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MONETARY POLICY IMPLEMENTATION FRAMEWORKS: A COMPARATIVE ANALYSIS

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We compare two stylized frameworks for the implementation of monetary policy. The first framework relies only on standing facilities, whereas the second framework relies only on open-market operations. We show that the Friedman rule cannot be implemented when the central bank uses standing facilities only. For a given rate of inflation, we show that standing facilities unambiguously achieve higher welfare than just conducting open-market operations. We conclude that elements of both frameworks should be combined. Also, our results suggest that any monetary policy implementation framework should remunerate both required and excess reserves.

Keywords: Monetary Policy Implementation, Corridor System, Standing Facilities, Open-Market Operations

1. INTRODUCTION

In this paper, we compare the performance of two frameworks for the implementation of monetary policy. In the first framework, the central bank (CB) operates a channel system. In a channel system, the CB offers two facilities: One lending facility where it stands ready to lend funds against collateral and one deposit facility where it accepts and remunerates deposits. The CB does not intervene in any other way. In a second framework, the CB conducts open-market operations (OMOs) and does not provide access to any facilities. Our analysis makes it possible to clearly identify the costs and benefits of these two polar cases and it suggests ways in which these systems can be combined.

In practice, CBs often adopt a mix of these two approaches, but with different emphases. Some systems rely primarily on the use of facilities, whereas other systems rely on OMOs and may not provide facilities. For example, the Federal

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Reserve uses OMOs to implement its monetary policy but does not operate facilities. Although the discount window is a source of credit for banks, it is not considered a regular source of funding.[1] At the other end of the spectrum are CBs that run "narrow corridors," such as the Bank of Canada or the Reserve Bank of Australia. In such systems, the credit and deposit facilities operated by the CB play a preeminent role. Our goal is to shed light on some of the welfare costs and benefits of different approaches to implementing monetary policy. We are also interested in monetary policy implementation because the Federal Reserve has received authority to pay interest on reserves as of October 2008. This authority gives the Federal Reserve the opportunity to modify how it implements monetary policy in important ways.[2]

We base our analysis on a variant of the model of Berentsen and Monnet (2008), hereafter BM. They use a tractable general equilibrium model of monetary policy implementation using only facilities (a pure channel system). In this model, agents hold money to trade. Modeling the reasons that agents (or banks) hold money is important because the monetary policy implementation framework can modify their incentives to hold it. We depart from BM's assumption that agents pledge goods as collateral. Instead, we introduce government bonds as eligible collateral. Agents can exchange money for bonds, with each other and with the CB, in a bonds market. This market allows agents to adjust their money holdings after they observe a signal about whether or not they are likely to need money. As in BM, we consider monetary policy implementation frameworks with a daily reserve maintenance period, in which the level of required reserves is normalized to zero.

We obtain two main results. First, a pure channel system is unable to achieve the efficient allocation. The intuition is simple: In our model, welfare increases as the growth rate of the money supply (and, thus, inflation) decreases. Therefore, the money stock should shrink at the rate of time preference because this policy, called the Friedman rule, eliminates the opportunity cost of holding money. With a pure channel system, the money stock only decreases when agents borrow at the CB's lending facility. Indeed, the interest rate these agents pay the central bank reduces the supply of money. Conversely, the money stock increases when agents deposit at the CB's deposit facility. In this case, the interest rate that the CB pays the agents increases the supply of money. The change in the money stock is a combination of these effects. In our model, the stock of money shrinks when the lending rate is higher than the deposit rate. However, the central bank cannot set its lending and deposit rates in a way that lowers the rate of growth of the money supply enough to achieve the Friedman rule. When the opportunity cost of holding money becomes sufficiently low, agents suffer little cost from carrying money, and they will not want to borrow from the CB if the interest rate is too high. This effect will limit the CB's ability to lower the money supply further. As a result, the money growth has to be strictly higher than the Friedman rule. Hence, a pure channel system is incompatible with efficiency.

