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MetZet Classic Papers

Stationary flow near fronts

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Abstract
In 1906, the Austrian scientist Max Margules published a paper on temperature stratification in resting and non-accelerated moving air. The paper derives conditions for stationary slopes of air mass boundaries and was an important forerunner of frontal theories. Its formulation of relations between changes in density and geostrophic wind across the front is basically a discrete version of the thermal wind balance equation. The paper was highly influential and is still being cited to the present day. This paper accompanies an English translation of Margules' seminal paper. We conclude here our "Classic Papers" series of the *Meteorologische Zeitschrift*.

Keywords: history of meteorology, thermal wind, fronts

1 Introduction

The *Meteorologische Zeitschrift* celebrates its 150th anniversary this year in 2016 (see Emeis, 2008). In 1866, the journal of the Austrian Meteorological Society published its first volume, with Julius Hann and Karl Jelinek as editors. Two decades later, the journal became the *Meteorologische Zeitschrift* after merging with the publication of the German Meteorological Society. In 1906, in honor of the 40-year anniversary of the editorship of Julius Hann, a special volume of the *Meteorologische Zeitschrift* appeared. The volume, known as the "Hann Volume", comprised works of the most renowned meteorologists of that time, including Georg von Neumayer, Alfred Angot, Leon Teisserenc de Bort, Cleveland Abbe, Felix Exner, Richard Assmann, Wladimir Köppen, Josef Maria Pernter, Gustav Hellmann and others. Among the published papers was one by Max Margules, entitled "Über Temperaturschichtung in stationär bewegter und in ruhender Luft" (Margules, 1906a), a translated version of which is published in this issue. Here, we comment on that paper and put it into a historical and present-day context.

Max Margules is considered as one of the founding fathers of theoretical meteorology, and his paper, which formulates the thermal-wind equation in discrete form for the flow near fronts, has found its way into fluid dynamics textbooks in the atmospheric and oceanic sciences. We therefore have chosen this paper to conclude our Classic Papers series, which started in 2009 (Brönnimann, 2009). In consideration of the 150th anniversary of the Austrian precursor journal, it is apt to end the series with a paper that appeared in the "Hann Volume" on the occasion of the 40 year anniversary of the journal, and with a paper written by an Austrian meteorologist.

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In the following, we briefly provide some biographical notes on Max Margules. The "pre-frontal" era of frontal research is then sketched, after which the 1906 paper is introduced. We then discuss the subsequent development of frontal theories, and end with some brief conclusions.

2 Biographical sketch

Max Margules (Fig. 1) was born on 23 April 1856 in Brody, Galicia, at the very east of the Austro-Hungarian Empire (now Ukraine) and grew up in the Jewish community of that city, where he received his basic education (see Höflechner, 2002, for the following). He went on to study Mathematics, Physics and Chemistry at the University of Vienna and obtained his doctorate in 1876. His work dealt mainly with electro- and hydrodynamics. After an assistantship at the "Zentralanstalt für Meteorologie und Geodynamik" (ZAMG, the Austrian Weather Service), he continued his studies at the University of Berlin with a fellowship. From Berlin, he applied for his habilitation in Vienna and obtained the *Venia Legendi* (permission to lecture) in 1880. He planned (and partly held) lectures on various topics, but relinquished his *Venia* in 1882 to return to the ZAMG, where he stayed until his early retirement in 1906. It seems that Margules did not prosper in the academic environment in Vienna as there was no one who could have served as a mentor (Stefan was primarily at the Academy, Boltzmann was in Graz and later Munich, Lang worked on other topics, and Exner was arguably too young).

Margules stayed at the ZAMG for 24 years, where most of Margules' papers on meteorology date from this period. He was promoted from assistant to adjunct, and then to secretary. In his spare time, he continued to work on broader physical and chemical topics. Margules retired prematurely in 1906 (at the age of 50)

Figure 1: Photograph of Max Margules (source: ZAMG).

from service. One of the reasons might have been disagreements with the director of the Institute, while another suggestion is that he did not feel sufficiently recognized by the community (Wisniak, 2003).

