</td>
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<tr>
<td></td>
<td>-20 -10 0 10 20</td>
</tr>
</tbody>
</table>

**GLONASS–M**

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Conclusions

- The definition of the angular argument \( \Delta u = u_{\text{sat}} - u_{\text{Sun}} \) instead of \( u_{\text{sat}} \) allows a better interpretation of estimated parameter series, e.g., w.r.t. the elevation of the Sun above the orbital plane.
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- The definition of the angular argument \( \Delta u = u_{sat} - u_{Sun} \) instead of \( u_{sat} \) allows a better interpretation of estimated parameter series, e.g., w.r.t. the elevation of the Sun above the orbital plane.

- Adding twice-per-revolution terms in \( D \)-component improves the orbit solution, in particular for satellites with stretched bodies.
Estimated Solar Radiation Pressure

Conclusions

• The definition of the angular argument \( \Delta u = u_{sat} - u_{Sun} \) instead of \( u_{sat} \) allows a better interpretation of estimated parameter series, e.g., w.r.t. the elevation of the Sun above the orbital plane.

• Adding twice-per-revolution terms in \( D \)-component improves the orbit solution, in particular for satellites with stretched bodies.

• Even if the sin-terms are not necessary according to theory they are needed for representing real satellite trajectories.
The Empirical CODE Orbit Model

\[ D = D_0 + \sum_{i=1}^{n_D} D_{2i,c} \cos(2i \cdot \Delta u) \]
\[ + D_{2i,s} \sin(2i \cdot \Delta u) \]  \hspace{1cm} (4)

\[ Y = Y_0 \]

\[ B = B_0 + \sum_{i=1}^{n_B} B_{2i-1,c} \cos((2i - 1) \cdot \Delta u) \]
\[ + B_{2i-1,s} \sin((2i - 1) \cdot \Delta u) \]

- In practice the expansion is only used up to \( n_D = n_B = 1 \).
The empirical CODE Orbit Mode (ECOM) as shown in Equation 4 on slide 65 was developed in Arnold et al., 2015.

The extension considers in particular the effect on satellites with stretched bodies (e.g., GLONASS, Galileo, QZSS).
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It is an extension of the classical ECOM as introduced by Beutler et al., 1994. \((n_D = 0 \text{ and } n_B = 1)\)

The ECOM is widely used within the IGS.

In the semi-analytical approach the ECOM is also often in use to compensate for the deficiencies of the introduced a priori models.
Shadow Effects

- If the elevation of the Sun $\beta$ becomes smaller than a certain angle $\beta_0$, so called **eclipse phases** occur where the satellite is **not** illuminated by the Sun.

  During eclipse, the force caused by the solar radiation needs to be switched off in the orbit model during the eclipse phase.
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• The limit $\beta_0$ is computed by $\beta_0 = \arcsin \frac{a_{\text{Earth}}}{a_{\text{sat}}}$

  (with $a_{\text{Earth}} = 6380$ km):

  - GLONASS $a = 25500$ km $\beta_0 = 14.5^\circ$
  - GPS $a = 26560$ km $\beta_0 = 13.9^\circ$
  - Galileo $a = 30000$ km $\beta_0 = 12.3^\circ$
  - QZSS $a = 42000$ km $\beta_0 = 8.7^\circ$
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• The period where the satellite crosses the shadow of the Earth takes about one hour for a GNSS satellite in a MEO orbit.
Shadow Effects

- Umbra
- Penumbra

[Diagram showing shadow effects with umbra and penumbra areas]
A satellite flying into the shadow area behind the Earth crosses the penumbra in such a short interval that it can be neglected.
• A satellite flying into the shadow area behind the Earth crosses the penumbra in such a short interval that it can be neglected.
• The penumbra is on the other hand essential for the shadow generated by the Moon.
Other Radiation Pressure Effects

The biggest contribution comes from the

- **solar (or direct) radiation pressure**

  GPS: $\approx 250$ m  
  Galileo: $\approx 350$ m  
  QZSS: $\approx 700$ m

Maximal influence of the effect on the orbit after one day of orbit integration.
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Indirect radiation pressure effects due to solar radiation are

- **reflected/re-emitted by the Earth (Albedo effect)**
  - GPS: \( \approx 1 \text{ m} \)
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- reflected/re-emitted by the Moon
  currently neglected for GNSS satellites

Maximal influence of the effect on the orbit after one day of orbit integration.
Precise Orbit Determination for GNSS Satellites

