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Nuclear Physics B 914 (2017) 623–641

NUCLEAR PHYSICS B

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λ-Deformations of left–right asymmetric CFTs

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Received 24 October 2016; accepted 26 November 2016

Available online 30 November 2016

Editor: Stephan Stieberger

Abstract

We compute the all-loop anomalous dimensions of current and primary field operators in deformed current algebra theories based on a general semi-simple group, but with different (large) levels for the left and right sectors. These theories, unlike their equal level counterparts, possess a new non-trivial fixed point in the IR. By computing the exact in λ two- and three-point functions for these operators we deduce their OPEs and their equal-time commutators. Using these we argue on the nature of the CFT at the IR fixed point. The associated to the currents Poisson brackets are a two-parameter deformation of the canonical structure of the isotropic PCM.

1. Introduction

We are interested in a two-dimensional conformal field theory (CFT) which possesses two independent current algebras generated by $J_a(z)$ and $\bar{J}_a(\bar{z})$ which are holomorphic and anti-

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http://dx.doi.org/10.1016/j.nuclphysb.2016.11.022

holomorphic, respectively. The index a takes values $a = 1, 2, \ldots, \dim G$, where G is the non-Abelian group of the CFT. The singular part of the holomorphic currents OPE reads [1,2]

$$J_a(z)J_b(w) = \frac{\delta_{ab}}{(z-w)^2} + \frac{f_{abc}}{\sqrt{k_L}}\frac{J_c(w)}{z-w} + \cdots \,, \tag{1.1}$$

where the level k_L is a positive integer. A similar expression holds for the OPEs between the antiholomorphic currents $\bar{J}_a(\bar{z})$, but with a different level k_R. Of course, the OPE $J_a(z)\bar{J}_b(\bar{w})$ is regular.

We now consider this $G_{k_L} \times G_{k_R}$ current algebra theory perturbed with a classically marginal operator bilinear in the currents. In the Euclidean regime the action reads

$$S = S_{\text{CFT};k_L,k_R} - \frac{\lambda}{\pi}\int \mathrm{d}^2 z\, J_a(z)\bar{J}_a(\bar{z}) \,. \tag{1.2}$$

An explicit example of such an action is the bosonized non-Abelian Thirring model action (for a general discussion, see [3,4]), namely the WZW model two-dimensional CFT perturbed by the above current bilinear. In that case, however the two levels are equal, i.e. $k_L = k_R$. In what follows we will not need an explicit form for the action $S_{\text{CFT};k_L,k_R}$ of the unperturbed CFT.

The unequal level case has some very interesting features, not present when $k_L = k_R$. The basic one is that under RG flow the theory reaches a new fixed point in the IR which lies within the perturbative domain. In contrast, in the equal level case the RG flow drives the theory to a strong coupling regime. In recent years, a lot of progress has been made in the left–right symmetric case. This started with the construction [5] of the all-loop effective action corresponding to (1.2). The corresponding σ-model was shown to be integrable [5]. Subsequently, the extension of this construction to cosets [5–7] and to supergroups [6,7] took place, while the computation of the general RG flow equations using the effective action and gravitational methods was performed in [8,9]. In parallel developments, these models, named generically as λ-deformed, were embedded in specific cases to supergravity [10–14]. In addition, a classical relation to η-deformations [15–17] and in [18–20], via Poisson–Lie T-duality [21] and appropriate analytic continuations was uncovered in [22–26]. More recently the computation of the all-loop anomalous dimensions and of current and primary field operators was performed [27,28]. In view of the very interesting and totally different behavior under the RG-flow that we will shortly see, it is natural to push similar investigations in the case of left–right asymmetry, as well.

In our computation we will make use of the basic two- and three-point functions at the CFT point given for the holomorphic currents by

$$\langle J_a(z_1)J_b(z_2)\rangle = \frac{\delta_{ab}}{z_{12}^2}\,, \qquad \langle J_a(z_1)J_b(z_2)J_c(z_3)\rangle = \frac{1}{\sqrt{k_L}}\frac{f_{abc}}{z_{12}z_{13}z_{23}}, \tag{1.3}$$

where we employ the general notation $z_{ij} = z_i - z_j$. Similar expressions hold for the antiholomorphic currents with the replacement of k_L by k_R. Mixed correlators involving holomorphic and anti-holomorphic currents vanish. In addition, in order to compute higher order correlation functions, we will employ the Ward identity

$$\begin{aligned}\langle J_a(z)J_{a_1}(z_1)J_{a_2}(z_2)\cdots J_{a_n}(z_n)\rangle &= \frac{1}{\sqrt{k_L}}\sum_{i=1}^{n}\frac{f_{aa_ib}}{z-z_i}\langle J_{a_1}(z_1)J_{a_2}(z_2)\cdots J_b(z_i)\cdots J_{a_n}(z_n)\rangle \\ &+ \sum_{i=1}^{n}\frac{\delta_{aa_i}}{(z-z_i)^2}\langle J_{a_1}(z_1)J_{a_2}(z_2)\cdots J_{a_{i-1}}(z_{i-1})J_{a_{i+1}}(z_{i+1})\cdots J_{a_n}(z_n)\rangle\end{aligned} \tag{1.4}$$

and a similar one for the anti-holomorphic sector.

The CFT theory contains also affine primary fields $\Phi_{i,i'}(z,\bar{z})$ transforming in irreducible representations R and R' of the Lie algebra for G, with Hermitian matrices t_a and $\tilde{t}_a$. Under the action of the currents they transform as [2]

$$\begin{aligned} J_a(z)\Phi_{i,i'}(w,\bar{w}) &= -\frac{1}{\sqrt{k_L}}\frac{(t_a)_i{}^j\Phi_{j,i'}(w,\bar{w})}{z-w}\,, \\ \bar{J}_a(\bar{z})\Phi_{i,i'}(w,\bar{w}) &= \frac{1}{\sqrt{k_R}}\frac{(\tilde{t}_a)^{j'}{}_{i'}\Phi_{i,j'}(w,\bar{w})}{\bar{z}-\bar{w}}\,, \end{aligned} \tag{1.5}$$

where $[t_a,t_b]=f_{abc}t_c$ and $[\tilde{t}_a,\tilde{t}_b]=f_{abc}\tilde{t}_c$, with $i=1,2,\dots,\dim R$ and $i'=1,2,\dots,\dim R'$. These fields are also Virasoro primaries with holomorphic and antiholomorphic dimensions given by [2]

$$\Delta_R=\frac{c_R}{2k_L+c_G}\,,\qquad \bar{\Delta}_{R'}=\frac{c_{R'}}{2k_R+c_G}\,, \tag{1.6}$$

where c_R, $c_{R'}$ and c_G are the quadratic Casimir operators in the representations R and R' and in the adjoint representation. They are defined as

$$(t_at_a)_i{}^j=c_R\delta_i{}^j\,,\qquad (\tilde{t}_a\tilde{t}_a)_{i'}{}^{j'}=c_{R'}\delta_{i'}{}^{j'}\,,\qquad f_{acd}f_{bcd}=-c_G\delta_{ab}\,. \tag{1.7}$$

The Virasoro central charges are

$$C_L=\frac{2k_L\dim G}{2k_L+c_G}\,,\qquad C_R=\frac{2k_R\dim G}{2k_R+c_G}\,. \tag{1.8}$$

Of particular importance, especially in considerations in subsection 2.4, will be the adjoint representation $\Phi_{a,b}$ for which the representations matrices are $(t_a)_{bc}=(\tilde{t}_a)_{bc}=-f_{abc}$. Then we have that

$$\begin{aligned} J_a(z)\Phi_{b,c}(w,\bar{w}) &= \frac{1}{\sqrt{k_L}}\frac{f_{abd}\Phi_{d,c}(w,\bar{w})}{z-w}\,, \\ \bar{J}_a(\bar{z})\Phi_{b,c}(w,\bar{w}) &= \frac{1}{\sqrt{k_R}}\frac{f_{acd}\Phi_{b,d}(w,\bar{w})}{\bar{z}-\bar{w}}\,. \end{aligned} \tag{1.9}$$

