

# Supermultiplets of the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in the continuum limit

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The spectrum of  $\mathcal{N} = 1$  supersymmetric Yang-Mills theory, calculated on the lattice, is presented.

The masses have been determined on three different lattice spacings and extrapolated towards vanishing gluino mass. We present the extrapolation to the continuum limit which is consistent with the formation of degenerate supermultiplets.

*The 33rd International Symposium on Lattice Field Theory  
14 -18 July 2015  
Kobe International Conference Center, Kobe, Japan*

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## 1. Introduction

The Standard Model (SM) of particle physics works pretty well. Still there is what is perceived by many theorists as an imperfection of the theory: that is the so-called fine-tuning problem. The problem arises because the mass of the Higgs boson is much lighter than the Planck mass, *i.e.* the scale when quantum gravity becomes important. Because of this, large quantum contributions would make the mass of the Higgs huge, unless an incredible fine-tuning cancellation occurs. Supersymmetry (SUSY), introducing for every SM particle a superpartner, protects the mass of the Higgs eliminating the quadratic mass divergences responsible for the problem.

Assuming that the standard model of cosmology is correct, the Planck mission team concluded that the total mass/energy density of the known universe contains 4.9% of ordinary matter, 26.8% of dark matter and 68.3% of dark energy [1]. The main hypothetical particle candidates for dark matter are known as weakly interacting massive particles, WIMPs. In some supersymmetric models the lightest supersymmetric particle is stable and electrically neutral. Moreover, it interacts weakly with the particles of the Standard Model, becoming therefore a good candidate for dark matter.

SUSY is therefore a prominent candidate for an extension of the Standard Model, but unfortunately so far, after Run 1 of LHC, no direct evidence for SUSY has been found [2, 3].

It is worth noting that actually the first response to the fine-tuning problem was to propose that the symmetry breaking occurs *dynamically*. Such theories are generically called technicolor. Our collaboration has recently started to work on one of them [4].

Until a few years ago, the effects of SUSY on a theory have been studied only by means of perturbative or effective methods. However, SUSY can be studied also non-perturbatively using the lattice field theory approach, *i.e.* discretising the space-time onto a lattice and then performing Monte Carlo simulations.

Because SUSY is explicitly broken on the lattice, as a consequence of the absence of translational invariance, different approaches are considered in literature. Some groups, *e.g.* [5, 6], work on formulating theories which have exact lattice supersymmetry on the lattice. The approach of our collaboration is based on the idea that, by appropriate tuning of the parameters of the theory, SUSY has to be recovered in the continuum limit. In these proceedings we show that this program can be pursued. We have determined the mass spectrum of the theory, before extrapolating it to the chiral limit, and than to the continuum limit. The formation of degenerate supermultiplets, testifying the restoration of SUSY, is observed. This contradicts a spontaneous supersymmetry breaking, in principle possible, in the supersymmetric Yang-Mills (SYM) theory we are considering.

## 2. The properties of the theory

The theory we study is  $\mathcal{N} = 1$  SYM theory with gauge group SU(2). The theory contains gluons as bosonic particles and gluinos as their fermionic superpartners. The Lagrangian of the theory is invariant under a transformation which relates gluons,  $A_\mu(x)$  gauge fields, and gluinos,  $\lambda(x)$  fields:

$$A_\mu(x) \rightarrow A_\mu(x) - 2i\bar{\lambda}(x)\gamma_\mu\epsilon \quad (2.1)$$

$$\lambda^a(x) \rightarrow \lambda^a(x) - \sigma_{\mu\nu}F_{\mu\nu}^a(x)\epsilon, \quad (2.2)$$

where  $\varepsilon$  is a global fermionic parameter (consistent with the Majorana condition), parametrising the transformation.

The fundamental assumption about non-perturbative dynamics of SYM theory is that there is confinement of the fundamental colour charge and spontaneous chiral symmetry breaking. Both properties have been observed in our simulations [7].

In our theory there is only one Majorana adjoint flavour, therefore there is only a global abelian chiral symmetry  $U(1)_\lambda$ . This classical symmetry is broken at the quantum level, *i.e.* it is anomalous. The remnant symmetry is a  $Z_4$  one; this discrete symmetry is spontaneously broken to  $Z_2$ , as can be revealed by the condensation of the gluino  $\langle \lambda \bar{\lambda} \rangle$ .

Confinement manifests itself as a linear increase of the potential between two static sources. The consequence is that the spectrum of the theory consists of colourless hadronic bound states. In the SUSY limit, the masses of particles are organised in degenerate multiplets.