Introduction and Motivation

Overview on the GNSS Constellations

Effects Acting on Satellites and Related Models

Precise Orbit Determination for GNSS Satellites
  Precise Orbit Determination in Theory
  Precise Orbit Determination in Practise
  Methods of GNSS-Orbit Validation

GNSS Orbit Determination within the IGS
In order to consider the gravitational and non-gravitational perturbations described before we have to extend the initial version of the equation of motion, see eqn. (3), by a function $f$:

$$\ddot{\vec{r}} = -GM_E \frac{\vec{r}}{|\vec{r}|^3} + f(t, \vec{r}, \dot{\vec{r}}, Q_1, \ldots, Q_n),$$

with initial conditions

$$\vec{r}(t_0) = \vec{r}(a, e, i, \Omega, \omega, u_0; t_0) \quad \text{and} \quad \dot{\vec{r}}(t_0) = \dot{\vec{r}}(a, e, i, \Omega, \omega, u_0; t_0),$$

as well as $Q_1, \ldots, Q_n$ shall represent all known and unknown parameters of the force model (e.g., for the Earth’s gravity field or the solar radiation pressure).
Osculating Elements

The perturbations described by the function $f$ cause a permanent change of the orbital elements, the so-called osculating elements:

GPS satellite G12.
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Galileo satellite E12.
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Galileo satellite E14.
Osculating Elements

The perturbations described by the function $f$ cause a permanent change of the orbital elements, the so call osculating elements:

QZSS satellite J01.
Osculating Elements

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![Graph showing the change in right ascension of the ascending node over days 090 to 099 in 2016 for QZSS satellite J01.](image)
The actual orbit $\vec{r}'(t)$ is expressed as a truncated Taylor series:

$$\vec{r}'(t) = \vec{r}_0(t) + \sum_{i=1}^{m} \frac{\partial \vec{r}_0}{\partial P_i}(t) \cdot (P_i - P_{0,i})$$

with

- $\vec{r}_0(t)$ the a priori orbit,
- $\frac{\partial \vec{r}_0}{\partial P_i}(t)$ the partial derivative of the a priori orbit $\vec{r}_0(t)$ w.r.t. parameter $P_i$,
- $P_{0,i}$ the a priori parameter values of the a priori orbit $\vec{r}_0(t)$, and
- $P_i$ the parameter values of the improved orbit $\vec{r}'(t)$. 
The **actual orbit** $\vec{r}(t)$ is expressed as a truncated Taylor series:

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- $P_i$ \text{ the parameter values of the improved orbit } $\vec{r}(t)$.
Principle of Orbit Determination

The actual orbit \( \vec{r}(t) \) is expressed as a truncated Taylor series:

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\vec{r}(t) = \vec{r}_0(t) + \sum_{i=1}^{m} \frac{\partial \vec{r}_0}{\partial P_i}(t) \cdot (P_i - P_{0,i})
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The actual orbit $\tilde{r}(t)$ is expressed as a truncated Taylor series:

$$
\tilde{r}(t) = \tilde{r}_0(t) + \sum_{i=1}^{m} \frac{\partial \tilde{r}_0}{\partial P_i}(t) \cdot (P_i - P_{0,i})
$$

with

- $\tilde{r}_0(t)$ the a priori orbit,
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- \( P_{0,i} \) the a priori parameter values of the a priori orbit \( \vec{r}_0(t) \), and
- \( P_i \) the parameter values of the improved orbit \( \vec{r}(t) \).

A **least-squares adjustment** of GNSS tracking data \( L_1, \ldots, n \) yields corrections to the a priori parameter values \( P_{0,i} \). Using the above equation, the improved (linearized) orbit \( \vec{r}(t) \) may be computed.
The partial derivative of the observation $L_j$ w.r.t. orbit parameter $P_i$ may be expressed as

$$\frac{\partial L_j}{\partial P_i}(t) = (\nabla(L_j))^T \cdot \frac{\partial \vec{r}_0}{\partial P_i}(t)$$

with the gradient given by

$$(\nabla(L_j))^T = \left( \frac{\partial L_j}{\partial r_{0,1}} \quad \frac{\partial L_j}{\partial r_{0,2}} \quad \frac{\partial L_j}{\partial r_{0,3}} \right)$$

if the observations only depend on the geocentric position vector and are referring to only one epoch.
Partial Derivatives

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Partial Derivatives

The partial derivative of the observation $L_j$ w.r.t. orbit parameter $P_i$ may be expressed as

$$\frac{\partial L_j}{\partial P_i}(t) = \left(\nabla (L_j)\right)^T \cdot \frac{\partial \vec{r}_0}{\partial P_i}(t)$$