During his whole life, Margules was a loner and filled with bitterness. After retirement, Margules moved to Perchtoldsdorf in the South of Vienna, and lived from a modest pension. But in the troublesome years after the First World War, his pension subsided. Margules refused to accept any support from friends or the Austrian Meteorological Society. Even when awarded the Hann medal in 1919, he accepted the honour but not the prize. Eventually, Margules starved to death, in a sense deliberately, as Exner expressed in an obituary (Exner, 1920). He died on 4 October 1920 in Perchtoldsdorf.

Margules was one of the founders of theoretical meteorology, a forerunner of the cyclone theories that followed, and one of the most influential meteorologists of the early twentieth century. Coming from electrodynamics, he was interested in fluid dynamics and, specifically, oscillations in fluids and air, motions on rotating spheriods, and later energetic considerations, namely the energy of storms. Between 1890 and 1895, he wrote a number of influential papers on these subjects, providing a new perspective of thermodynamics and mechanics of the Earth's atmosphere. At the same time, he also worked on physico-chemical topics. In fact, he (strictly) worked as a meteorologist in the morning and a physicist in the afternoon. In 1895, he found (independent of Gibbs and Duhem) the relation between partial pressure and composition of a mixture of two liquids (Duhem-Margules-Equation, Margules, 1895).

Margules wrote several landmark papers on thermodynamics and mechanics of the atmosphere, for instance, on atmospheric tides, and he formulated a theory of the polar front and for pressure waves. In 1904, he published a paper on predicting pressure using the continuity equation (Margules, 1904b, see Lynch, 2001, 2003). Particularly well known are his papers on the energetics of weather systems (Margules, 1901; Margules, 1905, Margules, 1906b) and on atmospheric flow near fronts (Margules, 1904a, 1906a). The paper that is translated here in the "Classic Papers" series is one of the latter and appeared in a group of several papers in the year of his retirement (Margules, 1906abc). After retirement, he returned to chemico-physical questions and quit meteorology altogether.

It was only after the First World War, through the achievements of the Norwegian school of meteorology, that the work of Margules became widely recognized in the community (Wisniak, 2003). Today, his work is seen as a foundation of theoretical meteorology.

3 The "pre-frontal" era of front research

As soon as regular meteorological observations became available around the middle of the 19^{th} century, it was realized that the spatial and temporal temperature distribution at the Earth's surface was not always smooth but sometimes abrupt changes could be documented. Such rapid changes of temperature were commonly observed in conjunction with cyclones. Hence, the research of cyclones in the middle and second half of the 19^{th} century was accompanied by the research of fronts, although such discontinuities in the density or temperature field were not called "front" at this time ("pre-frontal" era). The cyclone theory of Dove (Dove, 1828), in particular, assumed two bodies of air with different temperature (and humidity) in which the opposing flows interact through shearing at the center of a barometric minimum.

The presumably first vertical section through a "front" was published by Loomis (1841). Because of the lack of upper-air observations, the front could only have been drawn intuitively, and was used as an argument in support of the cyclone theory of Espy (Kutzbach, 1979).

The availability of regular surface observations covering Europe and the USA after the establishment of national weather services allowed the development of many different operational and experimental forms of surface weather maps. Examples are the maps of Buys-Ballot (Buys-Ballot, 1854), where front-like discontinuities were analyzed with high precision, or the aesthetically appealing colored maps of Fitzroy (Fitzroy, 1863). Toward the end of the 19^{th} century, regular surface weather maps were plotted in which cold fronts, still called “squall lines”, were analyzed with a high degree of detail (see, e.g., DURAND-GREVILLE 1894).

Around 1890, MARGULES became interested in the topic of boundaries between air masses of different densities and carried out a series of his own experimental research campaigns, including one flight with a gas balloon and the setup of a mesonet in the area of Vienna. The mesonet consisted of stations at different altitudes so that he could document situations with significant temperature differences across small vertical increments and sudden changes of wind directions accompanied by temperature “jumps” (MARGULES, 1900). This observational evidence of the existence of surfaces of density/temperature discontinuities, and the general interest of the meteorological community in such phenomena, certainly motivated MARGULES to explain such discontinuities on a physical and mathematical basis and led to the publication of the 1906 paper.