The two-point correlators for the affine primaries are

$$\langle\Phi^{(1)}_{i,i'}(z_1,\bar{z}_1)\Phi^{(2)}_{j,j'}(z_2,\bar{z}_2)\rangle=\frac{\delta_{ij}\,\delta_{i'j'}}{z_{12}^{2\Delta_R}\,\bar{z}_{12}^{2\bar{\Delta}_{R'}}}\,, \tag{1.10}$$

where the superscripts denote the different representations the primaries belong to. In addition, the mixed three-point functions involving one current are given by

$$\langle J_a(x_3)\Phi^{(1)}_{i,i'}(x_1,\bar{x}_1)\Phi^{(2)}_{j,j'}(x_2,\bar{x}_2)\rangle=\frac{1}{\sqrt{k_L}}\frac{(t_a\otimes\mathbb{I}_{R'})_{ii',jj'}}{x_{12}^{2\Delta_R}\,\bar{x}_{12}^{2\bar{\Delta}_{R'}}}\left(\frac{1}{x_{13}}-\frac{1}{x_{23}}\right) \tag{1.11}$$

and

$$\langle\bar{J}_a(\bar{x}_3)\Phi^{(1)}_{i,i'}(x_1,\bar{x}_1)\Phi^{(2)}_{j,j'}(x_2,\bar{x}_2)\rangle=-\frac{1}{\sqrt{k_R}}\frac{(\mathbb{I}_R\otimes\tilde{t}_a^*)_{ii',jj'}}{x_{12}^{2\Delta_R}\,\bar{x}_{12}^{2\bar{\Delta}_{R'}}}\left(\frac{1}{\bar{x}_{13}}-\frac{1}{\bar{x}_{23}}\right)\,. \tag{1.12}$$

These correlators are non-vanishing as long as the representations R and R' are conjugate for the holomorphic and anti-holomorphic sectors separately. Then, the corresponding primary operators have the same conformal dimensions. We have that

$$\text{Reps (1) and (2) conjugate}: \ t_a^{(1)} = t_a \ , \quad \tilde{t}_a^{(1)} = \tilde{t}_a \ , \quad t_a^{(2)} = -t_a^* \ , \quad \tilde{t}_a^{(2)} = -\tilde{t}_a^* \ , \tag{1.13}$$

where the first equality is just a convenient renaming to avoid superscripts. In this paper we will not compute the λ-deformed three-point function involving only primary fields (for the $k_L = k_R$ case this was done in [28]). Also, correlators with two currents and one affine primary field are zero for $\lambda = 0$ and stay so in the deformed theory as well.

In the following sections, we will compute the two- and three-point function of currents and for primary fields in the left–right asymmetric case, but now for $\lambda \neq 0$. From these we will extract the corresponding anomalous dimensions, OPEs, equal time commutators and classical Poisson brackets. Such correlators will be denoted by $\langle \cdots \rangle_\lambda$ in order to distinguish them from those evaluated at the CFT point, that is for vanishing λ.

2. Current correlators and anomalous dimensions

2.1. Two-point functions

In this subsection, we will evaluate the correlator of two holomorphic or two antiholomorphic currents. From this we will read the anomalous dimensions of the currents to all-orders in the deformation parameter λ. At $\mathcal{O}(\lambda^n)$, the correlation function $\langle J_a(x_1) J_b(x_2)\rangle_\lambda$ involves the sum of the expressions

$$\begin{aligned}\langle J_a(x_1) J_b(x_2)\rangle_\lambda^{(n)} &= \frac{1}{n!}\left(-\frac{\lambda}{\pi}\right)^n \\ &\times \int \mathrm{d}^2 z_{1\ldots n} \langle J_a(x_1) J_b(x_2) J_{a_1}(z_1)\cdots J_{a_n}(z_n)\rangle \langle \bar{J}_{a_1}(\bar{z}_1)\cdots \bar{J}_{a_n}(\bar{z}_n)\rangle \ ,\end{aligned} \tag{2.1}$$

where $\mathrm{d}^2 z_{1\ldots n} = \mathrm{d}^2 z_1 \cdots \mathrm{d}^2 z_n$. The dependence on the levels k_L and k_R arises from the evaluation of the correlation functions appearing in the integrand, at the CFT point. The right hand side of the above expression will in general be a function of the x_i's as well as of the $\bar{x}_i$'s. For notational convenience we have omitted this dependence on the left hand side which we will consistently follow throughout this paper.

On general grounds, the two-point function can be cast in the form

$$\langle J_a(x_1) J_b(x_2)\rangle_\lambda = \frac{\delta_{ab}}{x_{12}^{2+\gamma_L}\, \bar{x}_{12}^{\gamma_L}} \ , \tag{2.2}$$

where the deformation is encoded in the anomalous dimension γ_L of the holomorphic current.[1] We will denote the anomalous dimension of the anti-holomorphic currents by γ_R. Also, (2.2) implies that the holomorphic and anti-holomorphic dimensions of J_a and $\bar{J}_a$ are $(1 + \gamma_L/2, \gamma_L/2)$ and $(\gamma_R/2, 1 + \gamma_R/2)$, respectively. The perturbative calculation up to order $\mathcal{O}(\lambda^3)$ results to the following expression

$$\gamma_L(k_L, k_R, \lambda) = \frac{c_G}{k_L}\lambda^2 - 2\frac{c_G}{\sqrt{k_L k_R}}\lambda^3 + \cdots \ , \tag{2.3}$$

[1] We will keep characterizing J_a and $\bar{J}_a$ as holomorphic and anti-holomorphic, respectively, even though when the deformation is turned on they are no longer.

where the ellipses denote higher order terms in the λ as well as in the $\frac{1}{k_{L,R}}$ expansion. This expression arises from an analogous computation in [28] for the left–right symmetric case just by keeping track of the factors of k_L and k_R, hence the omission of the details.

To proceed we need the exact in λ beta-function. This is found using results of [29][2]

$$\boxed{\frac{d\lambda}{dt} = -\frac{c_G}{2\sqrt{k_L k_R}} \frac{\lambda^2(\lambda - \lambda_0)(\lambda - \lambda_0^{-1})}{(1-\lambda^2)^2}\ , \qquad \lambda_0 = \sqrt{\frac{k_L}{k_R}}}\ , \tag{2.4}$$

where $t = \ln \mu^2$, with μ being the energy scale. There are three fixed points, at $\lambda = 0$, at $\lambda = \lambda_0$ and at $\lambda = \lambda_0^{-1}$ in which the beta-function vanishes. The first at $\lambda = 0$ is the usual UV stable fixed point, present in the left–right symmetric case, as well. However, the other two fixed points are new. To investigate their nature we will assume through out the paper with no loss of generality that $\lambda_0 < 1$. Then, it is easy to see that, the fixed point at $\lambda = \lambda_0$ is IR stable whereas that for $\lambda = \lambda_0^{-1} > 1$ is UV stable. The first of these points is reached from $\lambda = 0$ under an RG flow. The second one can only be reached from large values of λ. Flowing between the two fixed points involves passing thought the strong coupling region at $\lambda = 1$.[3] We will argue that the region with $\lambda > 1$ should be dismissed and therefore the fixed point at $\lambda = \lambda_0^{-1}$ is unphysical. These are new feature of the left–right asymmetric models not present in the left–right symmetric case. We will see that at these new fixed points the anomalous dimensions of the operators we will compute below are generically non-vanishing.

The above beta-function is well defined under the correlated limit in which $\lambda \to \pm 1$ and in addition the levels become extremely large. Specifically,

$$\lambda = \pm 1 - \frac{b}{(k_L k_R)^6}\ , \qquad k_{L,R} \to \infty\ , \tag{2.5}$$

where b is the new coupling constants and where the limit is taken for both signs independently. This is a direct analogue of the pseudochiral model limit $\lambda \to -1$ [28] which can be taken not only in the beta-function and anomalous dimensions, but also at the level of the all-loop effective action of [5] for $k_L = k_R$. It is interesting that this limit exists even though in the left–right asymmetric case we do not know the corresponding all-loop effective action. Note that one cannot take the non-Abelian T-duality limit $\lambda \to 1$ which exists for the left–right symmetric case only [5].[4] The distinction between the $\lambda \to 1$ and $\lambda \to -1$ limits ceases to exist in the left–right asymmetric case.

As in [27] we may deduce by examining the Callan–Symanzik equation the form of the anomalous dimension to all orders in λ. The appropriate ansatz is of the form

$$\gamma_L(k_L, k_R, \lambda) = \frac{c_G}{\sqrt{k_L k_R}} \frac{\lambda^2}{(1-\lambda^2)^3} f(\lambda; \lambda_0) + \cdots\ , \tag{2.6}$$

[2] We use Eq. (3.4) of that reference where in order to conform with our notation we let

$$k_{R,L} \mapsto 2k_{R,L}\ , \quad g \mapsto \frac{2\lambda}{\sqrt{k_L k_R}}\ , \quad C_{\text{Adj}} \mapsto c_G\ .$$

The logarithm of the length scale in β_g is replaced by $t = \ln \mu^2$ which effectively flips the overall sign.

[3] By strong coupling region we mean the region where the β-function develops poles.

[4] This limit is of the form (2.5) but with $(k_L k_R)^6 \mapsto k$ and also involves an expansion of the group element of the WZW model action.

where now the ellipses denote higher order terms in the large level expansion. The function $f(\lambda;\lambda_0)$ is a to be determined and should be analytic in the complex λ-plane. It may depend on the levels k_L and k_R as parameters only via their ratio since we are interested in the leading order behavior in the levels. This explains the presence of the parameter λ_0. Note also that this ansatz remains finite under the limit (2.5).