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if the observations only depend on the geocentric position vector and are referring to only one epoch. The gradient only depends on the type of observations used, whereas the second term is independent of the observation type and is related to the variational equations. This separates the observation-specific (geometric) part from the dynamic part.
Variational Equations

For each orbit parameter $P_i$ the corresponding variational equation reads as

$$\ddot{\vec{r}}_{P_i} = A_0 \cdot \vec{r}_{P_i} + A_1 \cdot \dot{\vec{r}}_{P_i} + \frac{\partial f_i}{\partial P_i}$$

with the $3 \times 3$ matrices defined by

$$A_{0[i,k]} \equiv \frac{\partial f_i}{\partial r_{0,k}} \quad \text{and} \quad A_{1[i,k]} \equiv \frac{\partial f_i}{\partial \dot{r}_{0,k}}$$
Variational Equations

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$f_i$ \quad $i$-th component of the total acceleration function

$r_0$, $\dot{r}_0$ \quad positions and velocities from the a priori orbit

$r_{0,k}$ \quad $k$-th component of the geocentric position $\vec{r}_0$
Variational Equations

For each orbit parameter $P_i$ the corresponding variational equation reads as

$$\dddot{\vec{r}}_{P_i} = A_0 \cdot \ddot{\vec{r}}_{P_i} + A_1 \cdot \dot{\vec{r}}_{P_i} + \frac{\partial f_i}{\partial P_i}$$

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For each orbit parameter $P_i$ the variational equation is a linear differential equation system of second order in time. Their solutions are all needed for orbit determination.
The variational equation is a linear, homogeneous system with initial values

\[ \vec{r}_{P_i}(t_0) \neq 0 \quad \text{and} \quad \dot{\vec{r}}_{P_i}(t_0) \neq 0 \quad \text{for} \quad P_i \in a, e, i, \Omega, \omega, u_0 \]
Variational Equations

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\]

and a **linear, inhomogeneous system** with initial values

\[
\vec{r}_{P_i}(t_0) = 0 \quad \text{and} \quad \dot{\vec{r}}_{P_i}(t_0) = 0 \quad \text{for} \quad P_i \in Q_1, \ldots, Q_n
\]
Variational Equations

The variational equation is a **linear, homogeneous system** with initial values

\[ \vec{r}_{P_i}(t_0) \neq 0 \quad \text{and} \quad \vec{r}'_{P_i}(t_0) \neq 0 \quad \text{for} \quad P_i \in a, e, i, \Omega, \omega, u_0 \]

and a **linear, inhomogeneous system** with initial values

\[ \vec{r}_{P_i}(t_0) = 0 \quad \text{and} \quad \vec{r}'_{P_i}(t_0) = 0 \quad \text{for} \quad P_i \in Q_1, \ldots, Q_n \]

Let us assume that the functions \( \vec{r}_{O_j}(t), \ j = 1, \ldots, 6 \) are the partials w.r.t. the six parameters \( O_j, \ j = 1, \ldots, 6 \) defining the initial conditions at time \( t_0 \).
The variational equation is a **linear, homogeneous system** with initial values

\[ \vec{r}_{P_i}(t_0) \neq 0 \quad \text{and} \quad \dot{\vec{r}}_{P_i}(t_0) \neq 0 \quad \text{for} \quad P_i \in a, e, i, \Omega, \omega, \nu_0 \]

and a **linear, inhomogeneous system** with initial values

\[ \vec{r}_{P_i}(t_0) = 0 \quad \text{and} \quad \dot{\vec{r}}_{P_i}(t_0) = 0 \quad \text{for} \quad P_i \in Q_1, \ldots, Q_n \]

Let us assume that the functions \( \vec{r}_{O_j}(t), \ j = 1, \ldots, 6 \) are the partials w.r.t. the six parameters \( O_j, \ j = 1, \ldots, 6 \) defining the initial conditions at time \( t_0 \). The ensemble of these six functions forms one complete system of solutions of the homogeneous part of the variational equation, which allows to obtain the solution of the inhomogeneous system by the method of "variation of constants".
Variational Equations

The solution and its first time derivative may be written as

\[ z_{P_i}^{(k)}(t) = \sum_{j=1}^{6} \alpha_{O_j,P_i}(t) \cdot z_{O_j}^{(k)}(t); \quad k = 0, 1 \]

with the coefficient functions defined by

\[ \alpha_{P_i}(t) = \int_{t_0}^{t} Z^{-1}(t') \cdot h_{P_i}(t') dt' \]