The low-lying spectrum of particles has been predicted constraining the form of the low-energy effective actions by means of the symmetries of the theory. In the seminal work [8] a first supermultiplet was described: it consists of a scalar ( $0^+$  gluinoball:  $a-f_0 \sim \bar{\lambda} \lambda$ ), a pseudoscalar ( $0^-$  gluinoball:  $a-\eta' \sim \bar{\lambda} \gamma_5 \lambda$ ), and a Majorana fermion (spin 1/2 gluino-glueball:  $\chi \sim \sigma^{\mu\nu} \text{Tr}[F_{\mu\nu} \lambda]$ ). A second supermultiplet was found in [9] introducing pure gluonic states in the effective Lagrangian. It consists of a  $0^-$  glueball, a  $0^+$  glueball, and a gluino-glueball. In that paper the authors argued that the glueball states, in the supersymmetric limit, are lighter than the gluinoball states. Other authors [10], using different arguments, and hints from ordinary QCD, deduce that the lighter states are instead gluinoballs. Our work is able to shed new light on this problem as well.

### 3. The theory on the lattice

The formulation we employ in our simulations is an improved version of what was first proposed in [11] where the Wilson-Dirac operator was considered. For the gauge part, we use the tree-level Symanzik improved gauge action including rectangular Wilson loops of perimeter's length six. To reduce the lattice artifacts also in the fermionic part we apply one level of stout smearing [12] to the link fields in the Wilson-Dirac operator. The configurations have been obtained by updating with a two step polynomial hybrid Monte Carlo (TS-PHMC) algorithm [13]. The integration of the fermionic variables yields a Pfaffian: the sign is taken into account by reweighting [14].

On the lattice there are three sources of SUSY breaking: 1) A non-zero gluino mass; it has to be introduced in numerical simulations but it breaks supersymmetry softly (it does not spoil the cancellation of quadratic divergences which SUSY is supposed to cure). 2) Finite volume effects; in [15] we have shown that using a box size of about 1.2 fm (in QCD units) the statistical errors and the systematic errors of the finite size effects, with our current numerical precision, are of the same order and hence the finite size effects can be neglected. 3) The discretisation of the space-time on a lattice and the consequent breaking of the translational invariance [16]. Because the generators of translation  $P_\mu$  appear directly in the relation which describes the algebra of the single conserved supercharge  $Q$ ,

$$\{Q_\alpha, Q_\beta\} \propto P_\mu \quad (Q_\alpha \text{ are Majorana spinors}), \quad (3.1)$$

than because the symmetry associated with the r.h.s is broken, also the one on the l.h.s, *i.e.* SUSY, is broken.

As discussed in [16] a way to circumvent this last problem and to realise complete supersymmetry on the lattice is to introduce in the Lagrangian nonlocal interactions and a nonlocal derivative. The approach followed in our work is different: the idea is that in the continuum limit Eq. (3.1) is valid again. Moreover, as shown in [11, 17] in SYM the fine-tuning of a single parameter is enough to recover not only supersymmetry but also to cancel the explicit chiral symmetry breaking induced by Wilson fermions. In practice, we have to tune the bare gluino mass so that the renormalised gluino mass vanishes. This has been verified in the past [18] using the Ward Identities. Currently, we monitor the mass of the unphysical adjoint pion  $a-\pi$ , which is defined by the connected contribution of the  $a-\eta'$  correlator. Using the OZI approximation it was possible to show [8] that the square of the adjoint pion mass is proportional to the mass of the gluino:  $m_{a-\pi}^2 \propto m_g$ . This relation has been obtained again recently in a partially quenched setup [19].

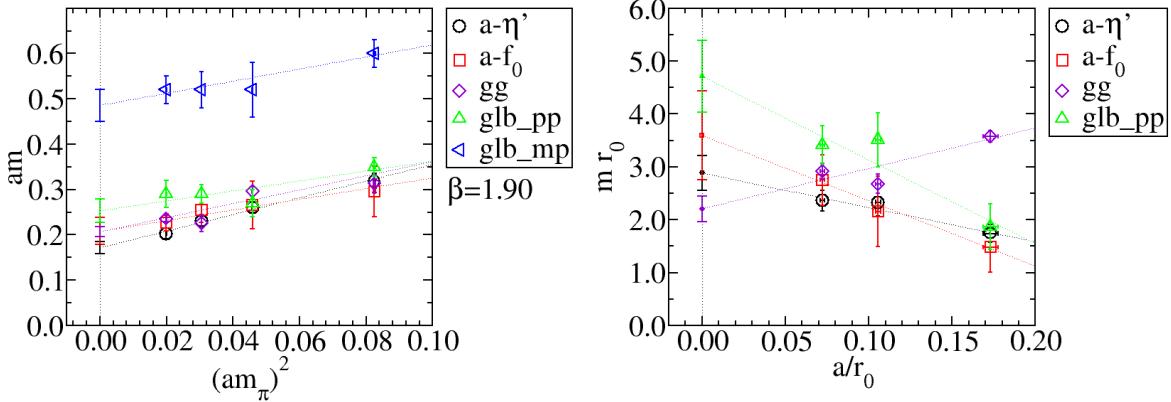
One of the main issues in the lattice approach is the setting of the scale: it is fundamental both for obtaining the observables in lattice units and for extrapolating to the continuum limit the numerical results. In the last years mainly two different observables have been considered: the Sommer parameter  $r_0$  [20] and the Wilson flow scale  $w_0$  [21]. In this work we have considered both these scales and studied them systematically [22].

