

Inflation Differentials in a Currency Area: Facts, Explanations and Policy

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Abstract The determinants of inflation differentials in a currency area are analyzed both from an empirical and a theoretical perspective. The empirical analysis shows that a sizeable dispersion of HICP inflation rates across euro-area countries arises mostly in the components based on non-traded goods. There is also a significant cross-country heterogeneity in the response to changes in a common latent factor which accounts for a large fraction of the dispersion in national inflation rates. A stylized model of a currency area is used to understand the interrelation among shocks, structures and policies in driving the data generating process. The model shows that the dynamic of the inflation differentials is largely driven by the variability of productivity in the non-tradable sector of the more flexible economy. Conversely, the output differentials is largely driven by the variability in the productivity of

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the tradable sector, also of the more flexible economy. Optimal policy is investigated together with an analysis of the optimal adjustment to adopting a common currency with initial incorrect real exchange rate parity.

Keywords Inflation dynamics · Monetary union · Price dispersion

1 Introduction

The analysis of price and inflation differentials in a currency area has attracted the attention of a considerable number of researchers in the recent past years.¹ Recent research was to a large extent spurred by the experience with the major change in the exchange rate regime and the implications for price formation that were triggered by the formation of the European Monetary Union. The initial years of EMU coincided with a change in the trend of inflation dispersion across European countries. In the run-up to monetary union, inflation rates across member countries had largely converged. Since the year 2000 a sizeable degree of dispersion across national inflation rates in euro area has been at times observed. The initial interpretation in the literature of such change in trend was to appeal to the Balassa-Samuelson effect. An explanation based on the Balassa-Samuelson effect seemed valid, taking into account in particular that national inflation rates were highest in the fastest-growing economies such as Ireland, Portugal and Spain. However, this narrow focus on a single explanatory factors was gradually superseded by broader explanations² in which the observed inflation differentials were accounted for by more complex interactions among three main factors: heterogeneity in structures, common and idiosyncratic shocks from both supply and demand sides, and the role of monetary policy. This paper analyses how these three dimensions interact to generate persistent differences in national inflation rates inside a monetary union and to draw possible implications for policy.

We first present evidence on the presence on dispersion in overall inflation rates and changes in CPI components based on the experience of European countries in the EMU. Our descriptive exercise shows that there has been a sizeable dispersion of HICP inflation rates across euro area countries. In a sectorial decomposition of this dispersion, we find that most of it occurs in the Service category in the HICP, even if also the Energy category has been a relevant source in some periods. This suggests that most of the sources of dispersion are in the components of the HICP that are more intensely based on non-traded goods. We then use a dynamic factor model to decompose the aggregate and sectorial dispersions between a common component driven by common factors and an idiosyncratic component. The heterogeneous response across countries to a change in the common factor can account for large

¹Our bibliography collects most of the recent and past works.

²See Blanchard (2001) for an early discussion in this respect.

fraction of the dispersion in national inflation rates. This is particularly the case for the Industrial good categories, even though the cross-country dispersion of the inflation rates of items in this category is quite low. However, it also applies to the Services categories, which is instead characterized by a high level of dispersion.

Our interpretation of the data generating process is the following. Countries in a currency area might have structural differences: price-setting mechanism, degree of competition in the goods and labour markets, preferences and technologies and others. Moreover, policy-makers might interact in this environment by setting their policy instruments; countries might experience idiosyncratic or common shocks. The interrelation between shocks, structures and policies drive the data generating process and help to explain which factors can be treated as common or idiosyncratic.

We build a stylized model of a currency area to aim at disentangling which shocks, structures and policies are important in explaining the qualitative features of the data. The model assumes two productive sectors, a traded and a non-traded goods sector, in each economy, with the price of traded goods across countries assumed equal from the law of one price. Thus, consumer price and inflation differentials arise in the model as a consequence of movements in the relative prices of non-traded goods.

Some noteworthy aspects of the theoretical model are noted: The model allows for the possibility of imperfect labour mobility across sectors. For this, imperfect substitutability between different types of sector-specific labour is introduced. This labour market friction generates wage differentials across sectors, which is a desirable feature from an empirical perspective (e.g. Ortega 2003) and introduces an amplifying effect on the relative price movements in response to sectorial shocks. However, regional mobility is not allowed in the model, even over the long run.

The model also incorporates market power in the traded goods sector. As has been recently pointed out in the literature (Benigno and Thoenissen 2003), a positive productivity gain in the tradable good sector has two counteracting effects on the overall price level, with upward pressure on the relative price of non-tradables offset by a decline in the terms of trade. Which effect dominates depends on the relative values of the elasticities of substitution between home and foreign tradables and between tradables and non-tradables and the strength of nominal rigidities.

In addition to supply-side disturbances, the model considers various fiscal shocks as well as shocks to the level of sectorial competition (as expressed by a markup, which in turn depends on the elasticity of substitution across varieties). An additional type of wedge between costs and prices is incorporated into the model with sector-specific ad valorem indirect taxes.

In this set up we consider both the flexible and rigid prices cases. The flexible-price version of the model by construction cannot capture any dynamic beyond the one implied by the shocks. However we can describe the long-run properties of the model. Contrary to the Balassa-Samuelson argument, we show analytically that an unbalanced country-specific productivity shock in the

traded sector is almost irrelevant for explaining consumer price differential. Instead, a balanced increase in productivity in one country has a negative and sizeable impact on the consumer price of the country relative to the rest of the area. Asymmetric demand shocks, driven by government purchases, can be also an important source of consumer price dispersion. There is otherwise no role for monetary policy or for area-wide shocks in explaining price dispersion.

These features are introduced and analyzed in the version of the model with nominal rigidities. Here we address a broader set of questions.

First we explore the role of common shocks (in the form of a monetary policy shock) in generating inflation differential. We show that a discretionary shock to monetary policy can itself create persistent deviations in consumer prices if there are considerable structural asymmetries across countries.

Second, we analyze the response of the inflation differential to the structural shocks in the system. We find that a modest degree of sticky prices can be sufficient to mute the short-run response to the shocks and to generate persistence well beyond the one implied by the stochastic process followed by the driving force. We find that sticky prices induce non-trivial dynamics and responses that contrast the ones of the flexible-price model.

Third, we show that the design of the monetary policy reaction function matters in shaping the dynamic of the inflation differential. The objectives and the rules followed by monetary policy may be critical in explaining the response and dynamic of price dispersion and other variables, following both common and idiosyncratic shocks.

We also consider the case of optimal monetary policy. We find that also under optimal policy sizeable and persistent inflation differentials still occur and that they are similar to those observed under conventional monetary policy rules based on aggregate inflation and output. Interestingly, we find that the responses of inflation differentials to various shocks under optimal policy are very close to the respective responses when monetary policy has an exclusive focus on maintaining price stability.

A fourth exercise performed in the model with nominal rigidities is the variance decomposition of endogenous variables with respect to the variability in the structural shocks. We find that the presence of different nominal rigidities in the two economies, notwithstanding the complete symmetry otherwise in the model and the shock structure, gives rise to interesting asymmetric responses in the variability of endogenous variables. Specifically, we find that the dynamic of the inflation differentials is largely driven by the variability of a single shock in a single region, namely in the productivity of the non-tradable sector in the more flexible economy. Conversely, the output differentials is largely driven by the variability in the productivity of the tradable sector, also of the more flexible economy. One effect of having optimal policy instead of a standard Taylor rule is to dampen somewhat the previous effects, creating a broader link between the variability in structural shocks and that in inflation and output differential.

The final exercise we perform is to compare the responses of the economy to the case where the countries form a monetary union using an exchange rate

parity which does not correspond to long-run equilibrium. This case is analyzed under three different policy regimes: A currency area in which monetary policy is conducted optimally; a currency area in which monetary policy targets the overall area average CPI inflation; and the case where the two economies keep independent currencies and in which policy is conducted optimally by the two separate monetary authorities, i.e. as to maximize joint welfare. We evaluate the welfare differences across these three regimes. Our results suggest that the costs of having an incorrect real exchange rate parity by 1% leads to a loss slightly below one tenth of percentage point of steady-state consumption in the case where policy is conducted optimally and to a loss slightly above one tenth of percentage point of steady-state consumption when policy follows the strict CPI inflation targeting procedure. These costs are two times higher than the ones usually found for business cycle movements.

The remainder of the paper is structured as follows: Section 2 presents the descriptive statistics on euro-area inflation differentials and proposes a dynamic factor model decomposition of it. Section 3 presents the structure of the model. Section 4 discusses the flexible-price version of the model while Section 5 analyzes the implications of the model under sticky prices. Section 6 analyzes the role of policy rules, including an approximation to the monetary policy optimal rule, in shaping inflation differentials. Section 7 reports the variance decomposition exercise. Section 8 addresses the question of what are the consequences of a country entering a monetary union with an incorrect exchange rate parity for overall inflation and inflation differentials, under both optimal and non-optimal monetary policy. The paper concludes with Section 9.

2 Euro Area Inflation Differentials

This section addresses three empirical questions concerning the euro area economy. How sizable are the inflation differentials in the euro area in comparison to other currency areas? Are the inflation differentials in the euro area a sectorial phenomena, i.e. tradable versus non-tradable? Are the inflation differentials in the euro area the result of idiosyncratic responses to common area-wide factors, or are they due to idiosyncratic (sector/country specific) factors?

A number of papers have recently provided empirical analysis of inflation differentials in the euro area. Alberola (2000) is one of the first analysis of post-EMU data specifically focusing on cross-country inflation differentials. Recently a more extensive review of empirical evidence and literature has been conducted by the ECB, (ECB 2003). The latter work surveys a variety of measures of price and cost developments at the national level in EU-12 during the 1999–2002 period and explores different possible macroeconomic determinants. Here we maintain a narrower focus on which are the stylized statistical facts that are more relevant for our analysis. For this, we provide descriptive statistics as well as perform econometric analysis based on dynamic factor models.

The data used is based on the harmonized index of consumption prices (HICP). These data are harmonized indices of consumer prices for a basket of all goods and services consumed by a country, expenditure-weighted. We analyze seasonally adjusted data for the area as a whole and for ten individual countries (all euro countries except for Greece and Luxemburg), for the period from January 1990 to February 2004. Excluding Greece and Luxembourg reflects the fact that HICP data series for these countries are shorter than for the other countries in the sample. Both for the euro area and the individual countries HICP we considered the overall HICP inflation index as well as the five main subcomponents in the HICP, namely Services, Industrial Good excluding Energy, Energy, Processed Food and Unprocessed Food.³

Notation To set notation, $\pi_{j,t}^i$ is the year on year growth rate of price in subindex j of country i and the aggregate euro area counterpart is $\pi_{j,t}^{euro}$. While π_t^i denotes the overall inflation rate in country i and π_t^{euro} is the overall euro area inflation rate. The inflation differential between country i and the euro area is denoted as:

$$\delta_{i,t} = (\pi_t^i - \pi_t^{euro});$$

while the dispersion of inflation is measured by the root mean squared around the euro area counterpart, as:

$$\Delta_t = \left(\frac{\sum_{i=1}^{10} \delta_{i,t}^2}{10} \right)^{1/2}.$$

The inflation differential between subindex j in country i and the euro area one is denoted as:

$$\delta_{j,i,t} = (\pi_{j,t}^i - \pi_{j,t}^{euro});$$

while the dispersion of inflation in the sub-component j is measured as:

$$\Delta_{j,t} = \left(\frac{\sum_{i=1}^{10} \delta_{j,i,t}^2}{10} \right)^{1/2}.$$

The overall dispersion Δ_t is not the weighted average of the sectorial dispersion given the nonlinearity of the transformation. We decompose the overall dispersion into the relative contribution of the different subcomponents. To this end, first, we assume that the sectorial country weights inside the HICP, $w_{j,i,t}$, are equal across countries, i.e. $w_{j,i,t} = w_{j,t} \forall i$, then we define the contribution of sector j to the overall inflation dispersion as:

$$\frac{w_j \times \Delta_{j,t}}{\sum_j w_j \times \Delta_{j,t}}.$$

³The weights of the five subindexes in the euro area HICP index in 2004 are 0.41, 0.31, 0.08, 0.12 and 0.08, respectively.

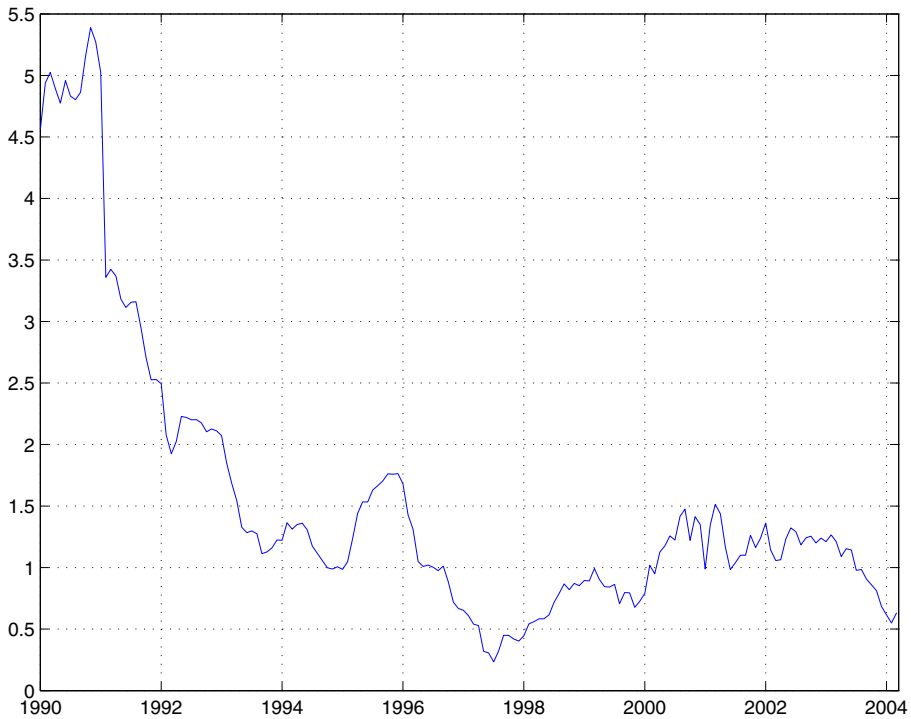


Fig. 1 Dispersion in HICP—y-o-y growth rate

Empirical Findings The cross-country dispersion in inflation has been declining during the 90s across euro area as documented in Fig. 1. Dispersion was above 4% in the early 90s while it reached its minimum at 0.26% at the end of 1997. After this point, the previous trend was inverted, starting to edge up again to around 1% between 1998 and 2002. In early 2004 it stood at around 0.64%.⁴

Compared with the degree of dispersion observed within some individual euro area countries, inflation dispersion within the euro area remains relatively high. In particular, the recent degree of dispersion within the euro area is around twice the comparable measures computed across the German *Länder*, the Spanish *Comunidades Autónomas* and the Italian cities. On the contrary, the recent euro area inflation dispersion is quite comparable to the one measured among the 14 US metropolitan areas. The inflation divergence among US cities stayed remarkably constant at around 1% for many years.