- \( \alpha_{P_i} \) column array defined by \( (\alpha_{O_1,P_i}, \ldots, \alpha_{O_6,P_i})^T \)
- \( Z \) 6 \times 6 matrix defined by \( Z[1,\ldots,3;j] = z_{O_j}, \ Z[4,\ldots,6;j] = \dot{z}_{O_j} \)
- \( h_{P_i} \) column array defined by \( (O^T, \frac{\partial f^T}{\partial P_i})^T \)
Variational Equations

Note that the solutions $z_{P_i}(t)$ of the variational equation and its time derivative may be expressed with the same functions $\alpha_{O_j, P_i}$ as a linear combination with the homogeneous solutions $z_{O_j}(t)$ and $\dot{z}_{O_j}(t)$, respectively. Therefore, only the six initial value problems associated with the initial conditions have to be actually treated as differential equation systems. Their solutions have to be either obtained approximately, or by numerical integration techniques.

All variational equations related to dynamical orbit parameters may be reduced to definite integrals. They can be efficiently solved numerically, e.g., by a Gaussian quadrature technique.

It must be emphasized that each additional orbit parameter requires an additional numerical solution of a definite integral.
Numerical Integration

Collocation algorithms (one particular class of numerical integration techniques) are subsequently used to briefly illustrate the principles of numerical integration:
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The original interval is divided into \( N \) integration intervals.
Numerical Integration

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The original interval is divided into $N$ integration intervals. For each interval $I_k$ a further subdivision is performed according to the order $q$ of the adopted method.
Collocation algorithms (one particular class of numerical integration techniques) are subsequently used to briefly illustrate the principles of numerical integration:

The original interval is divided into $N$ integration intervals. For each interval $I_k$ a further subdivision is performed according to the order $q$ of the adopted method. At these points $t_{kj}$ the numerical solution is requested to solve the differential equation system of order $n$. 
Numerical Integration

Initial value problem in the interval $t_k$ is given by:

$$\ddot{\vec{r}}_k = f(t, \vec{r}_k, \dot{\vec{r}}_k)$$

with initial conditions

$$\vec{r}_k(t_k) = \vec{r}_{k0} \quad \text{and} \quad \dot{\vec{r}}_k(t_k) = \dot{\vec{r}}_{k0}$$

where the initial values are defined as

$$\vec{r}_{k0}^{(i)} = \begin{cases} 
\vec{r}_0^{(i)} & k = 0 \\
\vec{r}_{k-1}^{(i)} & k > 0 
\end{cases}$$
Numerical Integration

The collocation method approximates the solution in the interval $I_k$ by:

$$\vec{r}_k(t) = \sum_{l=0}^{q} \frac{1}{l!} (t - t_k)^l \vec{r}^{(l)}_{k0}$$

The coefficients $\vec{r}^{(l)}_{k0}, l = 0, \ldots, q$ are obtained by requesting that the numerical solution assumes the initial values and solves the differential equation system at $q - 1$ different epochs $t_{kj}, j = 1, \ldots, q - 1$. This leads to the conditions

$$\sum_{i=2}^{q} \frac{(t_{kj} - t_k)^{l-2}}{(l - 2)!} \cdot \vec{r}^{(l)}_{k0} = f(t_{kj}, \vec{r}_k(t_{kj}), \dot{\vec{r}}_k(t_{kj})) \quad j = 1, \ldots, q - 1$$

They are non-linear but can be solved efficiently by an iterative procedure. See Beutler, 2005.
Transition Quasi-Inertial to Earth-fixed System

Contribution $Q(t)$:
Precession and nutation are caused by Moon and Sun and can be assumed to be known from their ephemeris.
Contributions $W(t)$ and $R(t)$:

The location of the rotation axis of the Earth is moving with respect to the Earth surface: polar motion.

The rotation velocity of the Earth also varies: Excess length of day.