#### 4. Mass spectrum numerical results

The main aim of our simulations is to verify the existence of a continuum limit with unbroken supersymmetry. This task is pursued measuring the low-lying mass spectrum of bound states: we expect in fact degeneracy inside the same supermultiplet.

We have already published results on the spectrum of the theory for two values of the lattice spacing. Simulations performed at  $\beta = 1.60$ , corresponding to a lattice spacing in QCD units of  $\sim 0.086$  fm, have been presented in [14]. In [23, 24] we decreased the value of the lattice spacing by  $\sim 40\%$  using  $\beta = 1.75$ . In [25] we presented our preliminary results at  $\beta = 1.90$ , where the lattice spacing was further reduced by  $\sim 30\%$ , *i.e.*  $a \sim 0.036$  fm. Here we present our final results at  $\beta = 1.90$ , with the extrapolation to the continuum limit of the spectrum. In Figure 1 (Left) we plot five bounds states for different values of the adjoint pion mass squared; they are linearly extrapolated to the chiral limit, according to the relation  $m_{a-\pi}^2 \propto m_g$ . This numerical extrapolation, at fixed lattice spacing, shows already degeneracy of the masses in the lowest supermultiplet within two standard deviations. Note that the glueball  $0^{++}$  and the gluinoball  $a-f_0$  have the same quantum numbers and they can be characterised by a strong mixing. The same is true for the glueball  $0^{-+}$  and the gluinoball  $a-\eta'$ . This means that it is actually difficult to say which of the two supermultiplets is more glueball-like or gluino-like [9, 10]. Moreover, to make things more complicated, the operators we have chosen to describe the two states can have a strong or a weak projection with one or the other state.

Now if two states (with two different energies) have a strong mixing, it seems reasonable that also the operators which describe them will have a strong overlap with the two states, and extracting the masses using these operators we will get the same mass from both of them, namely the one of the lightest state. Vice versa, if the mixing of the states is weak, it is possible that the



**Figure 1:** (Left) Mass spectrum (gg: gluino-glue; glb\_pp: glueball  $0^{++}$ ; glb\_mp: glueball  $0^{-+}$ ) at  $\beta = 1.90$  for four values of the  $a-\pi$  mass. The extrapolated chiral limit is shown as well. (Right) Mass spectrum extrapolated to the continuum limit, using  $r_0$ .

two operators will project differently in the two states, and extracting the mass from them we will have two masses with different values.

Looking at Figure 1 (Left) we can therefore say that the  $0^{++}$  and the  $a-f_0$  have a strong mixing and therefore a good overlap with the ground state; the  $0^{-+}$  and the  $a-\eta'$  have a weak mixing and the operators we are using for the  $0^{-+}$  have a strong overlap with the glueball-like state, but very weak with the gluino-like. This line of reasoning seem to indicate that the glueball states have an energy higher than the gluinoball as argued in [10].

In Figure 1 (Right), we show the spectrum of the theory extrapolated to the continuum limit using the inverse of the Sommer scale  $r_0$ . According to this plot we can finally claim that the theory shows restoration, within two standard deviations, of supersymmetry in the continuum limit. A similar plot has been obtained also using the scale  $w_0$ , showing an even better behaviour in the extrapolated continuum limit (even if also in that case we see degeneracy only within two standard deviations).