In [33] it was argued, using path integral arguments and manipulations, that the theory should be invariant under the transformation

$$\text{For } k_{L,R} \gg 1: \qquad k_L \mapsto -k_R\,, \quad k_R \mapsto -k_L\,, \quad \lambda \mapsto \frac{1}{\lambda}\,. \tag{2.7}$$

We emphasize that this statement is true without including the parity transformation $z \leftrightarrow \bar{z}$. In implementing the above in various expressions one should be careful and treat it as an analytic continuation when square roots appear. Hence, we better write $(k_L, k_R) \mapsto e^{i\pi}(k_R, k_L)$ and $1 - \lambda \mapsto e^{-i\pi}\lambda^{-1}(1-\lambda)$. One easily sees that this is a symmetry of the beta-function equation (2.4). Imposing it to be a symmetry of the anomalous dimension (2.6), i.e.

$$\gamma_L(-k_R, -k_L, \lambda^{-1}) = \gamma_L(k_L, k_R, \lambda) \quad \Longrightarrow \quad \lambda^2 f(\lambda^{-1}; \lambda_0^{-1}) = f(\lambda;\lambda_0)\,, \tag{2.8}$$

implying that $f(\lambda;\lambda_0)$ is a second order polynomial in λ with coefficients depending on λ_0 and related via the above symmetry. The matching of its coefficients with the perturbative result (2.3) gives $f(\lambda;\lambda_0) = \lambda_0(\lambda - \lambda_0^{-1})^2$. Hence we obtain for the holomorphic current' anomalous dimension the exact in λ expression

$$\boxed{\gamma_L(k_L, k_R, \lambda) = \frac{c_G}{k_R}\frac{\lambda^2(\lambda-\lambda_0^{-1})^2}{(1-\lambda^2)^3} + \cdots}\,. \tag{2.9}$$

Similar considerations lead to the anomalous dimension of the anti-holomorphic current with the result being

$$\boxed{\bar{\gamma}_R(k_L, k_R, \lambda) = \frac{c_G}{k_L}\frac{\lambda^2(\lambda-\lambda_0)^2}{(1-\lambda^2)^3} + \cdots}\,. \tag{2.10}$$

Notice that under the limit (2.5) both anomalous dimensions remain finite.

Obviously, for $\lambda = 0$ the anomalous dimensions vanish. However, this is no longer true for the other fixed points of the beta-function. We have that

$$\begin{aligned} &\lambda = \lambda_0: \qquad \gamma_L(k_L, k_R, \lambda_0) = \frac{c_G}{k_R - k_L}\,, \qquad \gamma_R(k_L, k_R, \lambda_0) = 0\,, \\ &\lambda = \lambda_0^{-1}: \qquad \gamma_L(k_L, k_R, \lambda_0^{-1}) = 0\,, \qquad \gamma_R(k_L, k_R, \lambda_0^{-1}) = \frac{c_G}{k_L - k_R}\,. \end{aligned} \tag{2.11}$$

The presence of non-vanishing anomalous dimensions at the new fixed points is indicative of the fact that the new CFT at these points are such that the original currents have acquired new characteristics. We will return to this point later.

To further check the validity of (2.9) we have performed a tedious perturbative computation at $\mathcal{O}(\lambda^4)$ whose details are presented in Appendix B. We found perfect agreement with the prediction of (2.9).

2.2. Current composite operators

In this subsection we will compute the two-point correlator of the composite operator that deforms the CFT. From this correlator we will extract the exact in the deformation parameter λ dimension of this operator. In particular, we will compute the dimension of the current-bilinear operator

$$\mathcal{O}(z,\bar{z}) = J_a(z)\bar{J}_a(\bar{z})\,, \tag{2.12}$$

which actually drives the deformation as in (1.2). We will do this in two different ways which will lead to the same result and are conceptually complementary.

In the first method we will use the geometry in the space of couplings as presented in [34] and used in the present context when $k_L = k_R$ in [27]. We have to evaluate the two-point function $G = \langle\mathcal{O}(x_1,\bar{x}_1)\mathcal{O}(x_2,\bar{x}_2)\rangle$ and read off the Zamolodchikov metric $g = |x_{12}|^4\langle\mathcal{O}(x_1,\bar{x}_1)\mathcal{O}(x_2,\bar{x}_2)\rangle$. It takes the form

$$G \sim |x_{12}|^{-4}\left(1+\gamma^{(\mathcal{O})}\ln\frac{\varepsilon^2}{|x_{12}|^2}\right), \quad g = g_0 - 2s\nabla_\lambda\beta + \mathcal{O}(s^2)\,, \tag{2.13}$$

where $s = \ln(|x_{12}|^2\mu^2)$ and the β-function is given by (2.4). The finite part of the two-point function was calculated as in [34] (see [27] for a detailed derivation) and reads

$$g_0 = \frac{\dim G}{(1-\lambda^2)^2}\,. \tag{2.14}$$

Upon this we built the connection that appears in the covariant derivative with respect to λ. In our case we have just one coupling constant λ and it turns out that the anomalous dimension of the composite operator is simply given by

$$\gamma^{(\mathcal{O})} = 2\nabla_\lambda\beta = 2\partial_\lambda\beta + \beta\frac{\partial_\lambda g_0}{g_0}\,. \tag{2.15}$$

Using (2.4) and (2.14), we find specifically that

$$\boxed{\gamma^{(\mathcal{O})}(k_L,k_R,\lambda) = c_G\lambda\,\frac{3(\lambda_0+\lambda_0^{-1})\lambda(1+\lambda^2) - 2(1+4\lambda^2+\lambda^4)}{\sqrt{k_Rk_L}(1-\lambda^2)^3} + \cdots}\,. \tag{2.16}$$

This respects the symmetry (2.7) since $\gamma^{(\mathcal{O})}(-k_R,-k_L,\lambda^{-1}) = \gamma^{(\mathcal{O})}(k_L,k_R,\lambda)$. Moreover, in the IR fixed point we find that

$$\gamma^{(\mathcal{O})}(k_L,k_R,\lambda_0) = \frac{c_G}{k_R-k_L}\,, \tag{2.17}$$

that is at the fixed point of the RG flow at $\lambda = \lambda_0$ the anomalous dimensions of the composite operator equals the sum of anomalous dimensions of J_a and $\bar{J}_a$, as it should be since the two CFTs are decoupled. Unlike, the left–right symmetric case for which $\gamma^{(\mathcal{O})}$ is strictly non-positive, here it doesn't have a definite sign for the RG in $\lambda \in (0,\lambda_0)$. It starts negative, then it develops a minimum and finally it reaches the positive value given above. This is expected since the perturbation is relevant and irrelevant near $\lambda = 0$ and near $\lambda = \lambda_0$, respectively.

It is instructive to also derive (2.16), using our method, i.e. low order perturbative results combined with the symmetry (2.7). The leading order one-loop contribution is

$$
\begin{aligned}
\langle \mathcal{O}(x_1,\bar{x}_1)\mathcal{O}(x_2,\bar{x}_2)\rangle_\lambda^{(1)} &= -\frac{\lambda}{\pi}\int \mathrm{d}^2 z \langle J_a(x_1)J_b(x_2)J_c(z)\rangle \langle \bar{J}_a(\bar{x}_1)\bar{J}_b(\bar{x}_2)\bar{J}_c(\bar{z})\rangle \\
&= \frac{c_G \dim G\,\lambda}{\sqrt{k_L k_R}\pi |x_{12}|^2}\int \frac{\mathrm{d}^2 z}{(x_1-z)(x_2-z)(\bar{x}_1-\bar{z})(\bar{x}_2-\bar{z})}\,, \\
&= -\frac{2c_G \dim G\,\lambda}{\sqrt{k_L k_R}|x_{12}|^4}\ln\frac{\varepsilon^2}{|x_{12}|^2}\,.
\end{aligned}
\tag{2.18}
$$

Turning next to the two-loop contribution we have that

$$
\begin{aligned}
\langle \mathcal{O}(x_1,\bar{x}_1)\mathcal{O}(x_2,\bar{x}_2)\rangle_\lambda^{(2)} &= \frac{\lambda^2}{2!\pi^2}\int \mathrm{d}^2 z_{12}\langle J_a(x_1)J_b(x_2)J_c(z_1)J_d(z_2)\rangle \\
&\quad\times \langle \bar{J}_a(\bar{x}_1)\bar{J}_b(\bar{x}_2)\bar{J}_c(\bar{z}_1)\bar{J}_d(\bar{z}_2)\rangle\,.
\end{aligned}
\tag{2.19}
$$