These variations are caused by mass redistributions in the Earth body, of the water on the surface of the Earth as well as within the Earth’s atmosphere.
Transition Quasi-Inertial to Earth-fixed System

The transition from the Earth-fixed $(x_E \ y_E \ z_E)^T$ into the quasi-inertial $(x_R \ y_R \ z_R)^T$ coordinate system is based on the following rotations:

1. $W(t)$: polar motion  
   (location of the rotation axis of the Earth)
2. $R(t)$: rotation of the Earth
3. $Q(t)$: nutation and precession  
   (rotation of the celestial pole)

$$
\begin{bmatrix}
 x_R \\
 y_R \\
 z_R
\end{bmatrix}
= Q(t) \cdot R(t) \cdot W(t) \cdot \begin{bmatrix}
 x_E \\
 y_E \\
 z_E
\end{bmatrix}
$$
Transition Quasi-Inertial to Earth-fixed System

- Origin of the terrestrial reference system
Transition Quasi-Inertial to Earth-fixed System

- Origin of the terrestrial reference system
- Center of mass of the Earth
Transition Quasi-Inertial to Earth-fixed System

- Origin of the terrestrial reference system
- Center of mass of the Earth
- Geocenter vector
The origin of the terrestrial reference frame is located in the long-term averaged position of the center of mass of the Earth. The geocenter vector points to the instantaneous center of mass.
The satellite orbit refers to the origin of the terrestrial reference system if the transformation from the terrestrial into the quasi–inertial system contains only rotations (Earth rotation parameters).
The satellite orbit need to refer to the center of mass of the Earth because the physics of celestial mechanics is based on the principle of gravitation.
Conclusion – the correct way is:
Transition Quasi-Inertial to Earth-fixed System

Conclusion – the correct way is:

Satellite positions in the terrestrial system
Transition Quasi-Inertial to Earth-fixed System

**Conclusion – the correct way is:**

Satellite positions in the terrestrial system

- Vector to the geocenter
Conclusion – the correct way is:

Satellite positions in the terrestrial system
  + Vector to the geocenter
Satellite positions w.r.t. the center of mass of the Earth
Conclusion – the correct way is:

Satellite positions in the terrestrial system
  + Vector to the geocenter
Satellite positions w.r.t. the center of mass of the Earth
  × Earth rotation parameters
Transition Quasi-Inertial to Earth-fixed System

Conclusion – the correct way is:

Satellite positions in the terrestrial system
  + Vector to the geocenter
Satellite positions w.r.t. the center of mass of the Earth
  × Earth rotation parameters
Satellite positions in inertial system (w.r.t. CoM)
Conclusion – the correct way is:

Satellite positions in the terrestrial system
  + Vector to the geocenter
Satellite positions w.r.t. the center of mass of the Earth
  × Earth rotation parameters
Satellite positions in inertial system (w.r.t. CoM)
  All orbit modelling...
Transition Quasi-Inertial to Earth-fixed System

Conclusion – the correct way is:

Satellite positions in the terrestrial system
  \[ \oplus \text{ Vector to the geocenter} \]
Satellite positions w.r.t. the center of mass of the Earth
  \[ \otimes \text{ Earth rotation parameters} \]
Satellite positions in inertial system (w.r.t. CoM)
  All orbit modelling...
Conclusion – the correct way is:

Satellite positions in the terrestrial system
+ Vector to the geocenter

Satellite positions w.r.t. the center of mass of the Earth
× Earth rotation parameters

Satellite positions in inertial system (w.r.t. CoM)

All orbit modelling.

Satellite positions in inertial system (w.r.t. CoM)
× Earth rotation parameters$^{-1}$
Conclusion – the correct way is:

Satellite positions in the terrestrial system
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Satellite positions w.r.t. the center of mass of the Earth
  × Earth rotation parameters
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  All orbit modelling. . .
Satellite positions in inertial system (w.r.t. CoM)
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Transition Quasi-Inertial to Earth-fixed System

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Satellite positions w.r.t. the center of mass of the Earth
  × Earth rotation parameters
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  All orbit modelling...
Satellite positions in inertial system (w.r.t. CoM)
  × Earth rotation parameters$^{-1}$
Satellite positions w.r.t. the center of mass of the Earth
  – Vector to the geocenter
Transition Quasi-Inertial to Earth-fixed System

Conclusion – the correct way is:

Satellite positions in the terrestrial system
   + Vector to the geocenter
Satellite positions w.r.t. the center of mass of the Earth
   × Earth rotation parameters
Satellite positions in inertial system (w.r.t. CoM)
   All orbit modelling...
Satellite positions in inertial system (w.r.t. CoM)
   × Earth rotation parameters⁻¹
Satellite positions w.r.t. the center of mass of the Earth
   − Vector to the geocenter
Satellite positions in the terrestrial system
Conclusion – the correct way is:

Satellite positions in the terrestrial system

+ Vector to the geocenter

Satellite positions w.r.t. the center of mass of the Earth

× Earth rotation parameters

Satellite positions in inertial system (w.r.t. CoM)

All orbit modelling...