⁴This finding is slightly different from the one reported in ECB (2003), due to the fact that we considered the dispersion of the ten indicated countries vis-à-vis the euro area while ECB (2003) considered also Greece and Luxemburg.

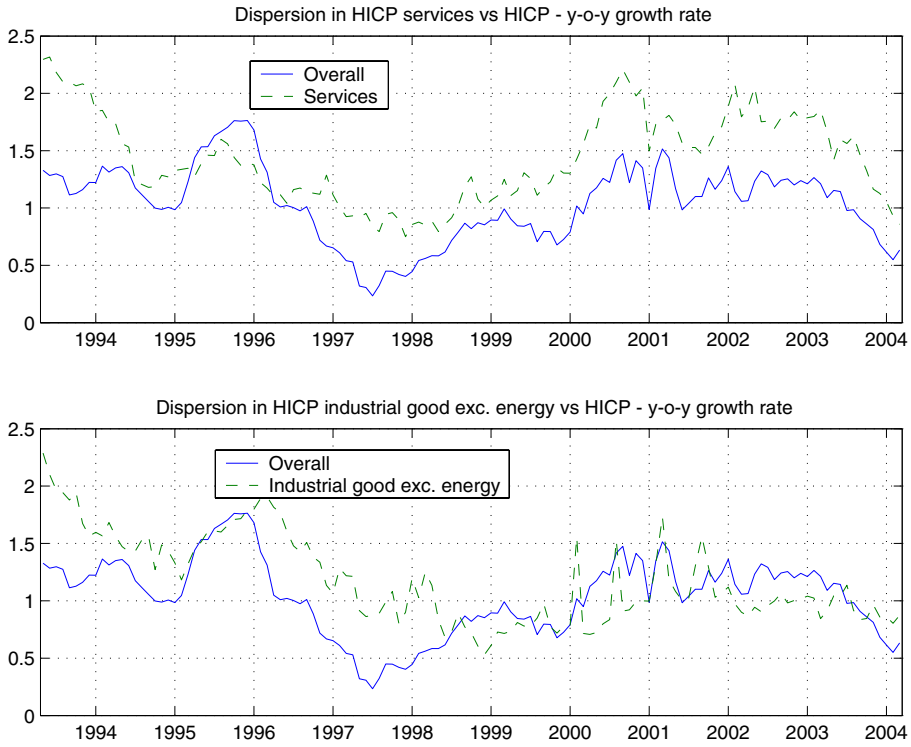


Fig. 2 Dispersion by sector

However the analogy is probably misleading, given that US cities are much smaller than EU nations, and their price indices tend to be more volatile.

Despite the possible similarities in the level of inflation dispersion between US and euro area, the high persistence of inflation differential is a characterizing feature of the euro area economy, as noted by Cecchetti et al. (2002). Indeed, in our data set the measured persistence of the inflation differential, $\delta_{j,i,t}$, is very close to the one of a unit root process on average across countries and sectors.⁵

The importance of sectorial pattern in explaining the overall dispersion is addressed in Figs. 2, 3, 4. Figures 2 and 3 present the dispersion for each of the five sectors, $\Delta_{j,t}$, plotted together with the overall dispersion, Δ_t . Finally, Fig. 4 presents the contribution of the sectors to the overall inflation dispersion.

Several conclusions can be drawn. First, all the components of inflation contributed to the very low dispersion observed in 1997–1998. Second, the increase on dispersion between 2000 and 2002 can mainly attributed to the dispersion

⁵If we measure the persistence of the differential process by the largest autoregressive roots of a fitted ARMA model, it turns out that the largest autoregressive root is 0.981 on average across sectors and countries.

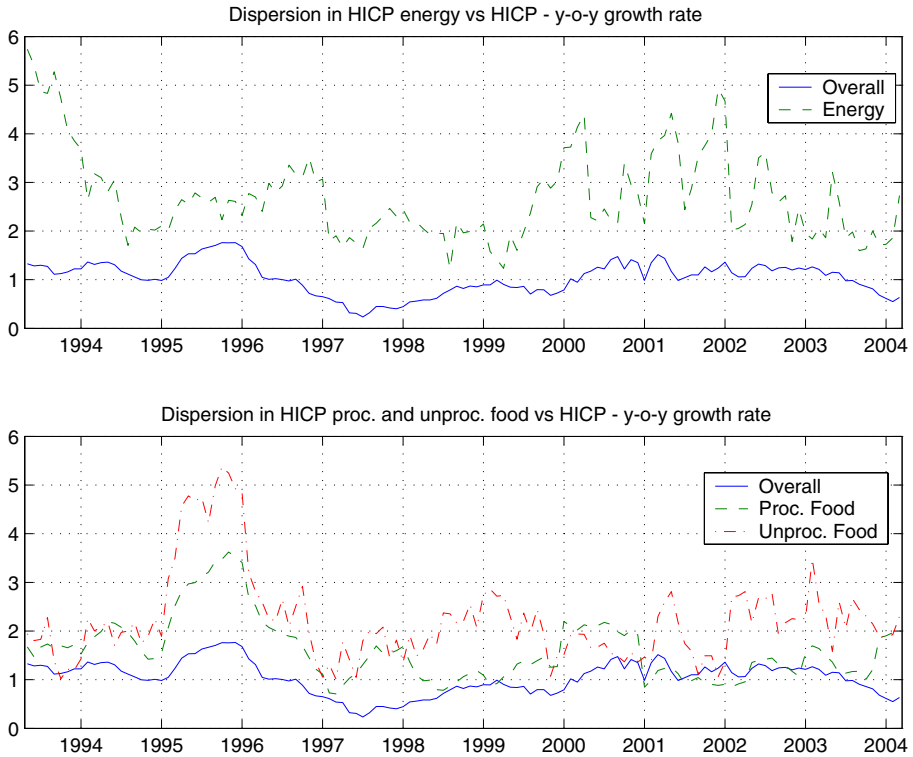


Fig. 3 Dispersion by sector

in Services and Energy sectors. Third, Fig. 2 shows that the dispersion in the Service sector has been almost always higher than the overall dispersion and its contribution has been increasing over time as indicated in Fig. 4, also in line with the increase of the weight of this subcomponent inside the HICP index. Fourth, differently to the Services, the Industrial Good excluding energy index presents a low degree of dispersion and its overall importance is decreasing. Finally, differently from the common wisdom, the dynamic of the Energy subindex is a major source of overall dispersion. This is due both to the large volatility of this subindex but also to considerable heterogeneity in the countries' response to shocks.

Interpreting those results some cautionary remarks should be made. The HICP is an index of final consumer prices for a basket of all goods and services consumed; so Industrial Good excluding energy price includes also the prices of (tradable) imported goods and a share of final sale services, such as the prices of any non-tradable marketing and other final consumption services; the services component included in the final Industrial good price might induce to overestimate the dispersion of this sector. On the contrary, this is not the case in the Services sector, where the value added deflator of Services almost

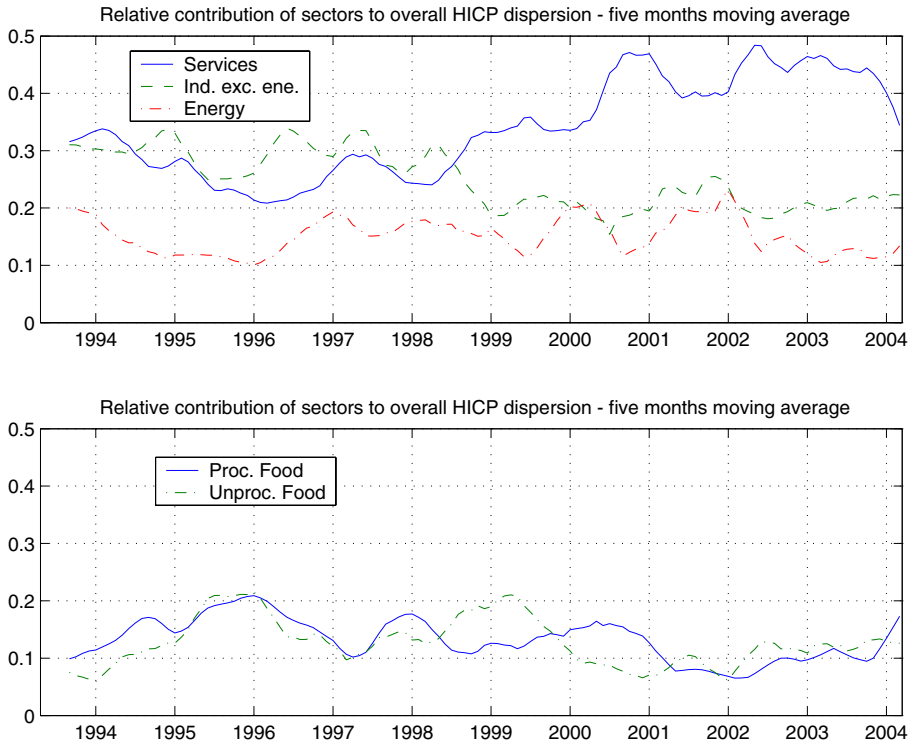


Fig. 4 Dispersion decomposition

entirely accounts for the final price of Services, as indicated by the input-output evidence produced by Sondergaard (2003) for some euro countries.

Finally, we address the last of the three questions stated at the outset—namely, whether the observed dispersion in inflation across euro area countries is due to different reaction to area-wide factors or the result of country/sectorial developments. To tackle this question we follow a similar approach to the one employed by Forni and Reichlin (2001) to decompose euro countries GDP growth into a European, a national, and a residual component. We estimate an approximate dynamic factor model⁶ on the inflation differential which allows us to decompose the differential in each countries and sectors as:

$$\delta_{j,i,t} = c_{ji} + \Phi_{ji}(L) \times u_t + \xi_{j,i,t} = \chi_{j,i,t} + \xi_{j,i,t} \tag{2.1}$$

where the first term of the right hand side is the average dispersion over the sample (ideally nil), the second term captures the effect of common area wide shocks, u_t , which is allowed to propagate across countries and sectors with

⁶See Stock and Watson (2002) and Forni et al. (2000).

a differentiate dynamic, and the last term captures an idiosyncratic dynamic, mainly associated to country or sectorial specific developments. We grouped the first and second term into $\chi_{j,i,t}$.

The approximate dynamic factor models exploits the cross-section dimension of a large panel of time series to identify and estimates the part of the time series driven by few common shocks, $\chi_{j,i,t}$. To this end, in order to have proper estimates of the euro area common components in Eq. 2.1, we augmented the sixty time series on differential, $\delta_{j,i,t}$, with 193 monthly macro-economic time series related to the ten euro area countries considered.⁷ The estimation has been performed on the sample period 1993.01–2003.06. In line with the finding of similar exercises, the results points to the presence of strong commonalities among the 253 variables and the estimated common factors account on average of around 50% of the variance of the 253 variables.⁸

Having an estimate of the two components in Eq. 2.1, we can first ask how much of the historical dynamic of differentials is accounted by the identified area wide factors. The table below reports the average across countries of the share of variance of the differentials, $\delta_{j,i,t}$, accounted by common shocks, both for the overall index and for the individual subindexes.

Overall	Services	Industrial	Energy	Proc. food	Unproc. food
0.66	0.58	0.67	0.52	0.65	0.42

It turns out that the Energy and the Services sector are the ones with the large idiosyncratic components, while the Industrial Good excluding Energy and the Processed Food sectors are the most common ones.

The importance of the common factors is also clear from Figs. 5, 6, 7 and 8, where we show how the dispersion in inflation can be decomposed into the part attributed to the common factor, $\chi_{j,i,t}$, and a remaining one, associated to the idiosyncratic part. Figure 5 presents the cross-country dispersion of the overall HICP and its decomposition. The common part is clearly responsible for the large part of the observed dispersion. The idiosyncratic part has a nil contribution to overall dispersion from 1994 to 2000, but it contributes positively to dispersion from 2000 onward. The Service sector presents a behavior similar to the overall index with large part of its dispersion explained by countries-specific reaction to common shocks. The increase in dispersion since 2000 is mainly associated to the common factor; even if idiosyncratic elements contributed positively both during 2000 and 2002. In the Industrial sector the commonality is even more striking, however it should be noted that the dispersion in this sector is relative low. By contrast, in the case of Energy

⁷The data considered are, inter alia, main and sectorial industrial production indexes, consumer and producer surveys, producer prices index, financial quantities, interest rates, trade statistics. The data has been seasonally adjusted and transformed to be stationary.

⁸The estimation procedure is based on the principal component decomposition of the variance-covariance matrix of the data. The estimation indicates the presence of five static factors in the models.

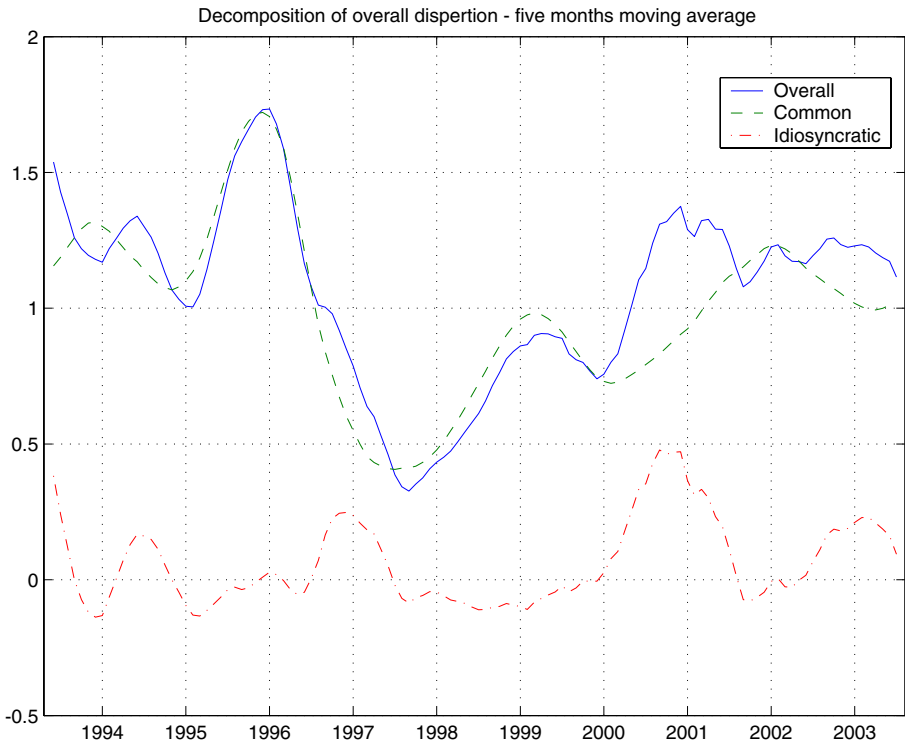


Fig. 5 Decomposition of overall dispersion

prices most of the dispersion is due to presence of idiosyncratic factors, see Fig. 8.