To proceed we evaluate the four-point function using the Ward identity (1.4)

$$
\begin{aligned}
\langle J_{a_1}(z_1)J_{a_2}(z_2)J_{a_3}(z_3)J_{a_4}(z_4)\rangle &= \frac{1}{k_L}\left(\frac{f_{a_1a_3e}f_{a_2a_4e}}{z_{12}z_{13}z_{24}z_{34}} - \frac{f_{a_1a_3e}f_{a_2a_4e}}{z_{12}z_{14}z_{23}z_{34}}\right) \\
&\quad + \frac{\delta_{a_1a_2}\delta_{a_3a_4}}{z_{12}^2 z_{34}^2} + \frac{\delta_{a_1a_3}\delta_{a_2a_4}}{z_{13}^2 z_{24}^2} + \frac{\delta_{a_1a_4}\delta_{a_2a_3}}{z_{14}^2 z_{23}^2}
\end{aligned}
\tag{2.20}
$$

and after some effort and heavy use of the identity

$$
\frac{1}{(x_1-z)(z-x_2)} = \frac{1}{x_{12}}\left(\frac{1}{x_1-z}+\frac{1}{z-x_2}\right),
$$

we find at leading order in $k_{L,R}$ that

$$
\langle \mathcal{O}(x_1,\bar{x}_1)\mathcal{O}(x_2,\bar{x}_2)\rangle_\lambda^{(2)} = \frac{3c_G \dim G\,\lambda^2}{|x_{12}|^4}\left(\frac{1}{k_R}+\frac{1}{k_L}\right)\ln\frac{\varepsilon^2}{|x_{12}|^2}\,. \tag{2.21}
$$

Either from the form of the Callan–Symanzik equation, or by demanding a well-defined behavior in the limit (2.5) and then by employing the symmetry (2.7) we find the all-loop expression

$$
\gamma^{(\mathcal{O})} = c_G\lambda\,\frac{3(\lambda_0+\lambda_0^{-1})\lambda(1+\lambda^2)-2(1+4a_2\lambda^2+\lambda^4)}{\sqrt{k_R k_L}(1-\lambda^2)^3}+\dots \tag{2.22}
$$

where the coefficient a_2 can not be determined by the symmetry arguments. One way to determine it is to further compute the $\mathcal{O}(\lambda^3)$ perturbative contribution. However, it is much easier to just demand for consistency that $\gamma^{(\mathcal{O})}$ equals the sum of the anomalous dimensions of the currents J_a and $\bar{J}_a$ at the fixed point $\lambda=\lambda_0$. This fixes $a_2=1$ and therefore (2.22) matches (2.16).

2.3. Three-point functions

Similarly to the two-point function case we use the perturbative result for the three-point function of holomorphic currents given by

$$
\langle J_a(x_1)J_b(x_2)J_c(x_3)\rangle_\lambda = \left[\frac{1}{\sqrt{k_L}}\left(1+\frac{3}{2}\lambda^2\right)-\frac{1}{\sqrt{k_R}}\lambda^3\right]\frac{f_{abc}}{x_{12}x_{13}x_{23}}+\cdots\,, \tag{2.23}
$$

which follows from the analogous computation in [28] in the left–right symmetric case by modifying appropriate the various terms to take into account the different levels. The ansatz for the all-loop expression takes the form

$$\langle J_a(x_1)J_b(x_2)J_c(x_3)\rangle_\lambda = \frac{g(\lambda;\lambda_0)}{\sqrt{k_L(1-\lambda^2)^3}}\frac{f_{abc}}{x_{12}x_{13}x_{23}} , \tag{2.24}$$

where as before $g(\lambda;\lambda_0)$ is everywhere analytic and should be determined by imposing the symmetry (2.7) and matching with the perturbative result (2.23). Imposing first the symmetry we obtain that

$$\lambda_0\lambda^3 g(\lambda^{-1};\lambda_0^{-1}) = -g(\lambda;\lambda_0) , \tag{2.25}$$

implying that $g(\lambda;\lambda_0)$ is a third order polynomial in λ with coefficients depending on λ_0. Simple algebra, taking also into account the perturbative result (2.23), gives that $g(\lambda;\lambda_0) = 1-\lambda_0\lambda^3$. Hence, the exact in λ three-point function of holomorphic function reads

$$\boxed{\langle J_a(x_1)J_b(x_2)J_c(x_3)\rangle_\lambda = \frac{1-\lambda_0\lambda^3}{\sqrt{k_L(1-\lambda^2)^3}}\frac{f_{abc}}{x_{12}x_{13}x_{23}} + \dots} \ . \tag{2.26}$$

In a similar fashion the three-point function for the anti-holomorphic currents is given by

$$\boxed{\langle \bar J_a(\bar x_1)\bar J_b(\bar x_2)\bar J_c(\bar x_3)\rangle_\lambda = \frac{1-\lambda_0^{-1}\lambda^3}{\sqrt{k_R(1-\lambda^2)^3}}\frac{f_{abc}}{\bar x_{12}\bar x_{13}\bar x_{23}} + \cdots} \ . \tag{2.27}$$

It remains to compute the three-point function for mixed correlators. Using the, appropriately modified, perturbative result of [28]

$$\langle J_a(x_1)J_b(x_2)\bar J_c(\bar x_3)\rangle_\lambda = \left(\frac{\lambda}{\sqrt{k_L}} - \frac{\lambda^2}{\sqrt{k_R}}\right)\frac{f_{abc}\bar x_{12}}{x_{12}^2\bar x_{13}\bar x_{23}} + \cdots , \tag{2.28}$$

and imposing the symmetry (2.7) to an appropriate ansatz for the all-loop result we obtain that

$$\boxed{\langle J_a(x_1)J_b(x_2)\bar J_c(\bar x_3)\rangle_\lambda = \frac{\lambda(\lambda_0^{-1}-\lambda)}{\sqrt{k_R(1-\lambda^2)^3}}\frac{f_{abc}\bar x_{12}}{x_{12}^2\bar x_{13}\bar x_{23}} + \dots} \ . \tag{2.29}$$

Similarly, for the other mixed three-point correlators we have that

$$\boxed{\langle \bar J_a(\bar x_1)\bar J_b(\bar x_2)J_c(x_3)\rangle_\lambda = \frac{\lambda(\lambda_0-\lambda)}{\sqrt{k_L(1-\lambda^2)^3}}\frac{f_{abc}x_{12}}{\bar x_{12}^2 x_{13}x_{23}} + \dots} \ . \tag{2.30}$$

2.4. The IR fixed point

The anomalous dimension $\gamma_R = 0$ at $\lambda = \lambda_0$ according to (2.11), so that $\bar J_a$ remains with dimension $(0, 1)$. This implies that $\bar J_a$ can be regarded as an antiholomorhic current not only at $\lambda = 0$ but also at $\lambda = \lambda_0$. In addition, at $\lambda = \lambda_0$ the prefactor on the r.h.s. of (2.27) becomes $1/\sqrt{k_R - k_L}$. This suggests that under an RG flow the new conformal point at $\lambda = \lambda_0$ is reached in the IR and at this point the anti-holomorphic current $\bar J_a$ generates the same current algebra but with a smaller level $k_R \mapsto k_R - k_L$.

The nature of the holomorphic current at $\lambda = \lambda_0$ is more delicate to determine. Its anomalous dimension $(1+\gamma_L/2, \gamma_L/2)$ with γ_L given in (2.11) implies, after recalling that we are in the

$k_{L,R} \gg 1$ regime, that it corresponds to an operator transforming in the adjoint representation for a current algebra at level $k_R - k_L$. Indeed, using the OPE's in (4.1) below evaluated at $\lambda = \lambda_0$, we easily see that

$$\begin{aligned}
J_a(x_1)J_b(x_2) &= \frac{\delta_{ab}}{x_{12}^2} + \frac{k_R + k_L}{\sqrt{k_L k_R (k_R - k_L)}} \frac{f_{abc} J_c(x_2)}{x_{12}} + \frac{1}{\sqrt{k_R - k_L}} \frac{f_{abc} \bar{J}_c(\bar{x}_2)\bar{x}_{12}}{x_{12}^2} + \cdots , \\
\bar{J}_a(\bar{x}_1)\bar{J}_b(\bar{x}_2) &= \frac{\delta_{ab}}{\bar{x}_{12}^2} + \frac{1}{\sqrt{k_R - k_L}} \frac{f_{abc} \bar{J}_c(\bar{x}_2)}{\bar{x}_{12}} + \cdots , \\
J_a(x_1)\bar{J}_b(\bar{x}_2) &= \frac{1}{\sqrt{k_R - k_L}} \frac{f_{abc} J_c(x_2)}{\bar{x}_{12}} + \cdots ,
\end{aligned} \tag{2.31}$$

where we emphasize that these OPE's are valid to leading order in the large level expansion (beyond the Abelian limit). Before proceeding, note also the curious fact that these OPE's are invariant under $(k_L, k_R) \mapsto -(k_R, k_L)$ which is the remnant of the symmetry (2.7) once we have fixed $\lambda = \lambda_0$.