Satellite positions in inertial system (w.r.t. CoM)

× Earth rotation parameters⁻¹

Satellite positions w.r.t. the center of mass of the Earth

– Vector to the geocenter

Satellite positions in the terrestrial system

:

Satellite positions may be related to the station coordinates
Development of the GPS satellite constellation
Development of the GPS satellite constellation
Development of the GLONASS satellite constellation
Development of the number of GLONASS tracking stations
Network used for the GNSS processing at CODE.

Status: June 2003
Network used for the GNSS processing at CODE.

Status: June 2004
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2005
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2006
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2007
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2008
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2009
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2010
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2011
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2012
Network used for the GNSS processing at CODE.

Status: June 2013
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2014
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2015
GNSS Stations in CODE Solution

Network used for the GNSS processing at CODE.

Status: June 2016
Development of the GLONASS orbit accuracy in the CODE final processing.
Development of the GLONASS orbit accuracy in the CODE final processing.
A successful ambiguity resolution is absolutely needed for a precise GNSS orbit determination.
Multi-Day Solutions

Orbit solution day $n$
Multi-Day Solutions

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$
Multi-Day Solutions

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

Orbit solution days $n - 1 \ldots n + 1$
Multi-Day Solutions

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

Extracted orbit for day $n$
Multi-Day Solutions

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

Extracted orbit for day $n$

Advantage of the ”Extracted orbit for day $n” with respect to the direct ”Orbit solution day $n” :

- better decorrelation between orbit and Earth rotation parameters.
- no (or at least less) degradation of the orbit at the end of the boundary.
- smoothed day boundary discontinuities (in particular if the satellite was only weakly observed).
Network used for the CODE MGEX solution: stations tracking GPS

Status: July 2016.
Tracking Situation in the MGEX Network

Network used for the CODE MGEX solution: stations tracking GLONASS

Status: July 2016.
Network used for the CODE MGEX solution: stations tracking Galileo

Status: July 2016.
Network used for the CODE MGEX solution: stations tracking BeiDou

Status: July 2016.
Network used for the CODE MGEX solution: stations tracking QZSS

Status: July 2016.
1. Fitting long arcs

- Orbit solution day $n - 1$
- Orbit solution day $n$
- Orbit solution day $n + 1$
1. Fitting long arcs

Orbit solution day \( n - 1 \)  Orbit solution day \( n \)  Orbit solution day \( n + 1 \)
1. Fitting long arcs

Orbit solution day \( n - 1 \)  
Orbit solution day \( n \)  
Orbit solution day \( n + 1 \)
1. **Fitting long arcs**

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$
Validation by Fitting Long Arcs

CODE MGEX solution for the year 2015

Median per satellite with associated quantiles
Validation by Fitting Long Arcs

CODE MGEX solution for the year 2015

Median per satellite with associated quantiles

RMS of long-arc fit [cm]

1d arc
3d arc

GPS
GLO
GAL
BDS
QZS
Validation by Fitting Long Arcs

CODE MGEX solution for the year 2015

Median per satellite with associated quantiles
Validation by Fitting Long Arcs

CODE MGEX solution for the year 2015

The multi-day long-arc solutions perform better than the one-day solutions for all satellites.
Validation by Fitting Long Arcs

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$
Validation by Fitting Long Arcs

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

Extracted orbit for day $n - 1$  Extracted orbit for day $n$  Extracted orbit for day $n + 1$
Validation by Fitting Long Arcs

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

Extracted orbit for day $n - 1$  Extracted orbit for day $n$  Extracted orbit for day $n + 1$
Disadvantage of the "Extracted orbit for day \( n \)" with respect to the direct "Orbit solution day \( n \)":

- The orbits extracted from the three-day arc are not independent anymore.
- An orbit fit over several days cannot be used as a real quality indicator anymore.
Validation by Orbit Overlaps

1. Fitting long arcs

Orbit solution day \( n - 1 \)  Orbit solution day \( n \)  Orbit solution day \( n + 1 \)

2. Orbit overlaps

Orbit solution day \( n - 1 \)  Orbit solution day \( n \)  Orbit solution day \( n + 1 \)
Validation by Orbit Overlaps

1. Fitting long arcs
   Orbit solution day \( n - 1 \)  Orbit solution day \( n \)  Orbit solution day \( n + 1 \)

2. Orbit overlaps
   Orbit solution day \( n - 1 \)  Orbit solution day \( n \)  Orbit solution day \( n + 1 \)
Validation by Orbit Overlaps