To conclude, the inflation differentials in the euro area are relevant and persistent. Our proxy of the non-tradable sector, namely the Services sector seems to make a particularly strong contribution to these differentials. By contrast, the dispersion in our proxy of the price index of tradable goods—namely industrial goods excluding energy—is relatively limited. A tentative overall conclusion from our attempt to understand the source of inflation differentials would indicate that they are largely associated to different responses of the ten euro area economies to common, area-wide shocks.

3 Model

We propose a stylized model of a currency area to investigate which shocks can be most important to drive the price dispersion and which structural features can rationalize the observed persistence. We model a currency area as composed by two countries of equal population size. Consumption preferences depend on non-traded and traded goods. In particular, each country is

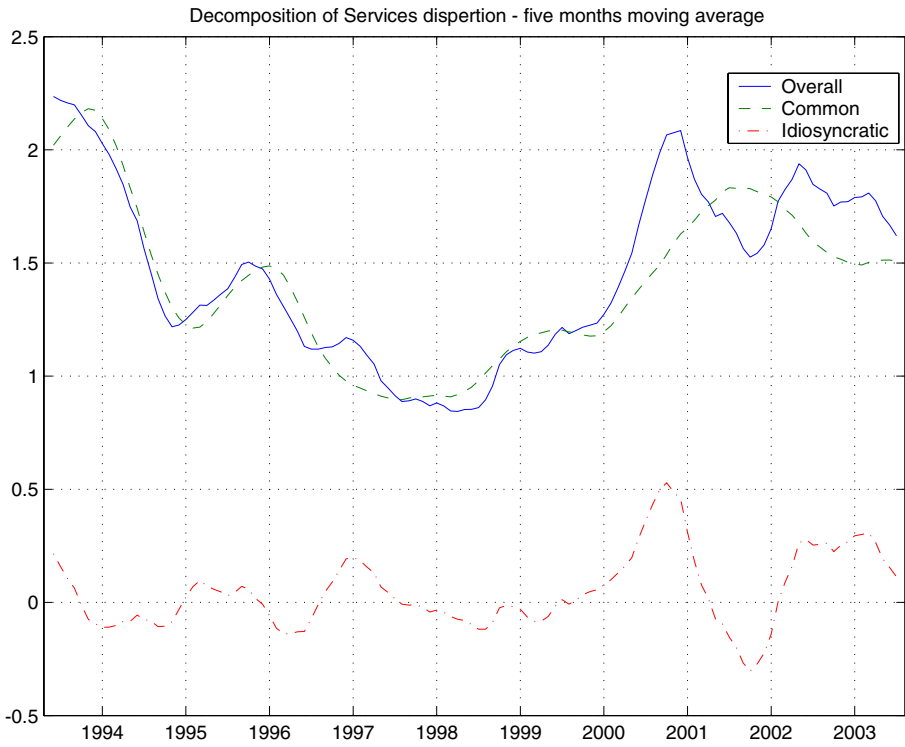


Fig. 6 Decomposition of services dispersion

specialized in the production of a bundle of traded goods. Financial markets are complete within and across countries. Labor is immobile across countries but imperfectly mobile within sectors of a country. There can be price rigidities in all sectors of the economy. The law of one price holds in the traded sector and the price dispersion at the consumer level depends on the movements of the relative prices of non-traded goods.

Our model is closely related to open-economy models, like Obstfeld and Rogoff (2000) who introduces non-traded goods in stochastic models with sticky prices. Benigno and Thoenissen (2003) use a similar model to study the real exchange rate behavior of UK with respect to the Euro area with the purpose of analyzing whether supply shocks can account for the real exchange rate appreciation in the late nineties. In reference to the recent literature on monetary policy in a currency area our model is closely related to Duarte and Wolman (2002, 2008). They further allow for price discrimination in goods that are tradeable. Moreover, they analyze the role of fiscal policy rules in reducing or amplifying inflation differentials. Indeed, in most of their work they assume that lump-sum forms of taxation are unavailable to the government. Our focus here is instead limited to the case in which lump-sum taxes are available. In Duarte and Wolman (2002) they find that their model can deliver more

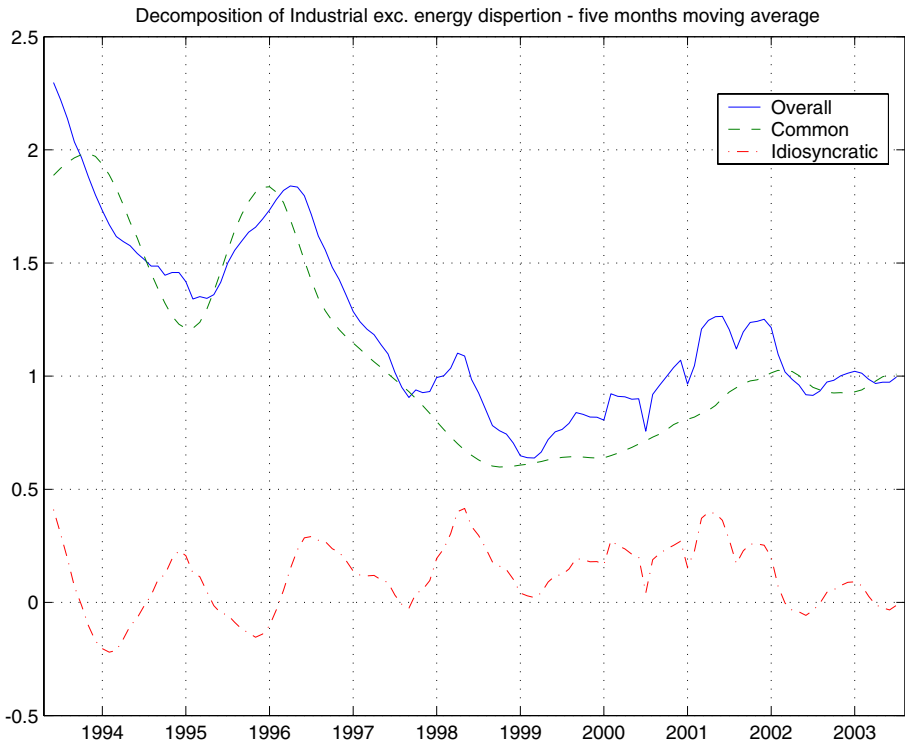


Fig. 7 Decomposition of industrial goods exc. energy dispersion

inflation dispersion than in the data following productivity shocks, while negligible dispersion following government spending shocks. Andrés et al. (2008) instead analyze a model with only traded goods, but they allow for price discrimination across countries due to different degrees of market competition. They show that their model can account for sizeable inflation differentials. In particular their findings are that the driving force of the price dispersion in the traded sector originates from the mechanism of price discrimination more than on price rigidity. They also find that the degree of openness of countries can play an important role. Angeloni and Ehrmann (2007) present a more stylized 12-country model of the euro area and in particular they focus on the role of past inflation in the aggregate supply equation. They find that this additional source of inflation persistence is important in driving up inflation dispersion in the currency area. There are other papers that have analyzed monetary models of currency areas, as Benigno (2004b), Beetsma and Jensen (2005) and Lombardo (2006).⁹ However these models do not allow for price dispersion at

⁹Our work is also related to model of small open economies as Natalucci and Ravenna (2002) that have investigated exchange rate policy for accession countries in the EU. See Soto (2003) for a small open-economy model with non-traded goods.

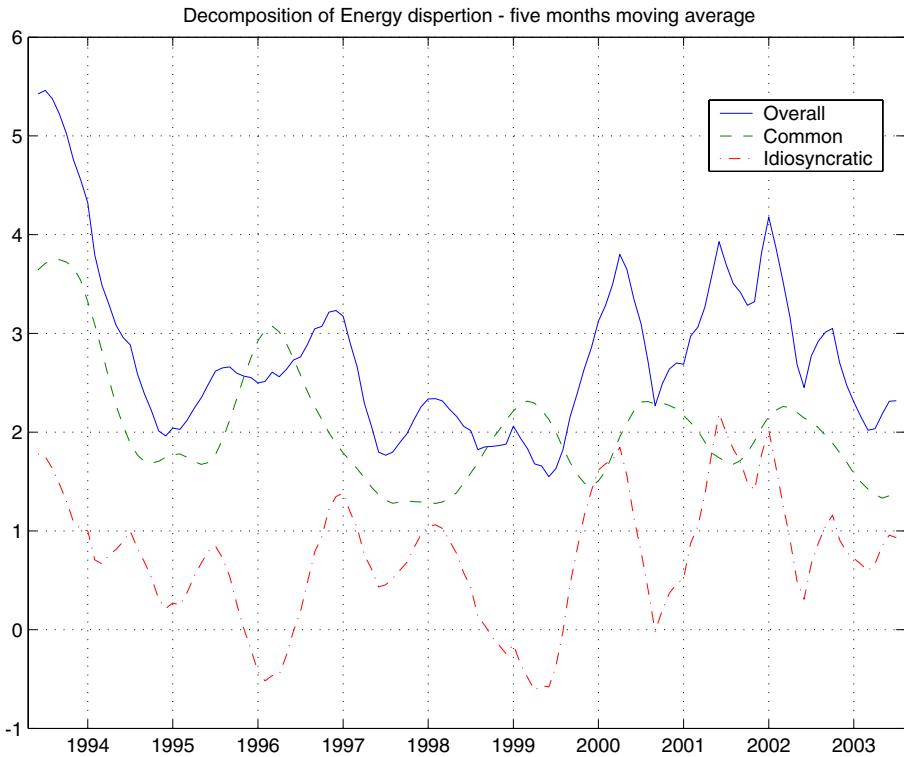


Fig. 8 Decomposition of energy dispersion

the consumer level and mainly focus on the role of the terms of trade and price stickiness in stabilizing asymmetric shocks.

3.1 Households

We consider a model of a currency area composed by two countries, Home (H) and Foreign (F). Each country is populated by a measure one of households. A generic household j belonging to either country H or F maximizes the following utility function:

$$U_t^j \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(L_s^j)],$$

where E_t denotes the expectation conditional on the information set at date t and β is the intertemporal discount factor, with $0 < \beta < 1$. Households derive utility from consumption and disutility from supplying hours of work.

There are two classes of goods in both economies: traded and non-traded goods. Each country produces a measure one of goods, a fraction γ (γ^* in region F)— with $0 < \gamma, \gamma^* < 1$ —of which is composed by traded goods. The

remaining fractions are non-traded goods. The consumption index C_i^j in region H is defined as a Dixit-Stiglitz aggregator of indexes of traded, C_T^j , and non-traded goods, C_N^j , as it follows

$$C^j \equiv \left[\omega^{\frac{1}{\varphi}} (C_T^j)^{\frac{\varphi-1}{\varphi}} + (1-\omega)^{\frac{1}{\varphi}} (C_N^j)^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}},$$

where φ is the elasticity of substitution between the bundles C_T and C_N with $\varphi > 0$ while ω denotes the share of traded goods in the general consumption basket, with $0 < \omega < 1$. (This share may be different for households in region F and it will be denoted by ω^* .) The traded goods are non-homogenous and differentiated in consumption preferences. They are also produced with different technologies. In particular the index C_T^j is defined as a Dixit-Stiglitz aggregator of the bundles of home-produced traded goods, C_H^j , and foreign-produced traded goods, C_F^j ,

$$C_T^j \equiv \left[n^{\frac{1}{\theta}} (C_H^j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}} (C_F^j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where θ , with $\theta > 0$, is the elasticity of substitution between the bundles C_H^j and C_F^j and n , with $0 < n < 1$, denotes the share of home-produced traded goods in the overall index of traded goods. In the foreign economy n^* , with $0 < n^* < 1$, denotes the share of foreign-produced traded goods in the overall basket of traded goods. The consumption bundles C_H^j and C_F^j are composed by the continuum of differentiated traded goods produced respectively in region H and F and are defined as

$$C_H^j \equiv \left[\gamma^{-\frac{1}{\sigma}} \int_0^\gamma c^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}, \quad C_F^j \equiv \left[\gamma^{*-\frac{1}{\sigma}} \int_0^{\gamma^*} c^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution among the differentiated goods. Similarly, C_N^j is the home consumption index of the continuum of differentiated non-traded goods:

$$C_N^j \equiv \left[(1-\gamma)^{-\frac{1}{\sigma}} \int_\gamma^1 c_N^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}},$$

with the same elasticity of substitution σ . In country F , γ^* replaces γ in the consumption bundles of non-traded goods.

Given the above consumption indices, we can derive the appropriate price indices. With respect to the general consumption index, we obtain the consumption-based price index P

$$P = \left[\omega (P_T)^{1-\varphi} + (1-\omega) (P_N)^{1-\varphi} \right]^{\frac{1}{1-\varphi}},$$

where P_T and P_N are given by

$$P_T = [n(P_H)^{1-\theta} + (1-n)(P_F)^{1-\theta}]^{\frac{1}{1-\theta}},$$

$$P_N = \left[(1-\gamma)^{-1} \int_{\gamma}^1 p_N(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}},$$

while P_H and P_F are given by

$$P_H = \left[\gamma^{-1} \int_0^{\gamma} p(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \quad P_F = \left[\gamma^{*-1} \int_0^{\gamma^*} p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}},$$

where $p(h)$, $p(f)$, $p_N(h)$ are respectively the prices in the common currency faced by households in country H for a generic home-produced traded good, a foreign-produced traded good and a domestic non-traded good. Similar indices are derived for country F with the appropriate modifications of the respective shares. Prices faced by foreign consumers are denoted with asterisks.

Given the consumption-based price indexes the generic home consumer j has the following demand of each of the home-produced traded goods

$$c^j(h) = \frac{n\omega}{\gamma} \left(\frac{p(h)}{P_H} \right)^{-\sigma} \left(\frac{P_H}{P_T} \right)^{-\theta} \left(\frac{P_T}{P} \right)^{-\varphi} C^j,$$

for $0 \leq h < \gamma$; of each of the foreign-produced traded goods

$$c^j(f) = \frac{(1-n)\omega}{\gamma^*} \left(\frac{p(f)}{P_F} \right)^{-\sigma} \left(\frac{P_F}{P_T} \right)^{-\theta} \left(\frac{P_T}{P} \right)^{-\varphi} C^j,$$

for $0 \leq f < \gamma^*$ and of each of the home-produced non-traded goods

$$c_N^j(h) = \frac{1-\omega}{1-\gamma} \left(\frac{p_N(h)}{P_N} \right)^{-\sigma} \left(\frac{P_N}{P} \right)^{-\varphi} C^j,$$

for $\gamma \leq h \leq 1$. Similar, but appropriately modified, demands hold in the foreign economy.