4 The 1906 paper

The 1906 paper on temperature stratification in resting and non-accelerated moving air took as its starting point the papers by HELMHOLTZ (1888ab), who formulated the conditions for stationary flow in the presence of horizontal density gradients in a rotating frame. Consider a situation in which a wedge of cold dense air with density, ρ_c lies below a warm, less dense air with density, ρ_w (Fig. 2). The surface separating the two air masses is inclined. Assume that we have stationary, zonal geostrophic flow u in the x-direction and hydrostatic conditions in both air masses (cf. BLUESTEIN, 1993). The conditions are thus described by the geostrophic equations for zonal flows of different density:

$$u_c = -\frac{1}{\rho_c f}\left.\frac{\partial p}{\partial y}\right|_c ; \quad u_w = -\frac{1}{\rho_w f}\left.\frac{\partial p}{\partial y}\right|_w \quad , \tag{4.1}$$

where f is the Coriolis parameter and p is pressure. The hydrostatic equations for two air densities are

$$\left.\frac{\partial p}{\partial z}\right|_c = -g\rho_c; \quad \left.\frac{\partial p}{\partial z}\right|_w = -g\rho_w \quad , \tag{4.2}$$

where g is acceleration of gravity. It is interesting to note that MARGULES used the then common tradition of meteorology, namely, to define the horizontal component of the pressure gradient contrary to the mathematical definition. Hence, he writes the geostrophic equation (his equation 1) for u with a positive sign, which requires some caution when interpreting his equations further.

Due to the impossibility of infinite pressure gradient forces, pressure must be continuous even at density discontinuities. This implies that, although pressure gradients are discontinuous at density discontinuities, the component of the pressure gradient parallel to the discontinuity must be equal on the dense and less dense side. This is expressed mathematically by the dynamic

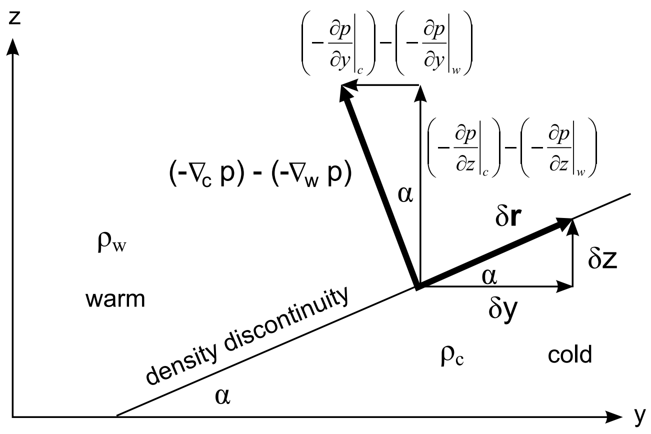

Figure 2: Sketch of a vertical section through two air masses with an inclined density discontinuity. See the text for further explanation.

boundary condition of MARGULS:

$$\begin{aligned}(\nabla_c p - \nabla_w p)\cdot\delta\mathbf{r} = &\left(\left.\frac{\partial p}{\partial y}\right|_c - \left.\frac{\partial p}{\partial y}\right|_w\right)\delta y \\ &+\left(\left.\frac{\partial p}{\partial z}\right|_c - \left.\frac{\partial p}{\partial z}\right|_w\right)\delta z \\ = 0 \quad .\end{aligned} \tag{4.3}$$

Because of the different vertical component of the pressure gradient (hydrostatic equation) on both sides of the discontinuity, $(\nabla_c p - \nabla_w p)$ cannot be zero, but must be perpendicular to the surface of the discontinuity (see Fig. 2). Note that in Fig. 2, the negatives of the gradient vector and its components have been plotted, because in the geostrophic as well as hydrostatic equation, the negative gradient components appear.