CODE MGEX solution for the year 2015

Median per satellite with associated quantiles
Validation by Orbit Overlaps

CODE MGEX solution for the year 2015

Median per satellite with associated quantiles
Validation by Orbit Overlaps

CODE MGEX solution for the year 2015

Along-track [cm]

Median per satellite with associated quantiles
Validation by Orbit Overlaps

CODE MGEX solution for the year 2015

Median per satellite with associated quantiles
Validation by Orbit Overlaps

CODE MGEX solution for the year 2015

Median per satellite with associated quantiles
Validation by Orbit Overlaps

\[ \text{Orbit solution day } n - 1 \quad \text{Orbit solution day } n \quad \text{Orbit solution day } n + 1 \]
Validation by Orbit Overlaps

Orbit solution day $n - 1$ Orbit solution day $n$ Orbit solution day $n + 1$

Extracted orbit for day $n - 1$ Extracted orbit for day $n$ Extracted orbit for day $n + 1$
Validation by Orbit Overlaps

Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

Extracted orbit for day $n - 1$  Extracted orbit for day $n$  Extracted orbit for day $n + 1$
Disadvantage of the "Extracted orbit for day $n$" with respect to the direct "Orbit solution day $n$":

- The orbits extracted from the three-day arc are not independent anymore.
- Day boundary discontinuities cannot be used as a real quality indicator anymore.
1. Fitting long arcs
   Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

2. Orbit overlaps
   Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

3. Comparison with independent measurements (e.g., SLR)
   • Consistency of the station coordinates between GNSS and SLR is required.
   • Biases of both techniques need to be known.
   • In case of problems an identification must be implemented to define which technique has caused the problem.
Validation by SLR Measurements

CODE MGEX solution (3d arc)

![Graph showing SLR residuals for Galileo E11, SVN E101, ECOM 1, and ECOM 2 from 2012 to 2016 with β in degrees on the y-axis and years on the x-axis. The residuals are in centimeters, with data points for each year and satellite represented.]
The new ECOM2 shows a clear improvement with respect to the old ECOM1 because of the stretched bodies of the Galileo satellites.
Validation by SLR Measurements

CODE MGEX solution (3d arc)

The new ECOM2 shows a clear improvement with respect to the old ECOM1 because of the stretched bodies of the QZSS satellites.
Validation by SLR Measurements

CODE MGEX solution (3d arc)

The ECOM2 decomposition is designed for the yaw-steering mode but not for the orbit normal mode.
Validation by SLR Measurements

CODE MGEX solution (3d arc)

Alternative coordinate systems are needed for the empirical orbit parameters.
Validation by SLR Measurements

CODE MGEX solution (3d arc)

Alternative coordinate systems are needed for the empirical orbit parameters.

SLR residuals [cm]

Year 2014 2015 2016

QZS-1, SVN J001 ECOM 1 ECOM 2 ECOM-N

SLR residuals [cm]
Validation by Checking the Clock Performance

1. Fitting long arcs
   Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

2. Orbit overlaps
   Orbit solution day $n - 1$  Orbit solution day $n$  Orbit solution day $n + 1$

3. Comparison with independent measurements (e.g., SLR)

4. Checking the performance of the GNSS satellite clock
   - Some of the GNSS satellites (Galileo, QZSS, GPS Block IIF) carry excellent clocks where a linear behaviour can be expected.
   - Orbit modelling problems (mainly in the radial component) may map into estimated satellite clock values.
Validation by Checking the Clock Performance

CODE MGEX solution (3d arc)

- Year 2013
- Year 2014
- Year 2015
- Year 2016

RMS of linear clock fit [ns]

|β| [deg]

Galileo E11, SVN E101
ECOM 1
ECOM 2
Validation by Checking the Clock Performance

Clock corrections of Galileo PRN E11, SVN E101

For Hour of DOY 14/100:

- ECOM 1
- ECOM 2

For Hour of DOY 14/180:

- ECOM 1
- ECOM 2
Validation by Checking the Clock Performance

CODE MGEX solution (3d arc)

Median per satellite with associated quantiles
Validation by Checking the Clock Performance

CODE MGEX solution (3d arc)

Median per satellite with associated quantiles

Not all GNSS satellite clocks perform well enough to serve for orbit validation purposes.
Handling of Repositioning Events

- Constellation keeping: GPS, GEO and IGSO satellites
- GPS Block IIF satellites during the injection procedure
Handling of Repositioning Events