Households get disutility from supplying labor to all the firms operating in their country of residence. The function $V(\cdot)$ is increasing and convex in an index of labor L^j . Each firm uses a specific labor factor and each household can supply all the varieties of labor used in the country to produce the continuum of traded and non-traded goods. Each household supplies a measure one of labor varieties, of which a fraction γ will be employed in the traded sector (γ^* in the foreign economy) and the remaining fractions in the non-traded sector. In particular we assume that L^j is a Dixit-Stiglitz aggregator of labor indices L_T and L_N in the traded and non-traded sectors, respectively, as it follows

$$L^j \equiv \left[\gamma^{\frac{1}{\phi}} (L_T^j)^{\frac{\phi-1}{\phi}} + (1-\gamma)^{\frac{1}{\phi}} (L_N^j)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}},$$

where $\phi > 0$ is the elasticity of substitution between labor in the traded and non-traded sector and L_T^j and L_N^j are composite index of the continuum of varieties supplied in both sectors

$$L_T^j \equiv \left[\gamma^{-\frac{1}{\nu}} \int_0^\gamma l_T^j(i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}}, \quad L_N^j \equiv \left[(1-\gamma)^{-\frac{1}{\nu}} \int_\gamma^1 l_N^j(i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}},$$

where $\nu > 0$ is the elasticity of substitution between varieties of labor within a sector. In contrast with standard models of traded and non-traded production, we are not necessarily assuming that labor is perfectly substitutable and mobile across sectors and instead we allow for wage differentiation across varieties of labor. The case of perfect substitutability, and perfect labor mobility, is nested under the assumption that both $\phi, \nu \rightarrow \infty$. In general, given wages $w_T(i)$ and $w_N(i)$ specific to the generic variety i in the respective traded and non-traded sector, we can write the following wage indexes associated with the above defined labor indices

$$W = \left[\gamma W_T^{1-\phi} + (1-\gamma) W_N^{1-\phi} \right]^{\frac{1}{1-\phi}},$$

$$W_T = \left[\gamma^{-1} \int_0^\gamma w_T^{1-\nu}(i) di \right]^{\frac{1}{1-\nu}}, \quad W_N = \left[(1-\gamma)^{-1} \int_\gamma^1 w_N^{1-\nu}(i) di \right]^{\frac{1}{1-\nu}}.$$

Given the relative wages and the choice of L^j , we can then characterize the household's labor decisions in the following way

$$l_T^j(h) = \left(\frac{w_T(h)}{W_T} \right)^{-\nu} \left(\frac{W_T}{W} \right)^{-\phi} L^j, \quad l_N^j(i) = \left(\frac{w_N(i)}{W_N} \right)^{-\nu} \left(\frac{W_N}{W} \right)^{-\phi} L^j,$$

for each variety of labor supplied for a generic firm in a traded and non-traded sector, respectively.

Each household faces the following flow budget constraint

$$B_t^j \leq A_t^j + W_t L_t^j + \Pi_t^j - P_t C_t^j + T_t^j \tag{3.2}$$

where A_t^j represents the beginning-of-period wealth that includes the bonds carried from the previous period. B_t^j is the end-of period portfolio that includes a wide selection of instruments that pay in each contingency that occurs. In particular they pay A_t in the particular contingency at date t . As of time $t - 1$, A_t is a random variable whose realization depends on the state of nature at time t .

Here it is assumed that there are complete financial market which implies that there exists a unique discount factor $Q_{t,t+1}$ with the property that the price in period t of a portfolio with random value A_{t+1} is

$$B_t = E_t[Q_{t,t+1} A_{t+1}],$$

where E_t denotes the expectation conditional on the state of nature at date t . In particular we define the short-term interest rate in the following way, as the

price of the portfolio that delivers one unit of currency in each contingency that occur one-period ahead, i.e.

$$\frac{1}{1 + i_t} = E_t[Q_{t,t+1}].$$

In Eq. 3.2, Π_t are aggregate profits of all the firms within a country. Profits are risk shared across households. T_t^j are transfers from the government to household j . The economy is a cashless-limiting monetary economy as discussed in Woodford (2003). The flow budget constraint of the consumer can be written as

$$E_t[Q_{t,t+1}A_{t+1}^j] \leq A_t^j + W_tL_t^j + \Pi_t^j - P_tC_t^j + T_t^j$$

and the consumer’s problem is further subject to the following borrowing limit condition in each contingency and date that the consumer will face.

$$A_{t+1}^j \geq - \sum_{s=t+1}^{\infty} E_{t+1} Q_{t+1,s} \{W_sL_s^j + \Pi_s^j + T_s^j\} > -\infty.$$

The borrowing limit condition together with the flow budget constraint imply the standard intertemporal budget constraint

$$\sum_{s=t}^{\infty} E_t Q_{t,s} [P_s C_s^j] \leq A_t^j + \sum_{s=t}^{\infty} E_t Q_{t,s} [W_s L_s^j + \Pi_s^j + T_s^j]. \tag{3.3}$$

Given the above decisions on how to allocate consumption and labor across all the varieties, the household j chooses the optimal path of the consumption index C_t and labor index L_t at all times and contingencies to maximize its utility under the intertemporal budget constraint. In particular the set of optimality conditions can be described by the set of Euler conditions

$$\frac{U_c(C_t^j)}{U_c(C_{t+1}^j)} = \frac{\beta}{Q_{t,t+1}} \frac{P_t}{P_{t+1}} \tag{3.4}$$

for each state of nature at time $t + 1$ looking ahead from time t . By choosing appropriately the distribution of initial state-contingent wealth, complete markets assure that consumption is perfectly equalized within households belonging to a country. Moreover, across countries, marginal utilities of consumption are proportional to the consumer-price differential

$$\frac{U_c(C_t)}{U_c(C_t^*)} = \varsigma \frac{P_t}{P_t^*} \tag{3.5}$$

for a positive factor of proportionality $\varsigma > 0$ which again depends on the initial wealth distribution. Under the assumption of complete financial markets, consumer-price differentials directly translate into consumption differentials, so that a country which experiences an increase in its consumer price level relative to another country should experience a fall in its own consumption relative to the other country. This complete-market assumption is a convenient

simplification but it comes at a cost of neglecting wealth distribution as an important channel through which price and inflation differentials can propagate into the economy and be amplified.

Real wages, computed using the general price and wage indices, are equated to the marginal rate of substitution between the labor index L and the consumption index C as

$$\frac{W_t}{P_t} = \frac{V_l(L_t)}{U_c(C_t)}. \tag{3.6}$$

Finally the last optimality condition is the exhaustion of the intertemporal budget constraint, i.e. Eq. 3.3 holds with equality at all times.

3.2 Firms

Regarding the supply side of the economy, we indeed assume that there is a continuum of firms, of measure one, which is producing the continuum of goods. In particular a fraction γ (γ^* in the foreign economy) is producing traded goods, while the remaining fractions are producing non-traded goods. Taking as representative the home economy, a generic firm producing in the traded sector is using the following technology $y^T(h) = A_T f(l(h))$, where A_T is a country- and sector-specific technological shock and $f(\cdot)$ is a standard concave production function in the specific variety of labor used in the production of good h . In the non-traded sector the technology is given by $y^N(h) = A_N f(l(h))$ for a generic firm h in the non-traded sector. Firms in both sectors are monopolist and set their prices considering the overall demand of their goods. In the traded sector, we assume that there is no price discrimination and that all the consumers of the area face the same price for the same variety of goods. In particular in the traded sector a generic firm h faces the following demand

$$y^T(h) = \frac{n\omega}{\gamma} \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\frac{P_H}{P_T}\right)^{-\theta} \left(\frac{P_T}{P}\right)^{-\varphi} C + \frac{n^*\omega^*}{\gamma} \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\frac{P_H}{P_T^*}\right)^{-\theta} \left(\frac{P_T^*}{P^*}\right)^{-\varphi} C^*$$

In the non-traded sector, a generic firm faces the following demand

$$y^N(h) = \frac{1 - \omega}{1 - \gamma} \left(\frac{p_N(h)}{P_N}\right)^{-\sigma} \left[\left(\frac{P_N}{P}\right)^{-\varphi} C_N + G \right],$$

where in particular G is an exogenous country-specific government-purchase shock that affects only the demand of non-traded goods. In both sectors, prices are sticky and staggered as in the Calvo’s style price-setting behavior.¹⁰ In

¹⁰See Calvo (1983).

particular a mass $1 - \alpha_T$ of firms in the traded sector ($1 - \alpha_N$ in the non-traded sector) with $0 \leq \alpha_T, \alpha_N < 1$, is allowed in each period to reset their prices. (In the foreign economy, we have respectively α_N^* and α_T^* with $0 < \alpha_T^*, \alpha_N^* < 1$.) In this case a generic firm h in the traded sector of country H sets its price in order to maximize the present discounted value of profits, taking in consideration that the price chosen at time t will remain the same at time s , with $s \geq t$, with a probability $(\alpha_T^i)^{s-t}$. The present discounted value of profits is

$$E_t \sum_{s=t}^{\infty} (\alpha_T)^{s-t} Q_{t,s} [(1 - \tau_{T,s}) \tilde{p}_t(h) \tilde{y}_{t,s}^T(h) - w_{T,s}(h) l_{T,s}(h)], \tag{3.7}$$

where $\tau_{T,t}$ is a sectorial country-specific time-varying proportional tax on sales in the traded sector, $\tilde{p}_t(h)$ denotes the price of the firm h chosen at date t and $\tilde{y}_{t,s}^T(h)$ is the total demand of firm h at time s conditional on the fact that the price $\tilde{p}_t(h)$ has not changed.

It can be shown that the optimal choice of the price satisfies

$$E_t \sum_{s=t}^{\infty} (\alpha_T \beta)^{s-t} U_c(C_s) \tilde{y}_{t,s}^T(h) \frac{1}{\mu_{T,s}} \frac{P_{T,s}}{P_s} \left[\frac{\tilde{p}_t(h)}{P_{H,t}} \frac{P_{H,t}}{P_{H,s}} \frac{P_{H,s}}{P_{T,s}} - mc_{t,s}^T(h) \right] = 0 \tag{3.8}$$

where the real marginal cost for firm j at time s conditional on the fact that the price $\tilde{p}_t(j)$ has not changed are defined by

$$\begin{aligned} mc_{t,s}^T(h) &\equiv \mu_{T,t} \frac{P_s}{P_{T,s}} \frac{w_{T,s}(h)}{P_s} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^T(h)/A_{T,s})) A_{T,s}} \\ &= \mu_{T,t} \frac{P_s}{P_{T,s}} \frac{W_s}{P_s} \left(\frac{W_{T,s}}{W_s} \right)^{1-\frac{\phi}{\nu}} \left(\frac{f^{-1}[(\tilde{y}_{t,s}^T(h)/A_{T,s})]}{L_s} \right)^{-\frac{1}{\nu}} \\ &\quad \times \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^T(h)/A_{T,s})) A_{T,s}} \end{aligned}$$

where $1/\mu_{T,t}$ is defined as

$$\frac{1}{\mu_{T,t}} \equiv \frac{(1 - \tau_{T,t})(\sigma - 1)}{\sigma}.$$

Given the Calvo’s mechanism, the evolution of the price index $P_{H,t}$ is described by the following law of motion

$$P_{H,t}^{1-\sigma} = \alpha_T P_{H,t-1}^{1-\sigma} + (1 - \alpha_T) \tilde{p}_t(h)^{1-\sigma}, \tag{3.9}$$

where indeed $1 - \alpha_T$ is the fraction of firms that can reset their prices. Following the same reasoning we can write the first-order condition for a generic firm in the non-traded sector as

$$E_t \sum_{s=t}^{\infty} (\alpha_N \beta)^{s-t} U_c(C_s) \tilde{y}_{t,s}^N(h) \frac{P_{N,s}}{P_s} \frac{1}{\mu_{N,s}} \left[\frac{\tilde{p}_{N,t}(h)}{P_{N,t}} \frac{P_{N,t}}{P_{N,s}} - mc_{t,s}^N(h) \right] = 0 \tag{3.10}$$

where

$$\begin{aligned}
 mc_{t,s}^N(h) &\equiv \mu_{N,s} \frac{P_s}{P_{N,s}} \frac{w_{N,s}(h)}{P_s} \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^N(h)/A_{N,s}))A_{N,s}} \\
 &= \mu_{N,s} \frac{P_s}{P_{N,s}} \frac{W_s}{P_s} \left(\frac{W_{N,s}}{W_s}\right)^{1-\frac{\phi}{v}} \left(\frac{f^{-1}[(\tilde{y}_{t,s}^N(h)/A_{N,s})]}{L_s}\right)^{-\frac{1}{v}} \\
 &\quad \times \frac{1}{f'(f^{-1}(\tilde{y}_{t,s}^N(h)/A_{N,s}))A_{N,s}}
 \end{aligned}$$

and

$$\frac{1}{\mu_{N,t}} \equiv \frac{(1 - \tau_{N,t})(\sigma - 1)}{\sigma},$$

where τ_N is a time-varying proportional tax on sales in the non-traded sector. The evolution of the price index $P_{N,t}$ is described by the following law of motion

$$P_{N,t}^{1-\sigma} = \alpha_N P_{N,t-1}^{1-\sigma} + (1 - \alpha_N) \tilde{p}_{N,t}(h)^{1-\sigma}, \tag{3.11}$$

where $1 - \alpha_N$ is the mass of firms that can change their prices in the non-traded sector. Similar price conditions can be obtained in the respective sectors of the foreign country.

3.3 Fiscal Policies

Each country has its own fiscal authority while there is a single monetary policy maker for the whole area. In each region, the government raises revenues from the distortionary sale taxes to finance the expenditure for domestic non-traded goods. Moreover, lump-sum taxes are available to balance the budget in each period. It follows that

$$\begin{aligned}
 0 &= \int_{\gamma}^1 p_{N,t}(h)g_t(h)dh + \int_0^1 T_t^j dj - \tau_{N,t} \int_{\gamma}^1 p_{N,t}(h)y_t^N(h)dh \\
 &\quad - \tau_{T,t} \int_0^{\gamma} p_t(h)y_t^T(h)dh
 \end{aligned}$$

for country H while

$$\begin{aligned}
 0 &= \int_{\gamma^*}^1 p_{N,t}^*(f)g_t^*(f)df + \int_0^1 T_t^{*j} dj - \tau_{N,t}^* \int_{\gamma^*}^1 p_t^{N*}(f)y_{N,t}^*(f)df \\
 &\quad - \tau_{T,t}^* \int_0^{\gamma^*} p_t^*(f)y_t^*(f)df
 \end{aligned}$$

for country F , where T_t^j and T_t^{*j} are lump-sum transfers to households in countries H and F respectively. The model is closed with the policy function chosen by the common monetary authority that will be specified later.