Our second result is that the channel system is better than OMOs for any rate of inflation that can be achieved in both frameworks. We show that if the inflation rate is high enough, a CB only needs a deposit facility to achieve the same allocation as a CB using OMOs. By using the lending facility and setting its rate slightly above the deposit rate, the CB can reduce inflation as described above. By discriminating between borrowers and depositors (i.e., charging them different rates), using both facilities creates a transfer from agents that borrow from the CB to agents that lend to the CB. This transfer increases welfare because it redistributes resources among risk-averse agents. However, such discrimination is not possible when the CB conducts OMOs, as the price of bonds is the same for all.

The recent literature on the implementation of monetary policy, as well as the literature on banks' reserves management problem initiated by Poole (1968), is mainly confined to analyzing the issue of monetary policy implementation within a given framework and does not contrast the performance of different systems in welfare terms. For example, the literature has been concerned with the behavior of the fed funds rate [see, for instance, Hamilton (1996), Furfine (1999)], or reducing the volatility of the interbank market rates [Whitesell (2006a, 2006b); Holthausen et al. (2007)]. Woodford (2000) argues that the CB can implement monetary policy even if it does not have direct control of the money supply. Goodfriend (2002) proposes a monetary implementation framework in which the CB pays interest on reserves at the policy rate and expands the supply of reserves considerably. Ennis and Weinberg (2007) also consider the benefits of paying interest on reserves and the impact it has on daylight credit in a simple model.

The remainder of the analysis proceeds as follows. In Section 2 we describe the model. In Section 3 we study an implementation framework that relies on OMOs. In Section 4 we study an implementation framework that relies on a channel. Section 5 compares the two frameworks, and Section 6 discusses our results and concludes.

2. THE ENVIRONMENT

There is a [0, 1] continuum of infinitely lived agents, which we also call banks. Their discount factor is β. In addition, there is a government and a CB. Time is discrete and three perfectly competitive markets open sequentially in each period. These markets—the bonds market, the goods market, and the settlement market—are described below. There are two assets in this economy, money and government bonds.

The government behaves like an automaton in our model. It issued a fixed number, $\bar{B}$, of infinitely lived bonds (consols) that pay a real return R each period. We assume that these bonds are book entry at the CB, so that they are illiquid.[3] The government simply finances the return using a lump-sum tax, without maximizing any objective function.

The bonds market opens first. In this market, agents and the CB trade money for bonds. Agents receive a signal $\varepsilon \in \{0, 1\}$ at the opening of the bonds market, before they trade. Agents who receive $\varepsilon = 1$ know they will need money in the next market with strictly positive probability, whereas agents with $\varepsilon = 0$ know

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they will have no need for money in the goods market. We let $\mu \in (0, 1)$ be the probability that $\varepsilon = 1$.[4]

Second, the goods market opens. In this market, agents can produce, consume, or trade a perishable good. However, agents cannot trade bonds with the CB or with each other.[5] Each agent receives a trading shock: With probability $1 - n$, an agent can consume but cannot produce; we refer to these agents as consumers. With probability n, an agent can produce but cannot consume; we call these agents producers. Consuming q units of goods in the second market generates utility $\varepsilon u(q)$, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$, and $u'(\infty) = 0$. Producing q units of output has a utility cost $c(q) = q$. The first-best allocation in the goods market is denoted q^* and satisfies $u'(q^*) = 1$. All trades are anonymous and agents' trading histories are private information.[6] Because producers require immediate compensation for their production effort, money is essential for trade.[7]

An agent who receives $\varepsilon = 0$ in the bonds market knows he or she does not need money in the following goods market. Indeed, such an agent is a producer with probability n, and never derives utility from consumption, because $\varepsilon = 0$. In either case, this agent does not need money to buy goods. In contrast, an agent with $\varepsilon = 1$ derives utility $u(q)$ from consuming q units of the good if he or she becomes a consumer, which happens with probability $1-n$. This agent will not need money if he or she becomes a producer. Hence, agents with $\varepsilon = 1$ bring money into the goods market in case they are consumers, but will not use the money if they turn out to be producers. We will see later that this creates a potential role for a deposit facility, where agents can earn interests on their idle money.