The dynamic boundary condition of MARGULES allows derivation of two very important conclusions given that the dense (cold) air can only sit below the less dense (warm) air due to static stability constraints: Firstly, knowing the densities of the two air masses, we can immediately derive from (4.3) that the difference between the horizontal components of the pressure gradients $\left(\left.\frac{\partial p}{\partial y}\right|_c - \left.\frac{\partial p}{\partial y}\right|_w\right)$ must be positive, and hence, the vertical wind shear $(u_w - u_c)/\delta z$ must also be positive. In addition, according to Fig. 2, the horizontal wind shear $(u_c - u_w)/\delta y$ must be negative, which means that we must mandatorily assume a cyclonic horizontal wind shear at a front. Secondly, we can derive the angle of inclination of a front from the density differences and the wind shear across a frontal surface according to

$$\begin{aligned}\tan\alpha = \frac{\delta z}{\delta y} = -\frac{\left.\frac{\partial p}{\partial y}\right|_c - \left.\frac{\partial p}{\partial y}\right|_w}{\left.\frac{\partial p}{\partial z}\right|_c - \left.\frac{\partial p}{\partial z}\right|_w} &= \frac{f}{g}\frac{u_w\rho_w - u_c\rho_c}{\rho_c - \rho_w} \\ &= \frac{f}{g}\frac{u_w T_c - u_c T_w}{T_w - T_c} \quad .\end{aligned} \tag{4.4}$$

MARGULES took typical data from his observations ($T_c = 273\,\mathrm{K}$, $T_w = 283\,\mathrm{K}$, $u_c = 0\,\mathrm{ms^{-1}}$; $u_w = 10\,\mathrm{ms^{-1}}$)

and was obviously pleased to find a very small angle of a little less than a quarter of a degree, or a frontal inclination of 1 km per 237 km. This was in good agreement with his observational findings that the progression of warming can take several hours within a few hundred meters of the surface.

In the second chapter, Margules considers a non-discontinuous frontal volume, where the density contrast is concentrated in a thin layer. One can then differentiate the frontal volume itself and end up with the thermal wind relation

$$\frac{\partial u}{\partial z} = \frac{u}{T}\frac{\partial T}{\partial z} - \frac{g}{fT}\frac{\partial T}{\partial y} \tag{4.5}$$

The first right-hand side term is orders of magnitude smaller than the second, so that with a very high degree of accuracy, one can write the finite difference form

$$\Delta_z u \cong -\frac{g}{f}\frac{\Delta_y T}{\bar{T}}\frac{\Delta z}{\Delta y} = -\frac{g}{f}\frac{\Delta_y T}{\bar{T}}\tan\alpha \quad , \tag{4.6}$$

where $\bar{T}$ is the arithmetic mean temperature of the two air masses. Finally,

$$\tan\alpha \cong \frac{f\bar{T}}{g}\frac{\Delta_z u}{\Delta_y T} \quad , \tag{4.7}$$

which compares well with the formula for a surface of density discontinuity. The inclination of a front increases with the wind shear across the front and decreases with the temperature contrast. This equation is today called the Margules relation based on the 1906 paper. The Margules relation is thus a version of the thermal-wind balance for a layered density.

In his paper, Margules also presents case studies to demonstrate that such conditions, which were otherwise not easily explained, do indeed exist. One has to keep in mind the limited knowledge on upper-level circulation at that time. Although Margules mentions balloons, back then there were almost no routine upper-air observations (see also Brönnimann and Stickler, 2013). A three-dimensional theoretical framework that allowed an improved interpretation of surface observations was therefore a significant achievement.

Margules (1906a) does not stop with the Margules relation. The paper continues by considering the mixing of air masses and by discussing energetic constraints, which ties in with Margules' other works on the energy of weather systems. Margules tried to constrain the energy of a weather system by considering the conversion of potential into kinetic energy. In the 1906 paper, he also specifically discusses the role of atmospheric moisture. He points out that the significantly different gas constants of dry air and water vapor lead to different densities of dry and moist air of the same temperature. This consideration may be seen as an early hint that for static stability considerations and, in general, for atmospheric thermodynamics, the virtual temperature introduced into meteorology by Guldberg and Mohn (1876), is the relevant quantity.