4 Price Differentials with Flexible Prices

In this section, we discuss the reasons for why price and inflation differentials can arise in the model outlined above. First, we note that we can write the ratio of the CPI prices of the two countries in the following way

$$\frac{P_t}{P_t^*} = \frac{P_{T,t}}{P_{T,t}^*} \frac{[\omega + (1 - \omega)T_{N,t}^{1-\varphi}]^{\frac{1}{1-\varphi}}}{[\omega^* + (1 - \omega^*)T_{N,t}^{*1-\varphi}]^{\frac{1}{1-\varphi}}}$$

where we have defined the relative price of non-traded with respect to traded goods in each country as $T_N \equiv P_N/P_T$ and $T_N^* \equiv P_N^*/P_T^*$. The ratio of the tradeable goods prices is instead given by

$$\frac{P_{T,t}}{P_{T,t}^*} = \frac{[nP_{H,t} + (1 - n)P_{F,t}]^{\frac{1}{1-\theta}}}{[n^*P_{F,t}^* + (1 - n^*)P_{H,t}^*]^{\frac{1}{1-\theta}}}$$

There can be several possible ways through which differences in consumer prices can arise among countries. One obvious reason has to do with the different composition of the consumption indices, due to differences in tastes. This can happen either because there can be home bias in the consumption of traded good, so that $n^* \neq 1 - n$ or because the share of traded goods in the overall consumption basket can vary across countries, $\omega \neq \omega^*$. Heterogeneity in consumption preferences can be a source of consumer-price differentials even if the prices of all the goods, traded and non-traded, are equalized across countries. In this work, we allow this channel to be in principle important to generate inflation differentials. Conversely, even if tastes are similar, price differentials at the single good level can produce differential at the consumer-price level. The above decomposition shows that this can happen either because of deviations from the law of one price for traded goods or because relative price of non-traded goods can vary across countries.

There are important reasons for why the law of one price might not hold for traded goods even in a currency area. Firms can price discriminate across different markets (countries) due to different degrees of competition in the markets or structural characteristics. Moreover, even if firms do not price discriminate and markets are characterized by similar structures and degrees of competition, there can be price deviations for traded goods that enter the consumption basket, stemming from the fact that traded goods usually carry some non-traded components (e.g. distribution costs) before reaching the consumer markets.¹¹ Sondergaard (2003) using input-output tables for France, Italy and Spain has shown that the traded sector relies more than others on intermediate inputs produced by other sectors in the economy; in particular total inputs from other sectors account for 60% of gross output in the

¹¹The importance of distribution sectors in explaining differential at the consumer-price level has been emphasized by Burstein et al. (2004) and Corsetti and Dedola (2005). Duarte and Wolman (2008) incorporate this feature in their currency-area model.

traded sector. This suggests that the distribution sector may be an important factor in explaining inflation differentials in a currency area. Movements in the prices of non-traded goods that enter in the production or transportation of traded goods can be an important source of price dispersion for traded goods at the consumer level. Moreover, taking in consideration intermediate stages of production de-emphasizes the importance of pure traded goods prices in explaining the fluctuations of the consumer price differential. The evidence of Altissimo et al. (2005) shows that inflation dispersion in the traded sector is much lower than in the non-traded sector, which suggests that in any case differences in the relative price of non-traded goods can be much more important in explaining the consumer price differential. In this work, we choose not to model any price differential in the traded sector to get more insight on what we believe can be a stronger channel. Another reasons for this modelling strategy has to do with the fact that we are interested in the normative analysis. We ask what are the desirable movements in CPI prices that a common monetary policymaker would like to generate in a currency area following several source of shocks. Deviations of the law of one price are usually thought to be of transient nature and perhaps are less important from a normative perspective. Indeed, one of the most popular and often advocated reason for why there can be long-lasting departures from PPP originating from non-traded goods prices is due to Balassa (1964) and Samuelson (1964). According to this view, countries that experience higher productivity growth in the traded sector will also show higher consumer prices. The reason is that productivity growth in the traded sector will translate into an increase in the overall wage in the economy, since prices of traded goods are tied internationally and there is perfect labor mobility. However, as discussed in Altissimo et al. (2005), we do not expect our model to display this feature since traded goods are not homogenous and terms of trade can play an important role in the adjustment as discussed in Benigno and Thoenissen (2003), Cova (2008), Duarte and Wolman (2008), Fitzgerald (2003), MacDonald and Ricci (2002).

5 Nominal Rigidities and Inflation Differentials

The flexible-price model provides useful benchmark. However, it has also obvious limitations. First, price differentials under flexible prices can arise only if there are asymmetric shocks across countries.¹² Second, under flexible prices persistence stems only from the intrinsic persistence of the exogenous shock process, although adding capital accumulation and incomplete financial

¹²This result hinges on the fact that we have imposed symmetry on the steady state of the model. Allowing for asymmetries in the steady-state position of countries, like for example different labor shares, would imply that balanced shocks to the whole area would have an effect on the price differential.

markets would create additional persistence. Finally, there is no role for monetary policy to affect price differentials.

We show that introducing sticky prices in the model can account for these three deficiencies: balanced shocks in the whole area create price and inflation dispersion; the model generates persistence; and monetary policy will matter for the determination of price dispersion and its degree of persistence. These three aspects of the model are shown below, after introducing the model’s log-linear equations under sticky prices.

In a log-linear approximations to the structural equilibrium conditions (in particular Eqs. 3.8, 3.9, 3.10 and 3.11 and the respective conditions for the foreign country) we obtain four aggregate supply (AS) equations for the respective traded and non-traded sector in each economy. These AS equations are familiar forms to the standard New-Keynesian AS equations of the closed-economy models of Galí and Gertler (1999) and Sbordone (2001). The inflation rate in each sector will depend on the real marginal cost in the sector and on the discounted value of the expected future sectoral inflation rate as follows

$$\pi_{H,t} = k_T(\widehat{mc}_{T,t}) + \beta E_t \pi_{H,t+1}$$

$$\pi_{N,t} = k_N(\widehat{mc}_{N,t}) + \beta E_t \pi_{N,t+1}$$

$$\pi_{F,t}^* = k_T^*(\widehat{mc}_{T,t}^*) + \beta E_t \pi_{F,t+1}^*$$

$$\pi_{N,t}^* = k_N^*(\widehat{mc}_{N,t}^*) + \beta E_t \pi_{N,t+1}^*$$

where we have defined $\pi_{H,t} = \ln P_{H,t}/P_{H,t-1}$, $\pi_{N,t} = \ln P_{N,t}/P_{N,t-1}$, $\pi_{F,t}^* = \ln P_{F,t}/P_{F,t-1}$, $\pi_{N,t}^* = \ln P_{N,t}^*/P_{N,t-1}^*$ and $k_j \equiv \frac{1-\alpha_j}{\alpha_j} \frac{1-\alpha_j\beta}{1+\frac{\vartheta\alpha_j}{\lambda}}$ where α_j can assume different values across sectors and countries (generically we have $\alpha_T, \alpha_N, \alpha_T^*, \alpha_N^*$) and $\vartheta \equiv 1 - \tilde{\lambda} - \frac{1}{v}$. Moreover the deviations of real marginal costs from the steady-state are given by

$$\begin{aligned} \widehat{mc}_{T,t} = & \mu_{T,t} + (1 - \gamma)\hat{T}_{N,t} + \frac{1}{2}\hat{T}_t + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{T,t} - \hat{L}_t) - \hat{A}_{T,t} \\ & + (1 - \tilde{\lambda})\hat{l}_{T,t}, \end{aligned}$$

$$\widehat{mc}_{N,t} = \mu_{N,t} - \gamma\hat{T}_{N,t} + \eta\hat{L}_t + \rho\hat{C}_t - \frac{1}{\phi}(\hat{l}_{N,t} - \hat{L}_t) - \hat{A}_{N,t} + (1 - \tilde{\lambda})\hat{l}_{N,t},$$

$$\begin{aligned} \widehat{mc}_{T,t}^* = & \mu_{T,t}^* + (1 - \gamma)\hat{T}_{N,t}^* - \frac{1}{2}\hat{T}_t + \eta\hat{L}_t^* + \rho\hat{C}_t^* - \frac{1}{\phi}(\hat{l}_{T,t}^* - \hat{L}_t^*) \\ & - \hat{A}_{T,t}^* + (1 - \tilde{\lambda})\hat{l}_{T,t}^*, \end{aligned}$$

$$\widehat{mc}_{N,t}^* = \mu_{N,t}^* - \gamma\hat{T}_{N,t}^* + \eta\hat{L}_t^* + \rho\hat{C}_t^* - \frac{1}{\phi}(\hat{l}_{N,t}^* - \hat{L}_t^*) - \hat{A}_{N,t}^* + (1 - \tilde{\lambda})\hat{l}_{N,t}^*.$$

We can manipulate the above aggregate supply equations to obtain the following form

$$\pi_{H,t} = k_H[\delta_{T,1}\hat{T}_{N,t} + \delta_{T,2}\hat{T}_t + \delta_{T,3}\hat{T}_{N,t}^R + (\tilde{\eta} + \rho)\hat{Y}_t^w + \delta'_{T,\xi}\xi_t] + \beta E_t\pi_{H,t+1}, \tag{5.12}$$

$$\pi_{N,t} = k_N[\delta_{N,1}\hat{T}_{N,t} + \delta_{N,2}\hat{T}_t + \delta_{N,3}\hat{T}_{N,t}^R + (\tilde{\eta} + \rho)\hat{Y}_t^w + \delta'_{N,\xi}\xi_t] + \beta E_t\pi_{N,t+1}, \tag{5.13}$$

$$\pi_{F,t}^* = k_F^*[\delta_{T,1}^*\hat{T}_{N,t}^* + \delta_{T,2}^*\hat{T}_t + \delta_{T,3}^*\hat{T}_{N,t}^{R*} + (\tilde{\eta} + \rho)\hat{Y}_t^w + \delta'_{T,\xi}\xi_t] + \beta E_t\pi_{F,t+1}^*, \tag{5.14}$$

$$\pi_{N,t}^* = k_N^*[\delta_{N,1}^*\hat{T}_{N,t}^* + \delta_{N,2}^*\hat{T}_t + \delta_{N,3}^*\hat{T}_{N,t}^{R*} + (\tilde{\eta} + \rho)\hat{Y}_t^w + \delta'_{N,\xi}\xi_t] + \beta E_t\pi_{N,t+1}^*, \tag{5.15}$$

where the coefficients δ are functions of the parameters of the model and ξ_t is a vector of all the shocks in the model. Moreover we note that

$$\hat{T}_{N,t} = \hat{T}_{N,t-1} + \pi_{N,t} - \left(\frac{1}{2}\pi_{H,t} + \frac{1}{2}\pi_{F,t}^* \right) \tag{5.16}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \tag{5.17}$$

$$\hat{T}_{N,t} - \hat{T}_{N,t}^* = (\hat{T}_{N,t-1} - \hat{T}_{N,t-1}^*) + \pi_{N,t} - \pi_{N,t}^*. \tag{5.18}$$

Table 1 presents the calibration of the parameters. The coefficient of risk aversion in consumer preferences is set to 2 as in Stockman and Tesar (1994) to get an intertemporal elasticity of substitution equal to 0.5. From Stockman and Tesar (1994), we borrow also the elasticity between traded and non-traded goods, $\varphi = 0.44$, and the share of traded goods in the consumption basket, $\gamma = 0.5$. The intratemporal elasticity of substitution between home and foreign traded goods is set such that $\theta = 1.5$ as in Backus et al. (1995). Consistent with several microeconomic studies, the Frisch elasticity of labor supply, $1/\eta$, is set

Table 1 Calibration of the parameters

$\beta = 0.99$	Intertemporal discount factor in consumer preferences
$\rho = -\bar{U}_{cc}\bar{C}/\bar{U}_c = 2$	Risk aversion coefficient in consumer preferences
$\varphi = 0.44$	Elas. of substitution between traded and non-traded goods
$\theta = 1.5$	Elas. of substitution between domestic and foreign traded goods
$\sigma = 7.88$	Elas. of substitution across goods within a sector
$\gamma = 0.5$	Share of traded goods in the consumption bundle
$\eta = \bar{V}_{ll}\bar{L}/\bar{V}_l = 0.25$	Inverse of the Frisch elasticity of labor supply
$\lambda = \bar{f}'\bar{l}/\bar{f} = 0.75$	Labor share
$1 - \tilde{\lambda} = -\bar{f}''\bar{l}/\bar{f}' = 0.25$	Curvature of the production function

to 4 and the labor share is set such that $\lambda = 0.75$. The discount factor β is assumed to be 0.99 and the elasticity of substitution for goods within a sector, σ , is set to 7.88 to imply a 15% mark-up as in Rotemberg and Woodford (1997).

5.1 Monetary Policy Shocks and Price Dispersion

Our first objective is to explore the importance of monetary policy shock in explaining consumer-price differentials. There is some evidence in the VAR literature that monetary policy shocks can be an important source of movements in the real exchange rate as discussed in Rogers (1999).¹³ We analyze the following issue. Consider the common monetary policymaker in the currency area that moves its instrument of policy in a discretionary way, how much price and inflation dispersion is going to be generated? In our context, we interpret a monetary policy shock as it is usually done in the VAR literature. We consider a one-time negative shock to the instrument of policy, the short-term nominal interest rate. This shock produces usually an hump-shaped positive response of output. We use the system of Eqs. 5.12 to 5.18 to determine the responses of relative prices and inflation rates to this hump-shaped positive response of output. To address this issue, we can then write Eqs. 5.12 to 5.18 as a system of the form

$$E_t \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} k_{t+1} \\ k_t \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \begin{bmatrix} k_t \\ k_{t-1} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} F_t$$

where $k_t = [\hat{T}_{N,t} - \hat{T}_{N,t}^*, \hat{T}_t, \hat{T}_{N,t}]$ and $F_t = [\hat{Y}_t^w, \xi_t]$ and A_j, B_j, C_j are matrices. Provided there are three stable eigenvalues of the above system, we can solve it obtaining the law of motion of the state variables as

$$k_t = -V_1^{-1}V_2k_{t-1} - V_1^{-1}\Lambda^{-1}E_t \sum_{T=t}^{\infty} \Lambda^{-(T-t)}V\tilde{C}F_T \tag{5.19}$$

where $V = [V_1 \ V_2]$ is the matrix of left eigenvectors associated with the unstable eigenvalues, where V_1 and V_2 are 3 by 3 matrices, Λ is a 3 by 3 diagonal matrices which contains the unstable eigenvalues and $\tilde{C} \equiv A^{-1}C$. In particular in the vector k_t we are interested to the variable $\hat{T}_{N,t} - \hat{T}_{N,t}^*$ which is proportional to the consumer price differential $\ln P_t/P_t^*$. The eigenvalues of the matrix $V_1^{-1}V_2$ characterize the intrinsic persistence that naturally arises when prices and relative prices are sticky across all sectors. In particular, under the assumption of symmetry in the rigidity across all the sectors in the area ($\alpha_T = \alpha_N = \alpha_T^* = \alpha_N^*$), a shock to the area output \hat{Y}_t^w does not produce any consumer price dispersion. Asymmetries in the structures of the AS equations, across countries and sectors, are critical for monetary policy to have a role

¹³See also Benigno (2004a) and Chari et al. (2002) on the issue whether a model with price rigidities and local currency pricing can account for monetary policy shocks to be important in capturing the volatility and persistence of real exchange rate.

in creating price dispersion. This is at the same time good and bad news, since it means that a monetary policy shock, in the way we interpreted it, does not create any consumer price dispersion but on the other side monetary policy is limited in its role to correct any unwanted price dispersion that can arise following other disturbances. In our experiment, we feed in Eq. 5.19 the estimated output response to a monetary shock found by Smets and Wouters (2003).¹⁴ As shown in the bottom chart of Fig. 9, a discretionary decrease in the interest rate produces an hump-shaped response of output. Output in the union increases up to 0.4% after 5 quarters and then converges back to the steady state. To perform this experiment, we need to calibrate additional parameters on top of the ones used in Table 1. We set all the elasticities in the aggregators of labor to infinity to get perfect labor mobility across sectors within a country. As already mentioned it is crucial to calibrate the degrees of rigidities that characterize the Calvo's mechanism of the four sector. We refer here to some micro and macro studies. On the micro side, Le Bihan and Sevestre (2004), by analyzing CPI micro-data for France, find that the average duration between price adjustment in service sector is 9.66 months while for food and goods is around 4 to 5 months. Costa Dias and Neves (2008) analyze consumer prices for Portugal and find relatively short "live" of posted prices, around three times a year on average with higher duration for services. Stahl (2004) shows that for producer prices in manufacturing sector the average duration can be of 9 months. On the opposite, there are studies that estimate AS equations for European countries, as Galí et al. (2001) and Benigno and Lopez-Salido (2006). They find that reasonable estimates of α for a country can be around 0.78.