In the goods market, the CB can operate facilities. Agents can borrow money at the CB's lending facility or deposit money at the CB's deposit facility, after they observe their idiosyncratic trading shock. All the CB's loans must be secured with bonds. The CB offers nominal loans at an interest rate i_l and promises to pay interest rate i_d on nominal deposits, with $i_\ell \geq i_d$.[8] The CB operates the facilities at zero cost. All contractual obligations are settled in the next settlement stage.

Third, and last, there is the settlement stage. In the settlement stage, agents can produce and consume a general good, settle their claims with the CB, and trade bonds with one another or the CB. General goods are produced solely from labor, according to a production technology with constant return to scale. Producing one unit of the consumption good generates one unit of disutility, whereas consuming one unit of it gives one unit of utility.[9] The lump-sum tax needed to finance the interest payment on the government's debt is levied during the settlement stage. We assume that only the government has the power to raise taxes. However, the CB can transfer money lump sum to agents during the settlement stage.

In the remainder of the paper, we assume that the CB operates only a subset of its monetary policy implementation tools. In the next section, the CB is active only in trading bonds and shuts down access to the facilities. In the following section, the CB is inactive in the bonds market but provides access to the facilities.

Our assumptions on the timing of events are motivated by CBs' practice. CBs that rely primarily on OMOs often intervene in markets early in the morning, before most of the banks' payment activity takes place. For instance, in the United States, the Desk at the Federal Reserve Bank of New York conducts its intervention around 10:30 a.m.[10] In contrast, banks can access facilities at any time during the day and even after the money market is closed in some instances. Therefore, banks that are short of money but still need to make an unexpected payment can do so by accessing the lending facility. For example, Hartmann et al. (2001) report that the euro area money market opens at around 8:00 A.M. and closes at around 5:45 P.M. Banks in the euro area can access the facilities at their national CB until 6:15 p.m., or 15 minutes after TARGET (the euro area Real-Time Gross Settlement system) closes.

To capture these institutional aspects, we assume that the CB conducts OMOs before banks know their exact need for money. Banks receive some information before they can trade with the CB, in the form of a noisy signal. Our bonds market is the equivalent of a secured interbank market. In contrast, banks can access facilities even after the payment system closes; i.e., after the goods market closes.

Nevertheless, we show in Section A.5 in the Appendix that our results are robust to a change in the timing of OMOs. We do so by allowing the CB to intervene in markets after agents have observed their shock. Although the equilibrium conditions are naturally somewhat affected, our results are unchanged.[11]

Consistent with CB practice, we assume that agents have to pledge collateral when they access the lending facility. Collateral protects CBs from the risk that a borrowing agent defaults. We do not introduce default risk explicitly in the model, but it would be straightforward to do so.[12]

In our model, there is no difference between a bank and an agent, and we use these terms interchangeably.[13] Our interpretation is that banks are intermediaries and each bank can serve exactly one agent, or depositor. Competition between banks implies that a bank must maximize the expected utility of depositors. Under this interpretation, the sole role of banks is to facilitate payments. If the bank's depositor needs money, then the bank accesses the bonds market or the facilities on behalf of the depositor. In this sense, there is no distinction between what a bank does and what an agent would do.

For a model of monetary policy implementation, this limited role for banks is sufficient. Hence, we do not assume that banks play any other role. Introducing another role for banks, such as the provision of liquidity insurance modeled by Diamond and Dybvig (1983), would be interesting but would complicate the model. This is left for future research.[14]

3. OPEN-MARKET OPERATIONS

In this section we study our economy when the CB engages in OMOs in the bonds market but does not operate facilities. Recall that the government behaves like an automaton: Having issued $\bar{B}$ bonds, it uses lump-sum taxes on the agents to raise the amount necessary to finance the real gross interest rate R. In other words, the

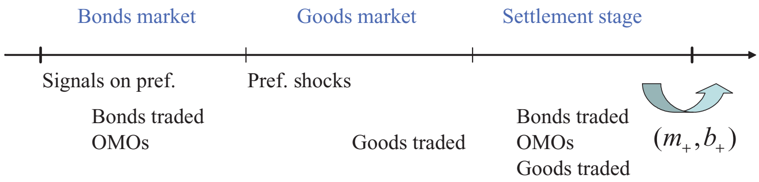

FIGURE 1. Timeline with OMOs.

government does not have an active policy regarding the supply of bonds. The timeline is provided in Figure 1.