The middle of the above OPE's is indeed a current algebra theory $G_{k_R - k_L}$, as advertized. The third line shows that J_a transforms non-trivially under $\bar{\bar{J}}_a$. From the form of this OPE and the anomalous dimensions that J_a has acquired it appears as if J_a is a composite operator of the form $J_a = \tilde{\Phi}_{b,a} \tilde{J}_b$, where $\tilde{\Phi}$ is a field transforming in the adjoint representation similar to (1.9) and $\tilde{J}_a$ generates a current algebra (the tilded symbol will be explained shortly). This interpretation should be considered with caution as far as the holomorphic sector is concerned. In that sector the theory is not a current algebra theory, in fact as we will argue shortly it is a coset CFT. As such it does not possess currents as holomorphic objects but their counterparts which are the non-Abelian parafermions [30] with dimensions deviating from unity by $1/k$-corrections. These have not been studied at the quantum level [31] as much as their Abelian counterparts [32]. Similarly, for $\tilde{\Phi}_{a,b}$ the left representation is approximately in the limit of large levels similar to the adjoint one. These comments explain the use of tilded symbols. Nevertheless, for the anti-holomorphic sector the above statement is exact and indeed one may check, using (1.9), with $k_R \mapsto k_R - k_L$ and the fact that the OPE between $\bar{J}_a$ and $\tilde{J}_b$ is regular, that the third of (2.31) is indeed reproduced. Finally, we have checked that the OPE in the first line of (2.31) is reproduced by appropriately normalizing the structure constants of the quasi-current algebra for $\tilde{J}_a$ and by using that $\partial \Phi_{c,a} \Phi_{c,b} \sim f_{abc} J_c$ and similarly for the ant-homomorphic derivative, as well. We will not present this computation here, not only because it is lengthy, but also since the purpose of the above is to reinforce related arguments, made in the past, on the nature of the CFT in the IR to which we now turn.

There has been already a suggestion in [29] based also on work in [36] that the RG flow is such that

$$G_{k_L} \times G_{k_R} \quad \stackrel{\text{IR}}{\Longrightarrow} \quad \frac{G_{k_L} \times G_{k_R - k_L}}{G_{k_R}} \times G_{k_R - k_L} . \tag{2.32}$$

This was suggested based on the fact that the sum of the central charges for the left and the right sectors is expected to lower in accordance with Zamolodchikov's c-theorem [35] and arguments that the difference of them should stay constant. One can argue that this is the case as follows. For $k_R > k_L$ the central charge for the anti-holomorphic Virasoro algebra is larger that for the holomorphic one (see (1.8)). Hence, the theory is chiral, there is a gauge anomaly and also the energy momentum tensor cannot be coupled anomaly free to a two-dimensional worldsheet metric. We remedy the situation by making the levels of the current algebra and Virasoro algebra central charges equal by adding chiral matter. These degrees of freedom do not participate in the RG flow and therefore the difference $k_R - k_L$, as well as that of that of the Virasoro central charges has to be invariant and their values in the IR be the same as in the UV.[5]

The suggestion in (2.32) clearly satisfies these requirements and was presented as a unique solution for the end point of the RG flow in the IR. This is further strongly reinforced by our findings as explained above. Nevertheless, these arguments are only suggestive and a much better justification would be to actually compute Zamolodchikov's c-function and its dependence on λ. This is left for future work.

Finally, note that, due to (2.7) the theory for $\lambda > 1$ and positive integers as levels of the two current algebras is equivalently to a theory with $\lambda < 1$ but now with negative levels of the current algebras. Hence, unitarity is violated and this makes this region unphysical. In accordance, the fixed point at $\lambda = \lambda_0^{-1}$ is excluded from our discussion.

3. Primary field correlators and anomalous dimensions

In this section, we calculate the all-loop anomalous dimensions of affine primary operators, as well as the three-point functions of one current (holomorphic or antiholomorhic) with two affine primary operators.

The perturbative computation up to $\mathcal{O}(\lambda^3)$ and to $\mathcal{O}(1/k_{L,R})$, for the two-point function of primary fields can be found by appropriately modifying the corresponding computation in [28] done in the left–right symmetric case. The result is

$$\begin{aligned}
&\langle \Phi^{(1)}_{i,i'}(x_1,\bar{x}_1)\Phi^{(2)}_{j,j'}(x_2,\bar{x}_2)\rangle_\lambda \\
&= \frac{1}{x_{12}^{2\Delta_R}\bar{x}_{12}^{2\bar{\Delta}_{R'}}}\Bigg[\left(1+\lambda^2\left(\frac{c_R}{k_L}+\frac{c_{R'}}{k_R}\right)\ln\frac{\varepsilon^2}{|x_{12}|^2}\right)(\mathbb{I}_R\otimes\mathbb{I}_{R'})_{ii',jj'} \\
&\quad - 2\lambda\frac{1+\lambda^2}{\sqrt{k_L k_R}}\ln\frac{\varepsilon^2}{|x_{12}|^2}(t_a\otimes t_a^*)_{ii',jj'}\Bigg] + \frac{1}{k_{L,R}}\mathcal{O}(\lambda^4)\,. \qquad (3.1)
\end{aligned}$$

Proceeding as in [28] there is a λ-independent matrix U chosen such that

$$(t_a\otimes t_a^*)_{IJ} = U_{IK}N_{KL}(U^{-1})_{LJ}\,, \qquad N_{IJ} = N_I\delta_{IJ}\,, \qquad (3.2)$$

where the N_I's are the eigenvalues of the matrix $t_a \otimes t_a^*$ and where we have adopted the double index notation $I = (ii')$. In the rotated basis

$$\widetilde{\Phi}^{(1)}_I = (U^{-1})_I{}^J\Phi^{(1)}_J\,, \qquad \widetilde{\Phi}^{(2)}_I = U_I{}^J\Phi^{(2)}_J\,, \qquad (3.3)$$

the correlator (3.1) becomes diagonal

$$\langle \widetilde{\Phi}^{(1)}_I(x_1,\bar{x}_1)\widetilde{\Phi}^{(2)}_J(x_2,\bar{x}_2)\rangle_\lambda = \frac{\delta_{IJ}}{x_{12}^{2\Delta_R}\bar{x}_{12}^{2\bar{\Delta}_{R'}}}\left(1+\delta_I^{(\Phi)}\ln\frac{\varepsilon^2}{|x_{12}|^2}\right)\,, \qquad (3.4)$$

where

[5] We note for completeness that in the left–right symmetric case the RG flow is driven to a strong coupling regime towards $\lambda = 1$. In this case a mass gap develops which is consistent with the fact that in that regime the description is better in terms of the non-Abelian T-dual of the principal chiral model (PCM) for the group G which should have a mass gap [37] being canonically equivalent [38,39] to the original PCM.

$$\delta_I^{(\Phi)} = -2\lambda \frac{1+\lambda^2}{\sqrt{k_L k_R}} N_I + \lambda^2 \left(\frac{c_R}{k_L} + \frac{c_{R'}}{k_R} \right) + \mathcal{O}(\lambda^4) . \tag{3.5}$$

To determine the exact anomalous dimension of the general primary field we first realize that we should include in the above expression the level-dependent part coming from the CFT dimensions of Δ_R and $\bar{\Delta}_{R'}$ in (1.6) up to order $1/k_{L,R}$. Hence the anomalous dimension is given by

$$\begin{aligned} \gamma^{(L)}_{I;R,R'}(k,\lambda)\big|_{\text{pert}} &= \frac{c_R}{k_L} + \delta_I^{(\Phi)} = \\ &= \frac{c_R}{k_L}(1+\lambda^2) + \frac{c_{R'}}{k_R}\lambda^2 - 2\frac{\lambda(1+\lambda^2)}{\sqrt{k_L k_R}} N_I + \mathcal{O}(\lambda^4) . \end{aligned} \tag{3.6}$$

Using the symmetry (2.7) and the corresponding transformation for the primary fields and representation matrices found in [28]

$$\Phi^{(1)}_{i,i'} \leftrightarrow \Phi^{(2)}_{i',i} , \qquad (t^{(1)}, t^{(2)}) \leftrightarrow (\tilde{t}^{(2)}, \tilde{t}^{(1)}) , \tag{3.7}$$

we have for the all-loop anomalous dimensions the relation

$$\gamma^{(L)}_{I;R,R'}(-k_R, -k_L, \lambda^{-1}) = \gamma^{(L)}_{I;R',R}(k_L, k_R, \lambda) . \tag{3.8}$$