In this work, we assume that one country has high rigidity and in particular a rigidity in the traded sector as in the work of Stahl (2004) equal to 0.67 (a duration of nine months) and a correspondingly rigidity in the non-traded sector such that $\alpha_N = 0.84$ so that the overall rigidity in this country is around 0.78. In the other country, we assume that the average duration in the traded sector as in line with the work of Costa Dias and Neves (2008) equal to 5 months and that in the non-traded sector is as in Le Bihan and Sevestre (2004) equal to 9 months. Our benchmark I calibration is then: $\alpha_T = 0.67$, $\alpha_N = 0.84$, $\alpha_T^* = 0.37$, $\alpha_N^* = 0.67$. We also propose two other possible cases: in benchmark II, we assume $\alpha_T = 0.67$, $\alpha_N = 0.84$, $\alpha_T^* = 0.6$, $\alpha_N^* = 0.75$, in benchmark III we assume $\alpha_T = 0.7$, $\alpha_N = 0.4$, $\alpha_T^* = 0.67$, $\alpha_N^* = 0.37$. In benchmark II and III with respect to I the two countries are more similar in terms of rigidities; in benchmark II the rigidities are increased in both sectors of country F , in benchmark III they are reduced in both sectors of country H .

The results are presented in the first three charts of Fig. 9, where the impulse responses of the log of the consumer-price differential and of the CPI inflation rates in both countries are plotted following the output shock.

¹⁴However, the model estimated by Smets and Wouters (2003) does not include the possibility of heterogeneity across countries.

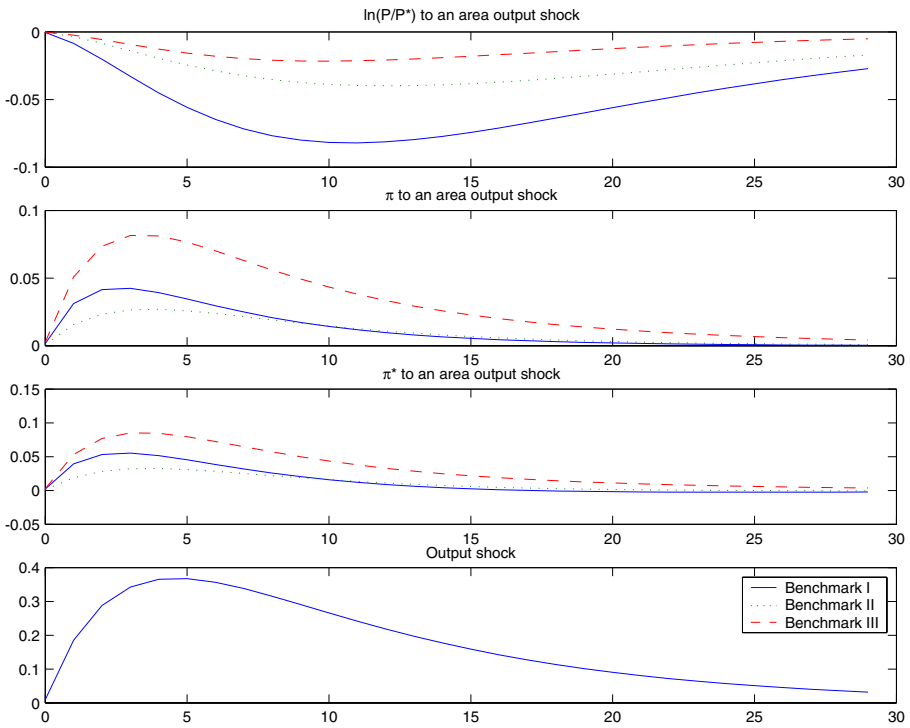


Fig. 9 Impulse response of output to a monetary policy shock (chart at the *bottom*) and impulse responses of price differentials ($\ln P/P^*$) and inflation rates (π and π^*) to an output shock

The impulse responses of the inflation rates are also consistent with VAR responses to a discretionary monetary policy shock. However, CPI inflation rate increases by a larger amount in the more flexible-price economy, but it is more persistent in the rigid economy. The overall picture that emerges is that the consumer-price differential is highly persistent. In the benchmark I case, the price differential has a peak after 10 quarters (i.e. after 5 quarters the peak of output). The differential reaches 8 basis point. This differential is of much smaller magnitude in the other two benchmarks, however the persistence is not altered by the different assumption. Overall a ‘discretionary’ monetary shock produces persistent price and inflation differentials although only of a small magnitude.

5.2 Nominal Rigidities and Price Dispersion

Under flexible prices, consumer-price differentials inherit the stochastic properties of the shocks. There is no persistence other than the one implied by the shocks. We now investigate how the introduction of sticky prices affects the response to the shocks and whether it increases persistence.

The answer depends on the policy rule used by the common monetary policymaker as we will further investigate in the next section. In this section, we assume that the policymaker sets its policy in a way to target a weighted average of the CPI inflation rates of the two countries, with weights given by the economic size of each country, in our case 1/2 for each country. The monetary policymaker then sets the average CPI inflation rate in the area equal to zero as

$$\frac{1}{2}\pi_t + \frac{1}{2}\pi_t^* = 0$$

at all times. The second important assumption in this experiment is the one on the persistence of the shocks. We assume that there are three possible degrees of persistence. Indeed we assume that shocks are autoregressive of type AR(1) as

$$s_t = \rho s_{t-1} + \varepsilon_t$$

for a generic shock s_t where ρ can be either 0 (for a white-noise process), or 0.9 for a persistence process or 1 for a unit-root process.¹⁵ We analyze impulse response functions to a one-time unitary increase (1% movement) in ε_t . The analysis of the response to a disturbance with zero correlation will be helpful to capture the persistence implicit in the model given by the combination of the stickiness of prices and policy rule; the analysis of more persistent process will be useful to understand how the persistence of the shock interacts with the persistence intrinsic in the model. Finally the analysis to permanent shocks can allow to investigate whether sticky prices are adding important transitional dynamics. In particular, the long-run response to a permanent shock exactly captures the flexible-price response, with the important caveat that short- and long-run are the same under flexible prices. Having in the same graph the flexible-price response would serve as an important benchmark for comparison.

Figure 10 presents the impulse responses of the consumer-price differential to productivity shocks. In the order (from the left to the right starting from the top), we analyze a one-time shock to the productivity in the home traded sector, to the productivity of the home non-traded sector, to a balanced increase in the productivity in both sectors in the home economy, to a balanced productivity increase in all the sectors of the area. As already discussed, a first presumption is that the stickiness of prices and, in particular, of relative prices can add persistence to the shock. And indeed this is the case when we focus on a white-noise shock. Under flexible prices the effect of this shock would disappear after one period, under sticky prices this happens after more or less 8 quarters. The second presumption is that sticky prices, while adding persistence, dampen the response to the shocks. Interestingly, this is not

¹⁵In the case of a unit-root process, we are allowed to use our log-linear approximation provided the shock lasts for a finite period of time.

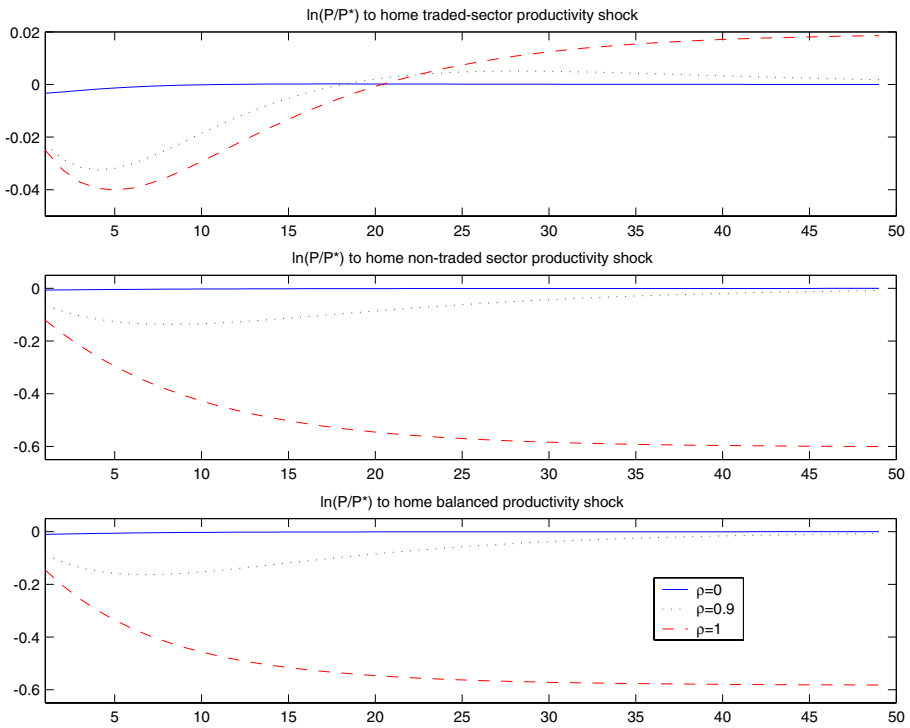


Fig. 10 Impulse response of price differentials ($\ln P/P^*$) to productivity shocks

necessarily a feature of our model. Indeed, by inspecting the impulse response to a productivity shock in the home traded sector, we see that actually sticky prices can revert the sign of the response and sometimes magnify it. Indeed, with our parametrization, the flexible-price response would require $\ln P/P^*$ to increase by two basis points. With sticky prices, we actually get a complete reversal of the Balassa-Samuelson effect with a relative magnification of the response which can achieve eighth basis points when shocks are more persistent. The intuition for this result is that even the terms of trade is very slow in the adjustment when sectorial prices are sticky. We note that under a more persistent shock, the response changes sign after 20 quarters. Indeed, when the shock is permanent it should reach the long-run positive value of two basis points. The picture is different when we analyze a shock to productivity in the non-traded sector or a balanced productivity shock in the home economy. These two cases are similar since the shock in the non-traded sector dominates. Indeed, we find that the existence of sticky prices mutes the response to these shocks. In particular the short-run response is negative and below or around -0.1% compared to -0.6% of the flexible-price model. The adjustment towards the long-run equilibrium proceeds slowly as in the previous case. A further interesting question to address is whether area-wide shocks can have important consequences. We remind that under flexible-price

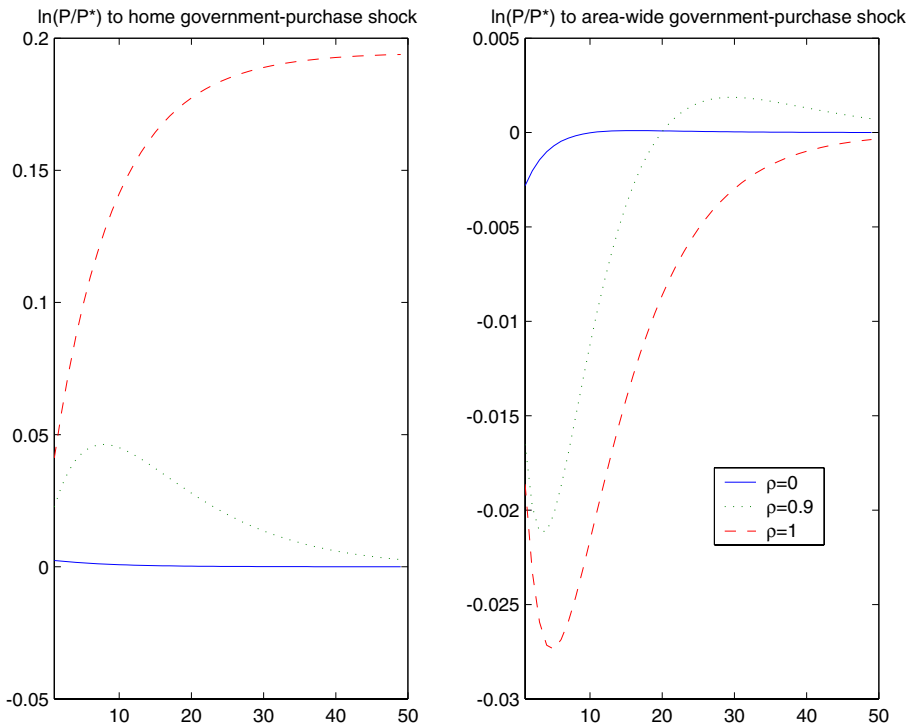


Fig. 11 Impulse response of price differentials ($\ln P/P^*$) to government shocks

these shocks have no effects on price differentials. We find that in the case of a balanced area-wide productivity shock, the deviations from full equalizations of the consumer price level are quantitatively insignificant.¹⁶

In Fig. 11, we focus on demand shocks. Namely, we consider a one-time shock to government purchase in the home country and then a common shock to both countries. The response to an asymmetric government purchase shock shows again the dampening effect of sticky prices as well as the additional source of inertia implied. Most interesting, we find that an area-wide governmental purchase shock can have some non-negligible effect on price differential. Given our parametrization, a symmetric government-purchase shock lowers the consumer price in the home economy relative to the foreign, since the home economy is more rigid. Indeed, a demand shock increases prices by a larger amount in the more flexible-price country. This is why consumer prices in the home economy fall below foreign. As a difference from other area-wide shock, government-purchase shocks affect only the non-traded sector, so it is not surprising that they can create larger asymmetric responses of prices, even if they are common to both countries.

¹⁶The symmetry imposed in the steady-state obviously plays an important role in this result.