First, it is useful to describe the evolution of the money stock when the CB conducts OMOs. Let M denote the stock of money at the beginning of the bonds market in a generic period.[15] We let B and b denote the stocks of bonds held by the CB and by agents, respectively, at the beginning of the bonds market; hence market clearing requires $\bar{B} = B + b$. During the bonds market, the CB buys Y bonds from the private sector at price η (the usual convention applies that a negative purchase is a sale). Therefore, the stock of money at the end of the bonds market is given by $M' = M + \eta Y$. In the settlement stage, agents and the CB get the interest rate on their bonds, and the CB buys bonds Y' at the price ρ and makes a lump-sum transfer $\tau \geq 0$ to agents. We restrict the lump-sum transfer to be positive, as we assume that the CB has no fiscal authority. Also, the CB rebates the interest on its bond holdings to the government, and we assume that the government redistributes it lump sum to the agents. Hence, the stock of money at the end of the settlement stage or at the start of the next period is $M_+ = M' + \rho Y' + \tau$. Therefore, the stock of money evolves according to the equation

$$M_+ = M + \eta Y + \rho Y' + \tau. \quad \mathbf{(1)}$$

Notice that money growth is endogenous, as prices η and ρ are equilibrium objects.

3.1. Equilibrium

We focus on symmetric and stationary equilibria in which all agents follow identical strategies and where real allocations are constant over time. Let ϕ denote the real price of money in the settlement stage in a generic period. In a stationary equilibrium, end-of-period real money balances are time-invariant, so that

$$\phi M = \phi_+ M_+. \quad \mathbf{(2)}$$

Define the growth rate of the money supply, i.e., inflation, as $\gamma \equiv M_+/M = \phi/\phi_+$.

We solve the equilibrium backward, first considering the settlement stage, then the goods market trades, and finally the bonds market. We let $Z(m, b)$, $W(m, b)$, and $V(m, b)$ denote the expected value of entering the bonds market, the goods market, and the settlement market, respectively, with m units of money and b bonds.

Settlement stage. In the settlement stage, the problem of an agent with portfolio (m, b) is

$$V(m, b) = \max_{h, m_+, b_+} -h + \beta Z(m_+, b_+),$$

$$\text{s.t.} \quad \phi m_+ + \phi\rho b_+ = h + \phi(m - T) + \phi(\tilde{R} + \rho)b + \phi\tau,$$

where h is hours worked in the settlement stage, and T is the lump-sum tax imposed by the government to finance the real interest rate R on bonds. It is convenient to work with the nominal interest rate instead, so that the government pays a nominal rate $\tilde{R}$ that is adjusted to inflation so that the real rate is $\phi\tilde{R} = R$. Using the budget constraint to eliminate h in the objective function, we can write

$$V(m, b) = \max_{m_+, b_+} \phi(m - T) + \phi(\tilde{R} + \rho)b + \phi\tau - \phi m_+ - \phi\rho b_+ + \beta Z(m_+, b_+)$$

$$= \phi(m - T) + \phi(\tilde{R} + \rho)b + \phi\tau + \max_{m_+, b_+}\{-\phi m_+ - \phi\rho b_+ + \beta Z(m_+, b_+)\}.$$

Notice that our assumption of a quasi-linear utility function implies that individual wealth can be pulled out of the maximization problem. As a consequence, agents' wealth does not affect their portfolio decisions. We obtain the first-order conditions for an interior solution:

$$\beta Z_{m_+} = \phi, \qquad (3)$$

$$\beta Z_{b_+} = \phi\rho. \qquad (4)$$

Because the marginal disutility of working is 1, the utility cost of acquiring one unit of money in the settlement stage is $-\phi$ and the utility cost of acquiring one unit of bonds in the settlement market is $-\phi\rho$. Because individual wealth does not affect the portfolio decision of an agent, his or her private history does not matter and all agents enter the following period with the same amount of money and the same quantity of bonds, as implied by equations (3) and (4). The envelope conditions are

$$V_m = \phi \quad \text{and} \quad V_b = \phi(\tilde{R} + \rho). \qquad (5)$$

Market clearing also requires that

$$B + Y + Y' = \bar{B} - b_+, \qquad (6)$$

and so $b_+ = b - (Y + Y')$.