Although Margules is seen as a pioneer of research on atmospheric fronts, the term "front" was introduced into meteorology only shortly after his death. Whereas in the famous publication of the "polar front" model by Bjerknes (1919), the thermal boundary lines, whose inclination was seen in good agreement to Margules' theory, were still denominated by the terms "steering line" and "squall line", the terms cold and warm front were used in 1921 by Bjerknes and Solberg (1921) for the first time. The success of the front concept and its operational application extends to the present, where surface weather maps still contain lines of significant weather along cold, warm and occluded fronts.

It took roughly 50 years after the pioneering work of Margules for new insights into frontal dynamics to be achieved by application of the quasi-geostrophic theory. In this way, frontogenesis and the corresponding secondary circulations at fronts could be described and understood (e.g. Sawyer, 1956, Eliassen, 1962). The true three-dimensional front structure in the troposphere and its relation to jet streams was also discovered at that time (see, e.g., Palmén and Newton, 1969). At a later stage, the sharpness of atmospheric fronts close to a discontinuity could be better understood and modeled by the application of semi-geostrophic theory (e.g. Bluestein, 1993). Frontal analysis on surface weather maps has now been applied for nearly 100 years, a success story hardly surpassed by any other conceptual model in meteorology. Whereas frontal analysis was carried out for long time with the help of a subjective procedure, keeping in mind the basic findings of Margules, e.g., a cyclonic kink in the pressure field, a cyclonic wind shift and a sudden temperature change, fronts today are more commonly located by automatic mathematical procedures (see, e.g., Steinacker, 1992, McKann and Whistler, 2001).

The 1906 paper is today cited in basic meteorological and fluid dynamics textbooks and is recognized as a basis for studying atmospheric fronts. The Margules relation is used in atmospheric sciences as well as oceanography as evidenced by the fact that, on Google Scholar, the 1906 paper has already received 64 citations, 33 of which are since 2000.

5 Conclusions

Max Margules' 1906 paper on temperature stratification in resting and non-accelerated moving air was an important step in the progress of theoretical meteorology, of which Margules was an important actor. The Margules relation remains an important form of the thermal-wind balance that describes geostrophic flow near sloping surfaces of large density gradients (fronts) and, as such, has become textbook knowledge. For Max Margules, this paper (together with two others published in the same year) marked the end – and culmination – of his work on atmospheric mechanics and thermodynamics. Margules was a tragic figure in his time.

With this paper, we conclude the "Classic Papers" series of the *Meteorologische Zeitschrift*. All translated papers as well as the accompanying articles are available (open access) from the journal's website.

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References

Bjerknes, J., 1919: On the structure of moving cyclones. – Geofysiske Publikasjoner **1**, 1–8.

Bjerknes, J., H. Solberg, 1921: Meteorological conditions for the formation of rain. Geofysiske Publikasjoner **2**, 3.

Bluestein, H.B., 1993: Synoptic-Dynamic Meteorology in Mid Latitudes. Vol. II. – Oxford University Press, 594 pp.

Brönnimann, S., 2009: Start of the "Classic Papers" series. – Meteorol. Z. **18**, 339–340.

Brönnimann, S., A. Stickler, 2013: Aerological observations in the Tropics in the Early Twentieth Century. – Meteorol. Z. **22**, 349–358.

Buys-Ballot, C.H.D., 1854: Erläuterungen einer graphischen Methode zur gleichzeitigen Darstellung der Witterungserscheinungen an vielen Orten. – Ann. Phys. Chem., Ergänzungsband 4.

Dove, H.W. 1828: Über mittlere Luftströme. – Ann. Phys. Chem. **13**, 584–585.

Durand-Greville, E., 1894: Le graines et les orages. – Ann. Bur. Centr. Meteor. de France, **1**.

Eliassen, A., 1962: On the vertical circulation on frontal zone. – Geofysiske Publikasjoner **24**, 147–160.

Emeis, S., 2008: History of the Meteorologische Zeitschrift. – Meteorol. Z. **17**, 685–693.

Exner, F., 1920: Max Margules. – Meteorol. Z. **37**, 322–324.