Following our standard procedure with an appropriate ansatz we find that

$$\boxed{\gamma^{(L)}_{I;R,R'}(k_L,k_R,\lambda) = \frac{1}{1-\lambda^2}\left(\frac{c_R}{k_L} + \frac{c_{R'}}{k_R}\lambda^2 - 2\frac{\lambda}{\sqrt{k_L k_R}} N_I \right)} , \tag{3.9}$$

and similarly that

$$\boxed{\gamma^{(R)}_{I;R,R'}(k_L,k_R,\lambda) = \frac{1}{1-\lambda^2}\left(\frac{c_{R'}}{k_R} + \frac{c_R}{k_L}\lambda^2 - 2\frac{\lambda}{\sqrt{k_L k_R}} N_I \right)} . \tag{3.10}$$

At the fixed point at $\lambda = \lambda_0$ none of the anomalous dimensions vanishes identical for all possible representations. Depending on the representations R, R' and the multiplicity number N_I, there seem to be values of λ for which the anomalous dimensions of specific fields may vanish. Finally, the two-point function for conjugate primary fields take the form

$$\boxed{\langle \widetilde{\Phi}^{(1)}_I(x_1,\bar{x}_1) \widetilde{\Phi}^{(2)}_J(x_2,\bar{x}_2) \rangle = \frac{\delta_{IJ}}{x_{12}^{\gamma^{(L)}_{I;R,R'}} \bar{x}_{12}^{\gamma^{(R)}_{I;R,R'}}}} . \tag{3.11}$$

For completeness, the mixed correlators involving two primary fields and one current can be evaluated in a similar fashion resulting at

$$\langle J_a(x_3) \Phi^{(1)}_{i,i'}(x_1,\bar{x}_1) \Phi^{(2)}_{j,j'}(x_2,\bar{x}_2) \rangle_\lambda = -\frac{(t_a \otimes \mathbb{I}_{R'})_{ii',jj'} - \lambda_0 \lambda (\mathbb{I}_R \otimes \tilde{t}_a^*)_{ii',jj'}}{\sqrt{k_L(1-\lambda^2)}\, x_{12}^{2\Delta_R - 1} \bar{x}_{12}^{2\bar{\Delta}_{R'}} x_{13} x_{23}} . \tag{3.12}$$

Similar reasoning leads to

$$\langle \bar{J}_a(\bar{x}_3) \Phi^{(1)}_{i,i'}(x_1,\bar{x}_1) \Phi^{(2)}_{j,j'}(x_2,\bar{x}_2) \rangle_\lambda = \frac{(\mathbb{I}_R \otimes \tilde{t}_a^*)_{ii',jj'} - \lambda_0^{-1} \lambda (t_a \otimes \mathbb{I}_{R'})_{ii',jj'}}{\sqrt{k_R(1-\lambda^2)}\, x_{12}^{2\Delta_R} \bar{x}_{12}^{2\bar{\Delta}_{R'} - 1} \bar{x}_{13} \bar{x}_{23}} . \tag{3.13}$$

4. OPEs and Poisson structure

Employing the two- and three-point functions of the currents and affine primaries found in the previous sections one may derive the OPE at leading order in the $1/\sqrt{k_{L,R}}$ expansion and exact in the deformation parameter λ

$$
\begin{aligned}
J_a(x_1)J_b(x_2) &= \frac{\delta_{ab}}{x_{12}^2} + a_L \frac{f_{abc}J_c(x_2)}{x_{12}} + b_L \frac{f_{abc}\bar{J}_c(\bar{x}_2)\bar{x}_{12}}{x_{12}^2} + \dots ,\\
\bar{J}_a(\bar{x}_1)\bar{J}_b(\bar{x}_2) &= \frac{\delta_{ab}}{\bar{x}_{12}^2} + a_R \frac{f_{abc}\bar{J}_c(\bar{x}_2)}{\bar{x}_{12}} + b_R \frac{f_{abc}J_c(x_2)x_{12}}{\bar{x}_{12}^2} + \dots ,\\
J_a(x_1)\bar{J}_b(\bar{x}_2) &= b_R \frac{f_{abc}\bar{J}_c(\bar{x}_2)}{x_{12}} + b_L \frac{f_{abc}J_c(x_2)}{\bar{x}_{12}} + \dots ,\\
J_a(x_1)\Phi^{(1)}_{i,i'}(x_2,\bar{x}_2) &= -\frac{(t_a)_i{}^m\Phi^{(1)}_{m,i'}(x_2,\bar{x}_2) - \lambda_0\lambda(\tilde{t}_a^*)_{i'}{}^{m'}\Phi^{(1)}_{i,m'}(x_2,\bar{x}_2)}{x_{12}\sqrt{k_L(1-\lambda^2)}} + \dots ,\\
\bar{J}_a(\bar{x}_1)\Phi^{(1)}_{i,i'}(x_2,\bar{x}_2) &= \frac{(\tilde{t}_a^*)_{i'}{}^{m'}\Phi^{(1)}_{i,m'}(x_2,\bar{x}_2) - \lambda_0^{-1}\lambda(t_a)_i^m\Phi^{(1)}_{m,i'}(x_2,\bar{x}_2)}{\bar{x}_{12}\sqrt{k_R(1-\lambda^2)}} + \dots ,\\
\Phi^{(1)}_I(x_1,\bar{x}_1)\Phi^{(2)}_J(x_2,\bar{x}_2) &= C_{IJK}\,\Phi^{(3)}_K(x_2,\bar{x}_2) + \dots ,
\end{aligned}
\tag{4.1}
$$

where C_{IJK} are the structure (numerical) constants of the affine primaries ring, and the various constants are given by

$$
a_L = \frac{1-\lambda_0\lambda^3}{\sqrt{k_L(1-\lambda^2)^3}} , \qquad b_L = \frac{\lambda(\lambda_0^{-1}-\lambda)}{\sqrt{k_R(1-\lambda^2)^3}} ,
$$
$$
a_R = \frac{1-\lambda_0^{-1}\lambda^3}{\sqrt{k_R(1-\lambda^2)^3}} , \qquad b_R = \frac{\lambda(\lambda_0-\lambda)}{\sqrt{k_L(1-\lambda^2)^3}} .
$$

Using the above relations, we can evaluate the equal-time commutators of the currents and affine primaries via the time-ordered limiting procedure

$$
[F(\sigma_1,\tau),G(\sigma_2,\tau)] = \lim_{\varepsilon\to 0}\left(F(\sigma_1,\tau+\varepsilon)G(\sigma_2,\tau) - G(\sigma_2,\tau+\varepsilon)F(\sigma_1,\tau)\right)
$$

and the limit representations of Dirac delta-function distribution

$$
\begin{aligned}
&\lim_{\varepsilon\to 0}\left(\frac{1}{\sigma-i\varepsilon} - \frac{1}{\sigma+i\varepsilon}\right) = 2\pi i\,\delta_\sigma ,\\
&\lim_{\varepsilon\to 0}\left(\frac{\sigma+i\varepsilon}{(\sigma-i\varepsilon)^2} - \frac{\sigma-i\varepsilon}{(\sigma+i\varepsilon)^2}\right) = 2\pi i\,\delta_\sigma ,\\
&\lim_{\varepsilon\to 0}\left(\frac{1}{(\sigma-i\varepsilon)^2} - \frac{1}{(\sigma+i\varepsilon)^2}\right) = -2\pi i\,\delta'_\sigma ,
\end{aligned}
$$

where $\delta_\sigma := \delta(\sigma)$.

Using the above $\mathcal{O}(1/\sqrt{k_{L,R}})$, we find for the currents

$$
\begin{aligned}
[J_a(\sigma_1),J_b(\sigma_2)] &= 2\pi\, i\,\delta_{ab}\delta'_{12} + 2\pi\, f_{abc}\left(a_L J_c(\sigma_2) - b_L\bar{J}_c(\sigma_2)\right)\delta_{12} ,\\
[\bar{J}_a(\sigma_1),\bar{J}_b(\sigma_2)] &= -2\pi\, i\,\delta_{ab}\delta'_{12} + 2\pi\, f_{abc}\left(a_R\bar{J}_c(\sigma_2) - b_R J_c(\sigma_2)\right)\delta_{12} ,\\
[J_a(\sigma_1),\bar{J}_b(\sigma_2)] &= 2\pi\, f_{abc}\left(b_L J_c(\sigma_2) + b_R\bar{J}_c(\sigma_2)\right)\delta_{12} ,
\end{aligned}
\tag{4.2}
$$

and for the primaries

$$
\begin{aligned}
&[\Phi^{(1)}_{i,i'}(\sigma_1), \Phi^{(2)}_{j,j'}(\sigma_2)] = 0\,, \\
&[J_a(\sigma_1), \Phi^{(1)}_{i,i'}(\sigma_2)] = -\frac{2\pi}{\sqrt{k_L(1-\lambda^2)}}\Big((t_a)_i{}^m\,\Phi^{(1)}_{m,i'}(\sigma_2) - \lambda_0\lambda(\tilde{t}^*_a)_{i'}{}^{m'}\,\Phi^{(1)}_{i,m'}(\sigma_2)\Big)\,\delta(\sigma_{12})\,, \\
&[\bar{J}_a(\sigma_1), \Phi^{(1)}_{i,i'}(\sigma_2)] = \frac{2\pi}{\sqrt{k_R(1-\lambda^2)}}\Big((\tilde{t}^*_a)_{i'}{}^{m'}\,\Phi^{(1)}_{i,m'}(\sigma_2) - \lambda_0^{-1}\lambda(t_a)_i{}^m\,\Phi^{(1)}_{m,i'}(\sigma_2)\Big)\,\delta(\sigma_{12})\,.
\end{aligned}
\tag{4.3}
$$