Goods market. The goods market operates as an anonymous Walrasian market. Therefore, agents make their decision taking the price of goods as given. Anonymity preserves the need for money. Upon entering the goods market, agents know their preference shock $\varepsilon \in \{0, 1\}$ and learn their trading shock: With probability n they are producers and with probability $1 - n$ they are consumers. The

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expected payoff of an agent with ε and portfolio (m, b) is denoted

$$W^{\varepsilon}(m,b) = (1-n)W^{\varepsilon,c}(m,b) + nW^{\varepsilon,p}(m,b). \tag{7}$$

Let q_p^{ε} denote the quantities produced in the goods market by agents with signal ε. Let q denote the quantity of good consumed by agents in the goods market (only agents with $\varepsilon = 1$ consume). Producers solve the following problem:

$$W^{\varepsilon,p}(m,b) = \max_{q_p^{\varepsilon}}\left[-q_p^{\varepsilon} + V(m + pq_p^{\varepsilon}, b)\right].$$

Using (5), we know that the value of an additional unit of money is $V_m = \phi$, so that the first-order condition reduces to

$$\phi p = 1. \tag{8}$$

At the margin, producers must be compensated for the cost of making an additional unit of good, which is 1. They receive p units of money, which can be transformed into ϕp units of the general goods in the next settlement stage. Therefore, production requires that $\phi p \geq 1$ and, in equilibrium, $\phi p = 1$, as otherwise producers would produce an infinite amount. Hence, producers are indifferent regarding the amount they produce and we can define $q_p \equiv q_p^1 = q_p^0$. Market clearing requires $q_p = (1-n)\mu q/n$. The marginal value of money and bonds for producers in the goods market are

$$W_m^{\varepsilon,p} = V_m = \phi \quad \text{and} \quad W_b^{\varepsilon,p} = V_b = \phi(\tilde{R} + \rho), \tag{9}$$

respectively. Producer can only use money at the settlement stage, so the value they assign to money must be the same in the goods market and in the subsequent settlement stage. This is also true of the marginal value of bonds. An agent with signal $\varepsilon = 0$ does not wish to consume during the current period, so $W^{0,c}(m,b) = V(m,b)$. Consumers solve the following problem:

$$W^{1,c}(m,b) = \max_q, \quad u(q) + V(m - pq, b),$$
$$\text{s.t.} \quad pq \leq m.$$

Using (5) and (8), the first-order conditions can be written as

$$u'(q) = 1 + \bar{\lambda}_m^1, \tag{10}$$

where $\phi\bar{\lambda}_m^1$ denotes the real Lagrange multiplier on the budget constraint.[16] If it is binding, then $u'(q) > 1$, which means that trades are inefficient. Otherwise, trades are efficient. Using the envelope condition and (10), the marginal value of money in the goods market for agents with signal $\varepsilon = 1$ is

$$W_m^{1,c} = \phi u'(q). \tag{11}$$

The marginal value of money has a straightforward interpretation. A consumer with an additional unit of money acquires $1/p$ units of goods yielding additional utility $u'(q)/p$. However, from (8), $1/p = \phi$, and so we obtain (11).