Fitzroy, R., 1863: The weather book. – Longman, Green, London.

Guldberg, C.M., H. Mohn, 1876: Etudes sur les mouvements de l'atmosphère. – Christiania, A.W. Brøgger, 99 pp.

Helmholtz, H., 1888a: Über atmosphärische Bewegungen (I). – Sitzber. Preuss. Akad. Wiss. **5**, 647–663.

Helmholtz, H., 1888b: Über atmosphärische Bewegungen (II). – Meteorol. Z. **5**, 329–340.

Höflechner, W., 2002: Materialien zur Entwicklung der Physik und ihrer „Randfächer" Astronomie und Meteorologie an den österreichischen Universitäten 1752–1938. – Institut für Geschichte der Karl-Franzens-Universität Graz, Graz, http://www-gewi.uni-graz.at/wissg/geschichte_der_physik/.

Kutzbach, G., 1979: The thermal theory of cyclones. – Historical Monograph Series, AMS, Boston, 255 pp.

Loomis, E. 1841: On the storm which was experienced throughout the United States about the 20th of December, 1836. – Trans. Am. Phil. Soc. **7**, 125–163.

Lynch, P., 2001: Max Margules and his tendency equation. – Historical note (Met Éireann), **5**, 18 pp.

Lynch, P., 2003: Margules' Tendency Equation and Richardson's Forecast. – Weather **58**, 186–193.

Margules, M., 1895: Über die Zusammensetzung der gesättigten Dämpfe von Mischungen. – Sitzber. Kais. Akad. Wiss. Wien, Math.-Naturwiss. Klasse II **104**, 1243–1278.

Margules, M., 1900: Temperaturstufen in Niederösterreich im Winter 1898/99. Anhang zum Jahrbuch der Central-Anstalt für Meteorologie 1899, **26**, Wien.

Margules, M., 1901: Über den Arbeitswert einer Luftdruckverteilung und die Erhaltung der Druckunterschiede. – Densksch. Kais. Akad. Wiss. Wien, Math.-Naturwiss. Klasse II **73**, 329–345.

Margules, M., 1904a: Luftbewegungen in einer rotierenden Sphäroidschale. Sitzungsber. Kais. Akad. Wiss. Wien, IIA **102**, 11–56.

Margules, M., 1904b: Über die Beziehung zwischen Barometerschwankungen und Kontinuitätsgleichung. – Ludwig Boltzmann Festschrift, Leipzig, J.A. Berth, p. 585–589.

Margules, M., 1905: Über die Energie der Stürme. – Jahrbuch der k. k. Centralanstalt für Meteorol. und Erdmagnetismus in Wien 1903, Appendix, 20 pp.

Margules, M., 1906a: Über Temperaturschichtung in stationär bewegter und in ruhender Luft (On temperature stratification in resting and non-accelerated moving air). – Meteorol. Z., Hann-Volume, 243–254. (translated and edited by Volken, E., A.M. Giesche, and S. Brönnimann. – Meteorol. Z. **25** (2016), DOI:10.1127/metz/2016/0833).

Margules, M., 1906b: Zur Sturmtheorie. – Meteorol. Z. **23**, 481–497.

Margules, M., 1906c: Über die Änderung des vertikalen Temperaturgefälles durch Zusammendrückung oder Ausbreitung einer Luftmasse. – Meteorol. Z. **23**, 241–244.

McKann, D.W., J.P. Whistler, 2001: Problems and solutions for drawing fronts objectively. – Meteorol. Appl. **8**, 195–203

Palmén, E., C.W. Newton, 1969: Atmospheric Circulation Systems. – Academic Press, New York.

Sawyer, J.S., 1956: The vertical circulation at meteorological fronts and its relation to frontogenesis. – Proc. Roy. Soc. **A234**, 346–362.

Steinacker, R., 1992: Dynamical aspects of frontal analysis. – Meteorol. Atmos. Phys. **48**, 93–103

Wisniak, J., 2003: Max Margules—A cocktail of meteorology and thermodynamics. – J. Phase Equilibria **24**, 103–109.