## 5. Influence of topology on the scale setting

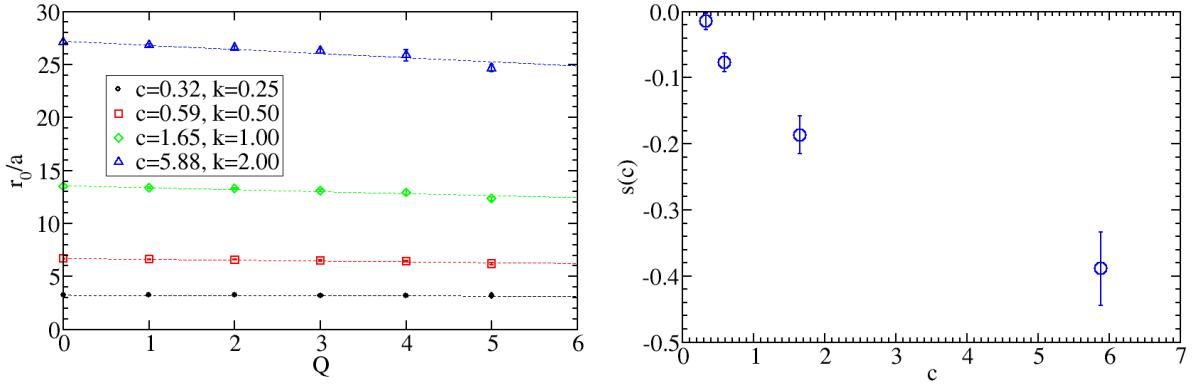
In a recent paper [22] we have investigated the relation between the scale, defined by  $w_0$  and  $r_0$ , and the topological charge, verifying a correlation between these two quantities. In this contribution we consider only the Sommer parameter case.

$r_0$  is defined as the distance  $r$  such that the strong force between a static quark-antiquark pair, multiplied by the squared distance, is equal to a specific reference value, typically 1.65. For a generic reference value  $c$  we have:

$$r_0^2 \left( \frac{\partial V}{\partial r} \right)_{r_0} = c . \quad (5.1)$$

In Figure 2 (Left) we have plotted the value of the scale parameter  $r_0$ , for different values of the reference parameter  $c$ , as a function of the topological charge  $Q$ . APE smearing has been used as smoothing procedure to suppress the short distance fluctuations of the lattice topological charge.

Data with the same value of  $c$  can be interpolated very well ( $\chi^2/\text{d.o.f.} \lesssim 1$ ) with a linear fit. The slope  $s(c)$  as a function of the reference parameter  $c$  is plotted in Figure 2 (Right). It



**Figure 2:** (Left) Linear fit of the dependence of  $r_0$  on the topological charge for different values of the parameter  $c$ . The value of the parameter  $k$  is the ratio between the interquark distance (quark-antiquark) for that value of  $c$  with respect the standard case  $c = 1.65$ . (Right) Slope coefficient  $s(c)$  as a function of the reference value  $c$ . Lattice  $32^3 \times 64$ ,  $\beta = 1.90$ ,  $\kappa = 0.14415$ .

is evident that increasing the value of  $c$  corresponds to increase also the value of the slope. This has important consequences in the precision we can determine the scale  $r_0$ : if the reference value is chosen too large, because of the slope which is large, the fluctuation in the value of  $r_0$ , going from one configuration to another, will be large (during the Monte Carlo simulation all topological sectors will be explored). The error which will characterise the scale parameter will be large. Of course the value of  $c$  cannot be taken too small otherwise the discretisation effects will strongly effect the scale determination.

The lesson we learnt from this analysis is that the value of the reference scale has to be chosen very carefully: there exist an optimal value which is at the same moment not affected by the discretisation effects and which is characterised by the smallest possible value of its statistical error. In [22] we have shown that this conclusion is true also for the Wilson parameter  $w_0$ .

## 6. Conclusion and Outlooks

In this work we have studied  $\mathcal{N} = 1$  supersymmetric Yang-Mills theory with gauge group  $SU(2)$ . We have determined the spectrum of the theory extrapolating the results first to the chiral limit and then to the continuum limit. We observe approximate degeneracy, within our systematic and statistical errors, of the mass spectrum in the low-lying supermultiplet indicating the recovery of SUSY in the continuum limit. In [26] we have shown that it is possible to check the SUSY realisation looking directly at the Witten index. Moreover we have studied the influence of topology on the scale setting, showing a correlation between the precision with which we can determine the scale and the reference value used in setting the scale. The next steps of this project include a study of the excited states and a deeper analysis of the mixing between glueball-like and gluino-like states. Moreover, we will soon move from  $SU(2)$  to the more realistic  $SU(3)$  gauge theory which is another important part of the supersymmetric extension of the Standard Model.

## Acknowledgements

This project has been supported by the German Science Foundation (DFG) under contract Mu

757/16. The authors gratefully acknowledge the computing time granted by the John von Neumann Institute for Computing (NIC) and provided on the supercomputers JUQUEEN and JUROPA at Jülich Supercomputing Centre (JSC). Further computing time has been provided by the compute cluster PALMA of the University of Münster.

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