6 Optimal Monetary Policy, Alternative Rules and Price Dispersion

In the previous section we have shown that a one-time monetary shock produces persistent but small price and inflation differentials. This section addresses the question of how the systematic component of the common monetary policy affects price dispersion and its persistence in response to the various shocks. We indeed compare the outcomes under different targeting and interest rate rules with the one that would be achieved under optimal policy. In this case, we consider that policy is chosen optimally to maximize a weighted average of the welfare of the two regions with equal weights

$$W_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{1}{2} U(C_s) + \frac{1}{2} U(C_s^*) - \frac{1}{2} V(L_s) - \frac{1}{2} V(L_s^*) \right] \quad (6.20)$$

under all the relevant structural equations of the model.¹⁷ We consider that the monetary policymaker is committed to maximize Eq. 6.20 at time t_0 under appropriate additional constraints given by a ‘timeless perspective’ commitment as discussed in Benigno and Woodford (2006a). These constraints are necessary to characterize the optimal policy problem in a recursive way and to obtain stationary policy rules. We aim to analyze the optimal allocation up to a first-order approximation to the solution of the non-linear problem. As shown in Benigno and Woodford (2006b), this can be seen as the solution of an equivalent problem in which a quadratic objective function is maximized under linear constraints—what is called a linear-quadratic (LQ) solution. Given the large dimension of our optimal policy problem, we use a Matlab-coded version of this LQ solution given by Altissimo et al. (2004). One of the advantage of using this LQ approach is that it will deliver a quadratic objective function through which we can make welfare comparisons.

We compare the optimal policy with other four monetary policy rules as described in the following table.

Rule 1 stabilizes a weighted average of the CPI inflation rate of the two countries, as in the previous section. Rule 2 is the standard Taylor rule and Rule 3 is a Taylor rule with a higher relative weight on output. Finally, Rule 4 considers an extreme case in which the CPI inflation differential across the two countries is closed to zero at all times.

Figures 12 and 13 show the impulse responses of the CPI inflation differential, the area-wide CPI inflation (a weighted average of the CPI inflation rates of the two countries) and of the area-wide output to: i) a shock to the productivity of the region H 's traded sector (Fig. 12); ii) a shock to region H 's government purchases (Fig. 13), respectively. We assume that the shocks are $AR(1)$ with correlation coefficient equal to 0.8.

¹⁷To simplify the analysis, we consider that all the monopolistic distortions are offset in the steady-state by appropriate taxation subsidies. However, we allow mark-ups to have small perturbations around this steady state.

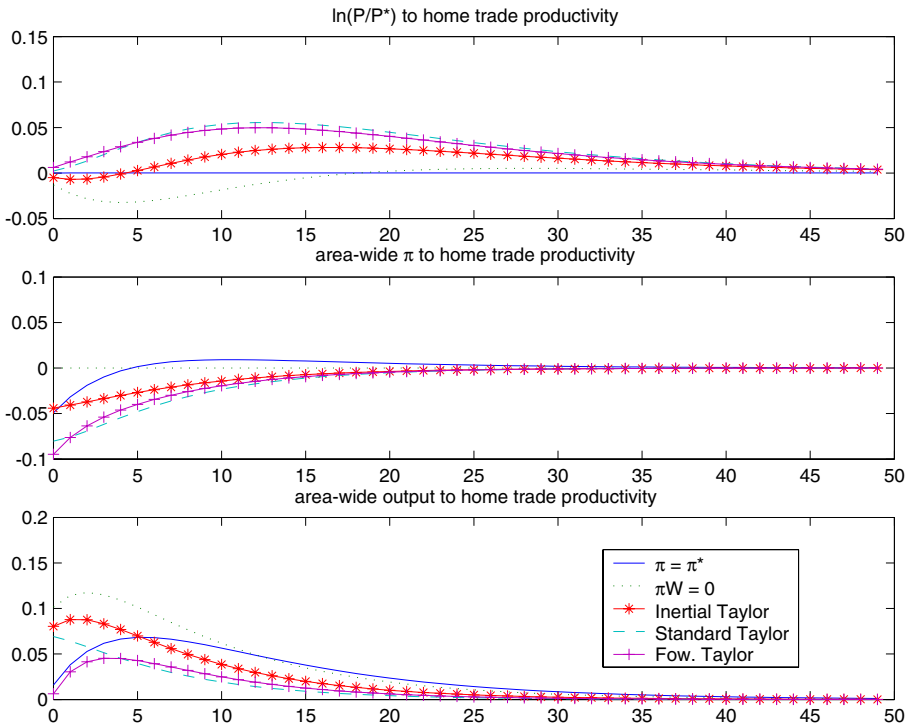


Fig. 12 Impulse responses of inflation differentials, $\Pi_t - \Pi_t^*$, area-wide inflation, Π_t^W , and area-wide output, $Y_{w,t}$ to a productivity shock in the traded sector in country H for alternative monetary policy rules

One feature that is common to all Figs. 12 and 13 is that the optimal rule tolerates sizeable and persistent inflation differentials, in particular for the two productivity shocks. The inflation differential is of the same sign and comparable in absolute value with the one produced by other rules for both productivity shocks. It is also broadly as persistent as under all the alternative rules for the two productivity shocks. By contrast, for the case of government shocks, the optimal rule generates a differential of smaller size than for most of the other rules. Overall, the comparison of Figs. 12 and 13 shows a high degree of alignment between the impulse responses under optimal policy and under the rule ensuring price stability, particularly as regards the responses under these two rules of inflation differentials and aggregate inflation.

It appears that permitting CPI inflation rates across the two countries to differ in a persistent way helps to stabilize overall inflation in the union as a whole. For productivity shocks specific to the traded sector the departure of inflation from the steady state under the optimal rule is relatively moderate. However the two Taylor rules and the policy that strictly closes the inflation differential are quite off in stabilizing the area inflation rate. Indeed, the short run is characterized by periods of deflation in which also the response of output

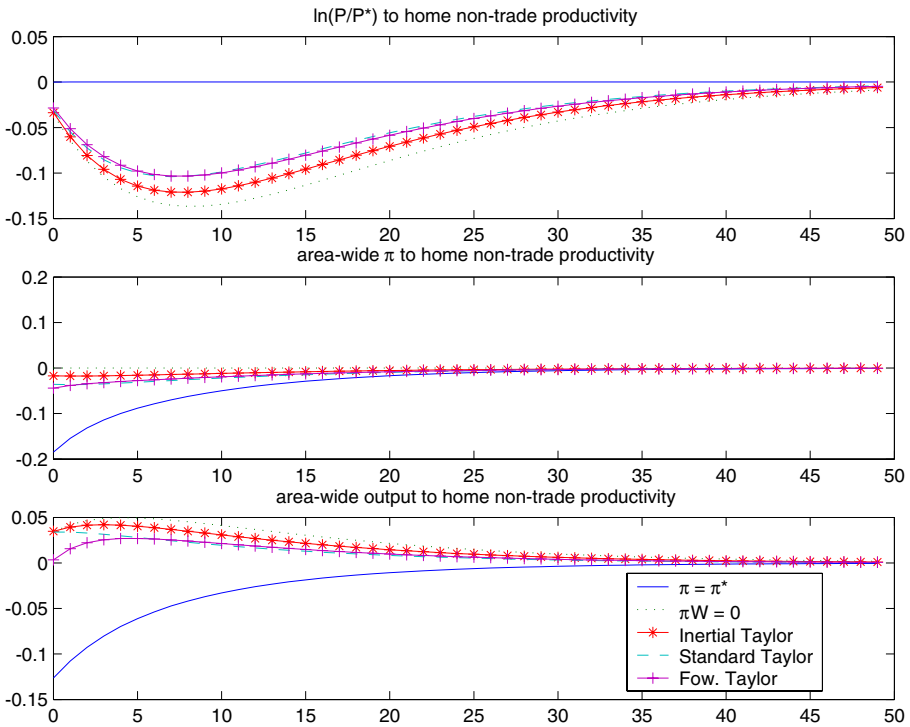


Fig. 13 Impulse responses of inflation differentials, $\Pi_t - \Pi_t^*$, area-wide inflation, Π_t^W , and area-wide output, $Y_{w,t}$ to a productivity shock in the non-traded sector in country H for alternative monetary policy rules

is muted compared to the optimal policy. A similar conclusion can be reached for the case of the productivity shock specific to the non-traded sector, where the optimal rule performs well both in terms of inflation stabilization and allows output to increase significantly. Note that in this cases small variations in the inflation differentials across the various rules considered can produce large deviations in the area inflation and output. This conclusion is shared more noticeably in the case of demand shocks, where the optimal policy allows to avoid any significant departure of inflation without producing any depressing effect on output.

7 Variance Decomposition of the Inflation Differential

The previous section has shown that the optimal response to the shocks allows in general for persistent and sometimes sizeable inflation differentials. We are now interested in evaluating in which proportion each of the shock contributes to the overall variability of the inflation differential implied by our model. In order to perform the variance decomposition exercise, we need to specify

Table 2 Monetary policy rules

(1)	$\pi_t^W = \frac{1}{2}\pi_t + \frac{1}{2}\pi_t^* = 0$
(2)	$r_t = 1.5 \pi_t^W + 0.5 y_t^W$
(3)	$r_t = \pi_t^W + 0.5 y_t^W$
(4)	$\pi_t = \pi_t^*$

and calibrate the stochastic properties of all the shocks considered in the model. While a fully fledged exercise would require the estimation of the whole model, here we resort to a simplifying short-cut and indeed assume that the productivity, the mark-up and the government shocks mimics the ones estimated by Smets and Wouters (2003) once aggregated across sectors and countries.

Starting from the productivity shock, we assume that each one of the four productivity shocks, $\{A_{T,t}, A_{N,t}, A_{T,t}^*, A_{N,t}^*\}$, follows independent $AR(1)$ process with lag coefficient of 0.823 and standard error of the innovation of 0.012; this ensures that the aggregate productivity, resulting from the weighted average of the countries/sectorial ones, exactly matches the process estimated by Smets and Wouters.

The shocks to government expenditure in each country are assumed to follow the same independent and very persistent $AR(1)$ process with coefficient on the lag term of 0.949 and standard deviation of the innovation of 0.0046.

Table 3 Variance decomposition under optimal policy

		$\pi - \pi^*$	$\frac{P_H}{P_F}$	$\frac{P_N}{P_N^*}$	$Y - Y^*$	$\frac{W}{P}$	$\frac{W^*}{P^*}$
Short run	Forecast error variance	0.0016	0.0110	0.0032	0.0112	0.0071	0.0069
	\hat{A}_T	0.03	0.33	0.03	0.31	0.14	0.19
	\hat{A}_N	0.26	0.02	0.26	0.04	0.02	0.06
	\hat{A}_T^*	0.02	0.62	0.02	0.55	0.46	0.37
Decomposition (fractions)	\hat{A}_N^*	0.65	0.00	0.65	0.00	0.22	0.19
	$\hat{\mu}$	0.00	0.01	0.00	0.02	0.02	0.01
	$\hat{\mu}^*$	0.04	0.02	0.04	0.06	0.14	0.18
	\hat{G}	0.00	0.00	0.00	0.01	0.00	0.00
	\hat{G}^*	0.00	0.00	0.00	0.00	0.00	0.00
Long run	Forecast error variance	0.0020	0.021	0.0163	0.033	0.0117	0.0117
	\hat{A}_T	0.03	0.40	0.01	0.39	0.24	0.19
	\hat{A}_N	0.30	0.02	0.42	0.05	0.22	0.04
	\hat{A}_T^*	0.01	0.57	0.00	0.52	0.31	0.28
Decomposition (fractions)	\hat{A}_N^*	0.62	0.00	0.57	0.02	0.17	0.41
	$\hat{\mu}$	0.00	0.00	0.00	0.00	0.01	0.01
	$\hat{\mu}^*$	0.03	0.01	0.00	0.01	0.05	0.06
	\hat{G}	0.00	0.00	0.00	0.01	0.00	0.00
	\hat{G}^*	0.00	0.00	0.00	0.01	0.00	0.00

Finally, for the mark-up shocks, we impose that they are common between tradable and non-tradable sectors in each economy and we also assume the same process for both economies but uncorrelated across economies. Furthermore, we impose that mark-ups are white noises with a standard deviation of 0.0066, in order to match the overall variability of the price and wage mark-up shocks of Smets and Wouters.

While the present calibration is admittedly ad hoc, it nonetheless provides some hints on the way in which different shocks contribute to the dynamic decomposition of the variance. Moreover, it allows to compare the decomposition for alternative monetary regimes. In this work, we consider the decomposition given by the optimal policy with respect to the first Taylor rule in Table 2.

Tables 3 and 4 present the variance decomposition for the cases of an optimal policy and Taylor rule, respectively. In each table, we show the forecast error variance in the short and long run—the latter coincides with the unconditional variance— for several variables of the model. In particular the first column considers the CPI inflation differential. We also present the fraction of the total variance to which each shock contributes for each of the respective variables.

Focusing on the CPI inflation differential we note that the long-run variance under optimal policy is equal to 0.0020 which implies a standard deviation of 0.44% quarterly which roughly corresponds to a standard deviation of 1.38%

Table 4 Variance decomposition under Taylor rule

		$\pi - \pi^*$	$\frac{P_H}{P_F}$	$\frac{P_N}{P_N^*}$	$Y - Y^*$	$\frac{W}{P}$	$\frac{W^*}{P^*}$
Short run	Forecast error variance	0.0016	0.0100	0.0035	0.0117	0.0038	0.0041
	\hat{A}_T	0.00	0.22	0.00	0.19	0.08	0.19
	\hat{A}_N	0.12	0.02	0.12	0.01	0.00	0.03
	\hat{A}_T^*	0.00	0.73	0.00	0.66	0.72	0.53
Decomposition (fractions)	\hat{A}_N^*	0.83	0.00	0.83	0.02	0.15	0.16
	$\hat{\mu}$	0.00	0.00	0.00	0.01	0.01	0.00
	$\hat{\mu}^*$	0.05	0.04	0.05	0.01	0.03	0.09
	\hat{G}	0.00	0.00	0.00	0.01	0.00	0.00
	\hat{G}^*	0.00	0.00	0.00	0.01	0.00	0.00
Long run	Forecast error variance	0.0022	0.0022	0.0182	0.0344	0.0056	0.0067
	\hat{A}_T	0.01	0.31	0.02	0.26	0.13	0.16
	\hat{A}_N	0.13	0.00	0.17	0.02	0.15	0.01
	\hat{A}_T^*	0.01	0.66	0.03	0.63	0.43	0.31
Decomposition (fractions)	\hat{A}_N^*	0.81	0.01	0.78	0.06	0.24	0.49
	$\hat{\mu}$	0.00	0.00	0.00	0.00	0.00	0.00
	$\hat{\mu}^*$	0.03	0.02	0.00	0.01	0.02	0.02
	\hat{G}	0.00	0.00	0.00	0.01	0.00	0.00
	\hat{G}^*	0.00	0.00	0.00	0.01	0.01	0.00

yearly.¹⁸ These numbers compare well with the empirical evidence for the euro area. Interestingly, under the sub-optimal Taylor rule the yearly standard deviation is slightly higher and equal to 1.45%. The important difference between the two policies is in how the shocks contribute to these variances. First we notice that productivity shocks matter for most of the total variability and in particular 90% of this variability is explained by the productivity shocks in the non-traded sectors. The striking difference between optimal and sub-optimal policies is in the importance of foreign versus domestic non-traded productivity shocks. Compared to the sub-optimal rule, optimal policy gives higher relevance to the productivity in the non-traded sector of country H and dampens the importance of the respective productivity in the other country. These results can be intuited by noting that in the current experiment the only elements of asymmetries across the sectors and countries are indeed the degrees of nominal rigidities. In particular we are assuming that the non-traded sector with lowest degree of nominal rigidity is that in the foreign economy, whose productivity indeed contributes mostly to the inflation differential. The optimal monetary policy corrects for this importance. Even productivity shocks in the non-traded sector of region H —which is the one with highest degree of rigidity— should matter and monetary policy should aim at this outcome.