Next, we take the classical limit of (4.2). The result is the two-parameter deformation of the Poisson brackets for the isotropic PCM [40] (in our conventions f_{abc} are imaginary)

$$
\boxed{
\begin{aligned}
&\{I^a_\pm(\sigma_1), I^b_\pm(\sigma_2)\}_{\text{P.B.}} \\
&\quad = -i\,\mathrm{e}^2 f_{abc}\Big((1\pm\rho)I^c_\mp(\sigma_2) - (1\mp\rho+2x(1\pm\rho))I^c_\pm(\sigma_2)\Big)\,\delta_{12} \pm 2\mathrm{e}^2\delta_{ab}\,\delta'_{12}\,, \\
&\{I^a_\pm(\sigma_1), I^b_\mp(\sigma_2)\}_{\text{P.B.}} = i\,\mathrm{e}^2 f_{abc}\Big((1+\rho)I^c_+(\sigma_2) + (1-\rho)I^c_-(\sigma_2)\Big)\,\delta_{12}\,,
\end{aligned}
}
\tag{4.4}
$$

where we have rescaled the currents as

$$
J_a \mapsto -\frac{1}{\mathrm{e}} I^a_+\,, \qquad \bar{J}_a \mapsto -\frac{1}{\mathrm{e}} I^a_-\,,
$$

and the various parameters are

$$
\begin{aligned}
&\mathrm{e}^2 = 4b_R^2(1-\rho)^{-2} = 4b_L^2(1+\rho)^{-2} = \frac{(\lambda_0^{1/2}+\lambda_0^{-1/2})^2}{\sqrt{k_L k_R}}\,\frac{\lambda^2}{(1-\lambda)(1+\lambda)^3}\,, \\
&x = \frac{1+\lambda^2}{2\lambda}\,, \qquad \rho = \frac{(1-\lambda_0)(1+\lambda)}{(1+\lambda_0)(1-\lambda)}\,.
\end{aligned}
\tag{4.5}
$$

These parameters are invariant under the transformation (2.7) and so is the above algebra. These brackets generalize Rajeev's extension [41] of the Poisson structure of the isotropic PCM.

It is interesting to note that these Poisson brackets are isomorphic to two commuting current algebras with levels $k_{L,R}$

$$
\boxed{\{\Sigma^a_\pm(\sigma_1), \Sigma^b_\pm(\sigma_2)\}_{\text{P.B.}} = -i\, f_{abc}\,\Sigma^c_\pm(\sigma_2)\delta_{12} \pm \frac{k_{L,R}}{2}\delta_{ab}\delta'_{12}}\,,
\tag{4.6}
$$

where

$$
\Sigma^a_\pm = \frac{k_{L,R}}{4}\big((1-\lambda)(1\pm\rho)+2\lambda\big)\Big(I^a_\mp - \frac{1}{\lambda}I^a_\pm\Big)\,.
$$

Note that in this decoupling form the symmetry (2.7) simply interchanges the two Poisson brackets in the algebra (4.6) since using the above basis change we see that $\Sigma_\pm \mapsto \Sigma_\mp$. Although the parameter λ does not appear in the algebra (4.6), we expect that the effective Hamiltonian expressed in terms of $\Sigma^a_\pm$ depends on λ, as in the isotropic case. This decoupled form of the algebra (4.6) has also been observed in [42].

5. Conclusions

In this work we investigate λ-deformed current theories based on a general semi-simple group but with a left–right asymmetry induced by the different levels in the left and right sectors of the theory. These left–right asymmetric theories are very interesting for several reasons. They possess a new non-trivial fixed point compared to the left–right symmetric case and there is a smooth RG flow form the undeformed current algebra theory in the UV to a new CFT in the IR (see, (2.32)), on the nature of which we gave strong arguments. To do so by computing the all-loop anomalous dimensions of the left and right currents, that of primary field operators in the aforementioned theories, as well as the exact in λ three-point functions involving currents and/or primary fields. Our computational method introduced in [27] and further developed in [28] combines low order perturbative results with symmetry arguments. The expressions for the aforementioned correlators allowed us to deduce the exact, in the deformation parameter, OPEs of the operators involved and from these OPEs their equal-time commutators. We have found that the associated currents' Poisson brackets are a two-parameter deformation of the canonical structure of the isotropic PCM which was found by purely classical algebraic methods [40]. Upon a suitable change of basis, this Poisson bracket structure is isomorphic to two commuting current algebras with levels $k_{L,R}$. In the isotropic case $k_L = k_R$, this Poisson structure was found to coincide with Rajeev's one-parameter family (with the parameter $\rho = 0$ in (4.4)) of the deformed Poisson brackets of the isotropic PCM [28] and is the canonical structure of the integrable λ-deformed σ-model [5]. It would be interesting to discover an effective action of the anisotropic non-Abelian Thirring model action as well.

Acknowledgements

K. Siampos' work was partially supported by the *Germaine de Staël* Franco-Swiss bilateral program 2015 (project No. 32753SG). K. Siampos and K. Sfetsos would like to thank the TH-Unit at CERN for hospitality and financial support during the final stages of this project.

Appendix A. Useful integrals

In this appendix we collect all the integrals which are needed in the calculations and are evaluated by the use of Stokes' theorem in the complex plane

$$\int_D \mathrm{d}^2 z\,(\partial_z U + \partial_{\bar z} V) = \frac{i}{2}\oint_{\partial D} (U\,\mathrm{d}\bar z - V\,\mathrm{d}z)\ . \tag{A.1}$$

In our regularization scheme we keep a small distance regulator ε when two points coincide (for further details on our regularization scheme the interested reader can consult [27,28])

$$\int \frac{\mathrm{d}^2 z}{(x_1 - z)(\bar z - \bar x_2)} = \pi \ln |x_{12}|^2\,, \qquad \int \frac{\mathrm{d}^2 z}{(x_1 - z)(\bar z - \bar x_1)} = \pi \ln \varepsilon^2\,, \tag{A.2}$$

$$\int \frac{\mathrm{d}^2 z}{(x_1 - z)^2(\bar z - \bar x_2)} = -\frac{\pi}{x_{12}}\,, \qquad \int \frac{\mathrm{d}^2 z}{(x_1 - z)(\bar z - \bar x_2)^2} = -\frac{\pi}{\bar x_{12}}\,, \tag{A.3}$$

$$\int \frac{\mathrm{d}^2 z}{(x_1 - z)^2(\bar z - \bar x_2)^2} = 0\,, \tag{A.4}$$

$$\int \frac{\mathrm{d}^2 z}{(z-x_1)(\bar{z}-\bar{x}_2)^2} \ln|z-x_1|^2 = \frac{\pi}{\bar{x}_{12}} \ln|x_{12}|^2 , \tag{A.5}$$

$$\int \frac{\mathrm{d}^2 z}{(z-x_1)^2(\bar{z}-\bar{x}_2)^2} \ln|z-x_1|^2 = \frac{\pi}{|x_{12}|^2} , \tag{A.6}$$

$$\int \frac{\mathrm{d}^2 z}{(z-x_2)^2(\bar{z}-\bar{x}_3)^2} \ln|z-x_1|^2 = \frac{\pi}{\bar{x}_{13}} \left(\frac{1}{x_{23}} - \frac{\bar{x}_{12}}{x_{12}\bar{x}_{23}} \right) , \tag{A.7}$$

$$\int \frac{\mathrm{d}^2 z}{(z-x_1)(\bar{z}-\bar{x}_1)^2} = 0 . \tag{A.8}$$

Appendix B. Current anomalous dimension at four-loop

In this appendix we would like to sketch the four-loop computation of the current operator anomalous dimension at leading order in the large $k_{L,R}$ expansion; denoted as $\gamma_L^{(4)}$. To proceed, we evaluate the four-loop contribution to the two-point function

$$\begin{aligned} \langle J_a(x_1) J_b(x_2) \rangle_\lambda^{(4)} = \frac{\lambda^4}{4!\pi^4} \int \mathrm{d}^2 z_{1234} \, &\langle J_a(x_1) J_b(x_2) J_{a_1}(z_1) J_{a_2}(z_2) J_{a_3}(z_3) J_{a_4}(z_4) \rangle \\ &\times \langle \bar{J}_{a_1}(\bar{z}_1) \bar{J}_{a_2}(\bar{z}_2) \bar{J}_{a_3}(\bar{z}_3) \bar{J}_{a_4}(\bar{z}_4) \rangle . \end{aligned} \tag{B.1}$$