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Handling of Repositioning Events

- Constellation keeping: GPS, GEO and IGSO satellites
- GPS Block IIF satellites during the injection procedure

- Two independent satellite arcs are assumed (before and after the event)
Handling of Repositioning Events

- Constellation keeping: GPS, GEO and IGSO satellites
- GPS Block IIF satellites during the injection procedure

- Two independent satellite arcs are assumed (before and after the event)
- The smallest distance between both arcs gives the epoch and magnitude of the event.
GPS Repositioning Events Estimated by CODE
GPS Repositioning Events Estimated by CODE

Velocity changes in mm/s
Introduction and Motivation

Overview on the GNSS Constellations

Effects Acting on Satellites and Related Models

Precise Orbit Determination for GNSS Satellites

GNSS Orbit Determination within the IGS
Precise GNSS satellite orbit determination is a challenging task requiring a global solution based on a well distributed network of stations.
Precise GNSS satellite orbit determination is a challenging task requiring a global solution based on a well distributed network of stations.

By 01. January 1994 the IGS was launched as an official service of the International Association of Geodesy (IAG).
Precise GNSS satellite orbit determination is a challenging task requiring a global solution based on a well distributed network of stations.

By 01. January 1994 the IGS was launched as an official service of the International Association of Geodesy (IAG).

IGS means:

- **International GPS Service for Geodesy and Geodynamics**
  January 1994
- **International GPS Service**
  May 1998
- **International GNSS Service**
  March 2005
**Final series** - ORB, ERP, CLK (300/30 sec. sampling), CRD
- available about two weeks after the end of the week
- GPS and GLONASS in compatible but independent series

**Rapid series** - ORB, ERP, CLK
- available at the day after the measurements, 17:00 UTC
- quality very close to the final products

**Ultra-rapid series** - ORB, ERP, (CLK, 300 sec. sampling)
- four updates per day, latency 3 hours
- contains 24 hours estimated and 24 hours predicted orbits
- GLONASS series on an experimental stage
Combined IGS Products

Analysis Center 1

Analysis Center 2

Analysis Center 3

... 

Analysis Center n
1. An unweighted mean orbit between the Analysis Centers is computed.
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2. The standard deviation of each contribution to this mean orbit is computed to assign a weight to each Analysis Center.
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2. The standard deviation of each contribution to this mean orbit is computed to assign a weight to each Analysis Center.
3. The combined IGS orbit consists of the satellite positions computed as the weighted mean of the positions contributed by the Analysis Centers.
1. An unweighted mean orbit between the Analysis Centers is computed.
2. The standard deviation of each contribution to this mean orbit is computed to assign a weight to each Analysis Center.
3. The combined IGS orbit consists of the satellite positions computed as the weighted mean of the positions contributed by the Analysis Centers.
1. An unweighted mean orbit between the Analysis Centers is computed.
2. The standard deviation of each contribution to this mean orbit is computed to assign a weight to each Analysis Center.
3. The combined IGS orbit consists of the satellite positions computed as the weighted mean of the positions contributed by the Analysis Centers.
4. The mean errors and the transformation parameters of the individual solutions with respect to the IGS orbit are made available every week for each day of the (preceding) week.
Final Orbit Quality from November 1993 – August 2016 as computed by the IGS Analysis Center Coordinator (smoothed weekly RMS values).
The consistency of the GNSS modelling between the individual Analysis Centers has significantly been increased during the last years.
The consistency of the GNSS modelling between the individual Analysis Centers has significantly been increased during the last years.

The biggest differences are currently in the GNSS satellite orbit modelling:

- All groups follow an empirical or semi-empirical approach where in most cases the parameters according to eqn. (4) are estimated.
- Significant differences exist in the a priori models that are introduced, e.g., for solar radiation pressure modelling.
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Many Analysis Centers focus currently on the development of their multi-GNSS processing capability.

The IGS needs also a multi-GNSS capable combination procedure.
Other Challenges

The GLONASS miracle:

In the first part of the year the old ECOM1 outperforms the new ECOM2. This changes when a new satellite occupies the same slot in the constellation.
Other Challenges

The GLONASS miracle:

$$\text{abs}(\beta) \geq 15^\circ$$

$$\text{abs}(\beta) < 15^\circ$$
Other Challenges

The GLONASS miracle:

The SLR residuals increase after two to three years of the satellites’ life time...?
THANK YOU
for your attention

Publications of the satellite geodesy research group:
http://www.bernese.unibe.ch/publist