Productivity shocks that affect the traded sector in general do not contribute much to the variability of the inflation differential since they are absorbed by terms of trade movements—indeed they are responsible for most of this volatility. Most important, they are the determinants of the variability of the output differential across countries. Productivity shocks are in general responsible for the variability of real wages too. Interestingly neither mark-up nor government-purchase shocks contribute significantly to the variability of the variables displayed in the table. For government-purchase shock the result is in line with Duarte and Wolman (2008).

8 Consequences of Incorrect Exchange Rate Parity

In this section, we consider the case of a country, H , that joins the union at a nominal exchange rate S_0 and real exchange rate $RE R_0 = S_0 P_0^*/P_0$. We assume that immediately after joining the union, it is understood that country H has a overall productivity level higher and incompatible with the current real exchange rate. In particular we assume that the new equilibrium real exchange rate, given by $RE R_\infty = S_\infty P_\infty^*/P_\infty$, should be depreciated by 1%.¹⁹

We compare the responses of the economy to this shock under three policy regimes. The first is the case of a currency area in which monetary policy is

¹⁸To convert on a yearly base we use a coefficient of 3.1 that takes into account of the autocorrelation of the series.

¹⁹To achieve this result we assume a permanent increase of 1/0.5127% in the productivity of both sectors of country H .

conducted optimally. The second considers again a currency area but in which monetary policy targets to zero the average CPI inflation of the area, i.e. $1/2 \cdot \pi_t + 1/2 \cdot \pi_t^* = 0$. The third regime is the case of economies with independent currencies in which policy is conducted optimally by the two separate monetary authorities in a way to maximize the cooperative objective function given by Eq. 6.20. We evaluate the welfare differences across these three regimes.

Joining a currency area represents a major change in the policy of a country, with respect indeed to the conduction of monetary policy. For the Lucas's critique, parameters in the reduced form of a model change when policy changes. However, dramatic changes in policy can have some effects on so called 'deep' parameters of a model. In our case, a monetary union can affect price and wage rigidities in a country, its degree of sectorial labor mobility, its trade pattern of goods and assets. As a first step, we keep all the structural parameters of the model constant. This is one of the reasons why this experiment is biased toward giving an upper bound on the costs of currency areas. There are other important reasons not developed here for this bias. Indeed, with independent currencies, monetary policy in both countries might not be necessarily conducted optimally and cooperatively. It can be the case that a currency area with an optimizing monetary authority can perform better than an economy with two separate monetary authorities acting in a non-cooperative way. Finally, there can be movements of the nominal exchange rate that are not directly related to the policy chosen or to the fundamental shocks of the model which can exacerbate the welfare costs when exchange rate is free to float.

Leaving aside these considerations, in our analysis the welfare under an optimally-designed currency area is always lower than the welfare under an optimally-designed floating exchange rate regime since in the latter case there is an extra degree of freedom over which policymakers can optimize.

In our experiment, we continue to assume that the monopolistic distortions are completely offset by appropriate subsidies in all sectors. Given this assumption and our calibration, the optimal cooperative policy under floating exchange rates coincides with the flexible price allocation. In this allocation the new equilibrium is achieved immediately after the new level of productivity is known. Table 5 discusses the decomposition of the long-run adjustment in the real exchange rate between the CPI inflation differential and the exchange rate depreciation or appreciation. In the floating exchange rate regime the 1% depreciation of the real exchange rate is brought about by a 2% depreciation of the nominal exchange rate which feeds into an increase of region H 's CPI

Table 5 Decomposition of the long-run adjustment in the real exchange rate $\Delta rer = \Delta s + \pi^* - \pi$

	Δs	π^*	π
Optimal policy under floating regime	2%	-0.5%	0.5%
Optimal policy in a currency area	0%	1.18%	0.18%
Area-CPI target in a currency area	0%	0.5%	-0.5%

prices of 0.5% (since the share of foreign traded good in the overall CPI basket is 1/4) and a proportional fall in foreign CPI prices of 0.5%. As shown in Fig. 14 the optimal cooperative policy with independent currencies requires zero producer price inflation in all sectors and in both countries. This implies that GDP inflation remains as well zero.

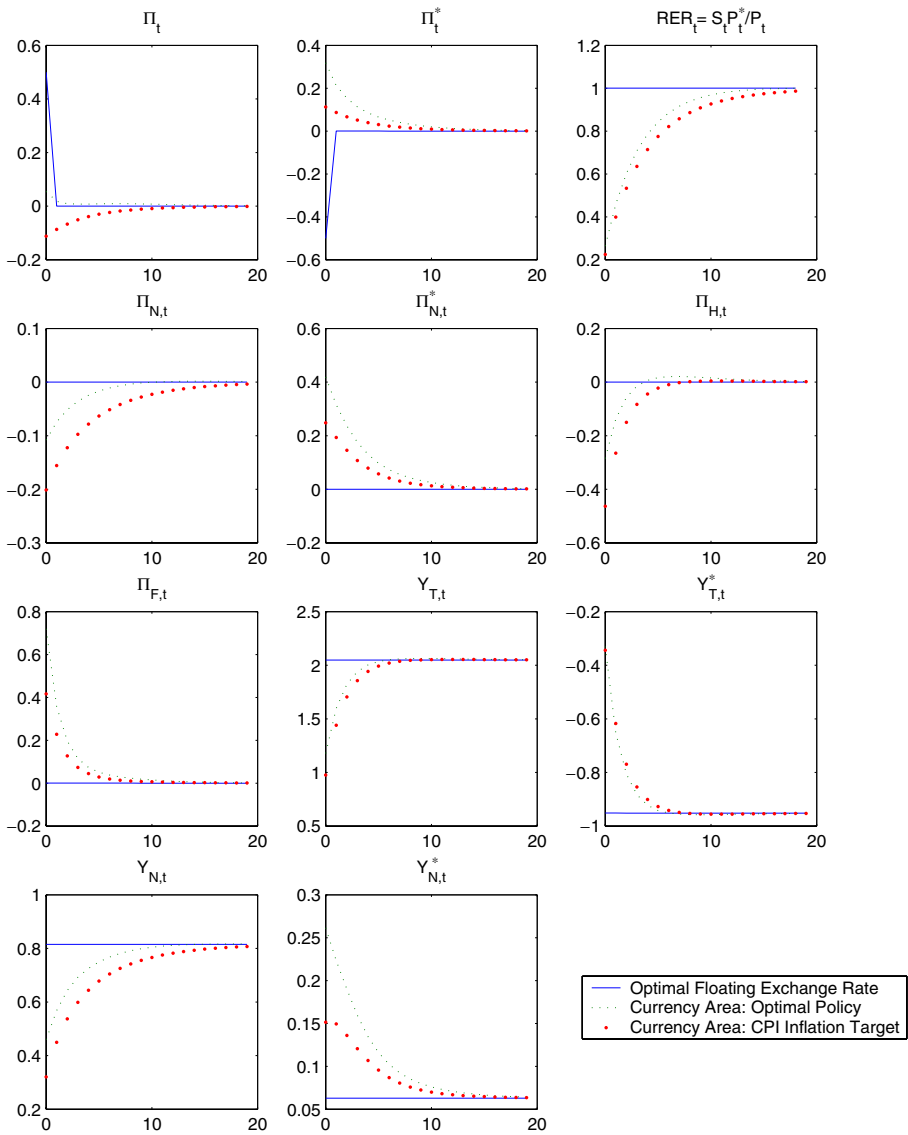


Fig. 14 Adjustment to a real exchange rate misalignment of selected variables under alternative monetary policy rules

On the opposite, in a currency area, all the adjustment should be brought about by movements in prices. First, we look at the optimal monetary policy. Country *H*, the one that experiences the favorable productivity shock, experiences a deflation in both the non-traded and traded sectors. Output increases in both sectors but slowly and below potential for at least 5 quarters. Country *F* instead benefits of an expansion and suffers of inflation in both of its sectors. The overall impact on CPI inflation is positive for both countries, but higher in country *F*. This is not surprising since foreign CPI should increase more than domestic CPI to achieve the new long-run equilibrium real exchange rate. Although country *H* experiences deflation and output below potential, its CPI increases because imported goods are now more expensive. Indeed, most of the adjustment is brought about by an increase in the prices of foreign traded goods. Overall, average CPI inflation in the area should show a positive sign for some quarters. In this case, optimal policy can be understood more simply as a strict target of the area CPI inflation over a medium horizon (around 6 quarters).

The real exchange rate moves slowly towards the new equilibrium level. CPI indices go up in both countries and more in country *F*, the opposite of what occurs under floating exchange rate—the difference being explained by indeed the movements of the nominal exchange rate.

We now consider the policy of strict targeting of the area CPI inflation in all periods. In this case CPI inflation cannot go up in both countries and should move instead specularly across the two countries. It follows that the adjustment of price differential to the new long-run equilibrium is slower. The deflation is stronger in both sectors of region *H* and output in these sectors is much more depressed and for longer periods below potential than under the optimal policy.

An important observation to draw from this example is that price differential at the consumer level may naturally arise as an equilibrium adjustment. However, it is not only important the dimension of the dispersion but also its dynamic. The fact that the same dispersion can be achieved with more adjustment of consumer prices of a country instead of another can have important welfare consequences. We study this issue more deeply.

We use our quadratic objective function to evaluate welfare differences between the three regimes. In particular we convert these differences in gains or losses of percentage of steady state consumption in an alternative economy in which consumption and labor are always constant. Our calculations show that under this experiment the costs of having the incorrect nominal and real exchange rates amounts to 0.093% of steady-state consumption when policy is conducted optimally and to 0.13% when policy follows the strict CPI inflation targeting procedure. These costs are two times higher than the ones Lucas (1987) found for business cycle movements. They are much more important if we consider that the experiment is built under the assumption of a one time—though permanent—shock that involves a misalignment of the real and nominal exchange rate of just 1%. By a parallel, a misalignment of 5% would

imply costs around 0.5% and 0.65% of steady state consumption under optimal policy and strict CPI inflation targeting policy, respectively.

We now analyze an alternative scenario in which the currency area involves more flexibility in the pricing decisions of firms. We assume a drastic reduction in the parameters measuring the degree of rigidities, α , and set them all equal to 0.37 implying an average duration of price contracts just above 1 quarter for all sectors. Our analysis is facilitated by the fact that the optimal cooperative monetary policy under floating exchange rate is independent of the assumption on the degrees of price rigidities. We find that under this new calibration, the costs of a 1% misaligned real exchange rate amounts to 0.036% of steady state consumption. They are reduced by more than a half with respect to the previous parametrization. Greater flexibility in the goods market can mitigate in a substantial way the costs of joining the union at the wrong nominal exchange rate. These results are in line with the high cost of nominal inertia that Canzoneri et al. (2004) found in closed-economy versions of this model. Our model would further suggest that the consideration of the slow adjustment in the intersectoral relative prices, because of sticky prices, can amplify more these costs. Interestingly when all the sectorial degrees of rigidity are equalized the optimal policy coincides with the strict targeting of the area CPI inflation. It is indeed the case, by inspection of Fig. 14 that another simple way through which optimal policy can be understood following this particular shock can be that of targeting a rigidity-adjusted weighted average of GDP inflation rates in the area which actually coincides with targeting average CPI inflation when all the sectors have the same degree of rigidity.²⁰

9 Conclusions

This paper provides an empirical and a theoretical analysis of the factors accounting for inflation differentials in an heterogeneous currency union. Some implications for policy are considered. This Section discusses some possible avenues for future work and points to the main limitations of the theoretical analysis.

There are two main limitations to the framework developed in this paper. First, the assumption of perfect international risk-sharing—introduced for analytical convenience—closes down one potentially important source of inflation differentials. Second, the model assumes away investment. This precludes the analysis of another potentially important source of differential inflation dynamics. For instance, it has sometime been argued that persistent inflation in a region within a monetary union could generate procyclical real interest rates: with a common nominal interest rate, the high-inflation countries will

²⁰This result is similar to Benigno (2004b), with the important qualification that his model considers only traded sector. As in Benigno (2004b) the appropriate inflation target involves prices of the sectors with nominal rigidities.

have lower real interest rates than their low-inflation fellow members. This could in principle amplify the local business cycle, endogenously generating inflation persistence and possibly local asset price booms. At the same time, a protracted positive inflation differential cumulates into real appreciation, which over time would offset the previous effect.

As regards avenues for future work, one additional exercise of interest that can be conducted within the current framework would be to use the model to account for the component of inflation differentials that arise as a consequence of the interaction between common external shocks and the heterogeneity among euro area members in terms of their trade linkages with non-EMU partner countries. Honohan and Lane (2003, 2004) have emphasized the relevance of this source of inflation differentials in the euro area, showing that movements in national trade-weighted multilateral exchange rates are significantly correlated with national inflation rates. Even if our model relates to a currency area with no external trade, the previous channel of inflation differentials stemming from common external shocks could be proxied in our framework by balanced productivity shocks which interact with heterogeneous country structures.

The framework developed in the paper can be extended to tackle the previous limitations. In addition, a number of extensions could be of interest. First, including a role for generating inflation differentials from other sources, like the presence of international price discrimination in the tradables sector and the existence of non-traded distribution services in determining the consumer prices of tradables would increase realism in the model. Another potentially interesting route would be to allow for firm-level and industry-level heterogeneity in productivity and trade costs, which could generate further sources of price dispersion and persistence, through the endogenous entry of firms and shifts in the non-traded/traded margin, alike to effects present in Bergin and Glick (2007).

Finally, a further avenue for future work is to analyze the interdependence between the optimal area-wide monetary policy rule and optimal fiscal policies at the national level. Related to the latter, an interesting extension could be to look at 'optimal' indirect tax rate: an intensely debated policy issue in a number of euro area countries has been whether price increases in various sectors should be offset by endogenous adjustment in the level of indirect taxation. The level of direct taxation on labour has also been discussed as a policy instrument: a rise in demand for local labour can in part be met by encouraging greater labour force participation and increased work effort through a reduction in labour taxes.

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