Employing the Ward identity (1.4) it is clear that the $\mathcal{O}(\lambda^4)$ the current anomalous dimension is of the form

$$\gamma_L^{(4)} = c_G \lambda^4 \left(\frac{\alpha_1}{k_L} + \frac{\alpha_2}{k_R} \right) , \qquad \alpha_1 + \alpha_2 = 4 , \tag{B.2}$$

where the last condition is such that we match the four-loop contribution the left–right symmetric case when $k_L = k_R = k$ which was computed in [28]. Hence, it is enough to find the contribution of the $1/k_R$-term. Hence,

$$\begin{aligned} \big\langle J_a(x_1) J_b(x_2) \big\rangle_\lambda^{(4)} \big|_{1/k_R-\text{term}} &= \frac{\lambda^4}{4!\pi^4 k_R} \int \mathrm{d}^2 z_{1234} \left(\frac{f_{a_1 a_3 e} f_{a_2 a_4 e}}{\bar{z}_{12}\bar{z}_{13}\bar{z}_{24}\bar{z}_{34}} - \frac{f_{a_1 a_4 e} f_{a_2 a_3 e}}{\bar{z}_{12}\bar{z}_{14}\bar{z}_{23}\bar{z}_{34}} \right) \\ &\times \left(\frac{\delta_{a a_1}}{(x_1 - z_1)^2} \big\langle J_b(x_2) J_{a_2}(z_2) J_{a_3}(z_3) J_{a_4}(z_4) \big\rangle \right. \\ &+ \frac{\delta_{a a_2}}{(x_1 - z_2)^2} \big\langle J_b(x_2) J_{a_2}(z_1) J_{a_3}(z_3) J_{a_4}(z_4) \big\rangle \\ &+ \left. \frac{\delta_{a a_3}}{(x_1 - z_3)^2} \big\langle J_b(x_2) J_{a_1}(z_1) J_{a_2}(z_2) J_{a_4}(z_4) \big\rangle + \frac{\delta_{a a_4}}{(x_1 - z_4)^2} \big\langle J_b(x_2) J_{a_1}(z_1) J_{a_2}(z_2) J_{a_3}(z_3) \big\rangle \right) , \end{aligned} \tag{B.3}$$

where we have disregarded a bubble term. The latter expression can be rewritten symbolically as

$$\langle J_a(x_1) J_b(x_2) \rangle_\lambda^{(4)} \Big|_{1/k_R-\text{term}} = \frac{\lambda^4}{4!\pi^4 k_R} \int \mathrm{d}^2 z_{1234} \, (I - II) \times (B + C + D + E) , \tag{B.4}$$

where we consider only the Abelian part in the holomorphic four-point function. To evaluate (B.4) it is a lengthy but straightforward computation which lays upon using the point splitting formula

$$\frac{1}{(x_1 - z)(z - x_2)} = \frac{1}{x_{12}} \left(\frac{1}{x_1 - z} + \frac{1}{z - x_2} \right) , \tag{B.5}$$

and the formulae of Appendix A. In addition, in accordance to our regularization scheme we follow a specific order in performing the integrations which we never violate, that is first the z_4 integration, then the z_3 one and so on and so forth.

Among all possible terms, let us consider one of the terms in (B.4)

$$\begin{aligned}\int d^2 z_{1234}\, I \times B &= \frac{1}{k_R}\int \frac{d^2 z_{1234}}{(x_1-z_1)^2}\langle J_b(x_2)J_{a_2}(z_2)J_{a_3}(z_3)J_{a_4}(z_4)\rangle \frac{f_{aa_3e}f_{a_2a_4e}}{\bar z_{12}\bar z_{13}\bar z_{24}\bar z_{34}}\\ &= -\frac{c_G\delta_{ab}}{k_R}\int \frac{d^2 z_{1234}}{(z_1-x_1)^2\bar z_{12}\bar z_{13}\bar z_{24}\bar z_{34}}\left(\frac{1}{(z_2-x_2)^2 z_{34}^2}-\frac{1}{(z_4-x_2)^2 z_{23}^2}\right)\\ \implies \quad \int d^2 z_{1234}\, I\times B &= -\frac{c_G\delta_{ab}}{k_R}\left[(\mathrm{i})-(\mathrm{ii})\right],\end{aligned} \tag{B.6}$$

with the apparent symbolic expression for the terms (i) and (ii). To evaluate the term denoted by (i) in (B.6), we integrate over z_4 and employ (B.5), (A.3), (A.8)

$$(\mathrm{i}) = \pi\int \frac{d^2 z_{123}}{(z_1-x_1)^2(z_2-x_2)^2\bar z_{12}\bar z_{13}\bar z_{23}z_{23}}\,. \tag{B.7}$$

Then we integrate over z_3 and use (B.5), (A.2)

$$(\mathrm{i}) = \pi^2\int d^2 z_{12}\,\frac{\ln\frac{|z_{12}|^2}{\varepsilon^2}}{(z_1-x_1)^2(z_2-x_2)^2\bar z_{12}^2}\,, \tag{B.8}$$

next we integrate over z_2 and apply (A.4), (A.6)

$$(\mathrm{i}) = \pi^3\int d^2 z_1\,\frac{1}{(z_1-x_1)^2|z_1-x_2|^2}\,. \tag{B.9}$$

Finally we integrate over z_1 and employ (B.5), (A.2), (A.3)

$$(\mathrm{i}) = -\frac{\pi^4}{x_{12}^2}\left(1+\ln\frac{\varepsilon^2}{|x_{12}|^2}\right). \tag{B.10}$$

Similarly we evaluate the term (ii) in (B.6) to find that

$$(\mathrm{ii}) = \frac{2\pi^4}{x_{12}^2}\left(1+\ln\frac{\varepsilon^2}{|x_{12}|^2}\right). \tag{B.11}$$

Plugging (B.10) and (B.11) into (B.6) we find

$$\int d^2 z_{1234}\, I\times B = \frac{c_G\delta_{ab}}{k_R}\frac{3\pi^4}{x_{12}^2}\left(1+\ln\frac{\varepsilon^2}{|x_{12}|^2}\right). \tag{B.12}$$

Working in a similar manner leads to

$$
\begin{aligned}
&\int d^2 z_{1234}\, I \times B = -\int d^2 z_{1234}\, II \times B = \frac{c_G \delta_{ab}}{k_R} \frac{3\pi^4}{x_{12}^2}\left(1 + \ln\frac{\varepsilon^2}{|x_{12}|^2}\right), \\
&\int d^2 z_{1234}\, I \times C = -\int d^2 z_{1234}\, II \times C = \frac{c_G \delta_{ab}}{k_R} \frac{\pi^4}{x_{12}^2}\left(1 + 3\ln\frac{\varepsilon^2}{|x_{12}|^2}\right), \\
&\int d^2 z_{1234}\, I \times D = -\int d^2 z_{1234}\, II \times E = \frac{c_G \delta_{ab}}{k_R} \frac{3\pi^4}{x_{12}^2} \ln\frac{\varepsilon^2}{|x_{12}|^2}, \\
&\int d^2 z_{1234}\, I \times E = -\int d^2 z_{1234}\, II \times D = \frac{c_G \delta_{ab}}{k_R} \frac{\pi^4}{x_{12}^2}\left(2 + 3\ln\frac{\varepsilon^2}{|x_{12}|^2}\right).
\end{aligned}
\tag{B.13}
$$

Plugging (B.13) into (B.4) we find that

$$
\langle J_a(x_1) J_b(x_2)\rangle_\lambda^{(4)}\Big|_{1/k_R-\text{term}} = \frac{c_G \delta_{ab}}{k_R\, x_{12}^2}\left(\frac{1}{2} + \ln\frac{\varepsilon^2}{|x_{12}|^2}\right), \tag{B.14}
$$

so $\alpha_2 = 1$ and therefore $\alpha_1 = 3$ in (B.2). Putting all these together we find that

$$
\gamma_L^{(4)} = c_G \lambda^4 \left(\frac{3}{k_L} + \frac{1}{k_R}\right), \tag{B.15}
$$

which is consistent with the $\mathcal{O}(\lambda^4)$ term in the expansion of (2.9).

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