Because bonds are illiquid in the goods market, the value of a bond is the same for all agents and equals its value in the subsequent settlement stage,

$$W_b^1 = W_b^0 = \phi(\tilde{R} + \rho). \tag{12}$$

Combining (7), (9), and (11), we can write the marginal values of money in the goods market as

$$W_m^1 = (1-n)\phi u'(q) + n\phi, \tag{13}$$

$$W_m^0 = \phi. \tag{14}$$

This has a straightforward economic interpretation. Agents with $\varepsilon = 1$ are consumers with probability $1-n$, in which case they spend money on the consumption good, with a marginal return equal to $\phi u'(q)$. With probability n, they are producers, in which case they hold onto their money until the following settlement stage, where it has a real value of ϕ. Therefore, the value of an additional unit of money for agents with $\varepsilon = 1$ is W_m^1, given by (13). Agents with $\varepsilon = 0$ derive no utility from consumption in the goods market, so they always keep their money to the following settlement stage, where again, it has a real value of ϕ. Therefore, the value of an additional unit of money for agents with $\varepsilon = 0$ is $W_m^0 = \phi$.

Bonds market. In the bonds market, agents receive a signal regarding their need for money in the following goods market. Agents with a signal $\varepsilon = 0$ know that they have no use for money, whereas agents with a signal $\varepsilon = 1$ will need money to consume with probability $1 - n$. An agent's expected lifetime utility when entering the bonds market with a portfolio (m, b) is

$$Z(m,b) = \mu W^1(m - \eta y^1, b + y^1) + (1-\mu)\, W^0(m - \eta y^0, b + y^0),$$

where y^ε, the quantity of bonds bought in the market by agents with signal ε, is chosen optimally as indicated below. Agents with signal ε solve

$$\max_{y^\varepsilon} W^\varepsilon(m - \eta y^\varepsilon, b + y^\varepsilon),$$
$$\text{s.t.} \quad -b \le y^\varepsilon \le m/\eta.$$

We impose the no–short selling constraint $-b \le y^\varepsilon$, so that an agent will not be able to sell more bonds than the amount he holds. We denote the real Lagrange multiplier on this constraint by $\phi\eta\lambda_b^\varepsilon$. Because agents need money to buy bonds, they can purchase at most m/η bonds. The real Lagrange multiplier on this constraint is $\phi\eta\lambda_m^\varepsilon$. The first-order condition is

$$W_b^\varepsilon - \eta W_m^\varepsilon - \phi\eta\lambda_m^\varepsilon + \phi\eta\lambda_b^\varepsilon = 0. \tag{15}$$

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Let us consider first the case of an agent with signal $\varepsilon = 1$. Using (13) to replace for W_m^1 and (12) to replace for W_b^1, we obtain an expression for the first-order condition:

$$(1-n)u'(q) + n - \frac{(\tilde{R}+\rho)}{\eta} = \lambda_b^1 - \lambda_m^1.$$

Because $u'(0) = +\infty$, it follows from the above equation that $\lambda_m^1 = 0$. Indeed, agents with $\varepsilon = 1$ leave the bonds market with a positive amount of money because they cannot acquire money in the goods market, as the CB does not operate a lending facility by assumption. The cash constraint does not bind for these agents and $\lambda_m^1 = 0$.

In contrast, let us consider the case of agents with $\varepsilon = 0$. Then using (14) and (12) to replace for W_m^0 and W_b^0, respectively, we obtain the first-order condition

$$\frac{(\tilde{R}+\rho)}{\eta} = 1 + \lambda_m^\varepsilon - \lambda_b^\varepsilon.$$

Notice that, in equilibrium, bonds should cost at least as much as what they yield, i.e., $\eta \geq \tilde{R} + \rho$, as otherwise the demand for bonds would be infinite. Hence, it cannot be that $\lambda_b^0 > \lambda_m^0 = 0$. Because agents with $\varepsilon = 0$ do not acquire money, their short-selling constraint does not bind and $\lambda_b^0 = 0$.

We can now combine the facts that $\lambda_m^1 = 0$ and $\lambda_b^0 = 0$ and use (15) to find an expression for the marginal value of bonds when entering the bonds market:

$$Z_b(m,b) = \mu\eta W_m^1 + (1-\mu)W_b^0. \quad \textbf{(16)}$$

When agents get the shock $\varepsilon = 1$, they can sell their bond for η units of money that will give them the marginal payoff W_m^1. Otherwise, they can keep their bonds. Similarly, agents with $\varepsilon = 1$ can use one unit of cash to acquire $1/\eta$ units of bonds, each with marginal value Z_b. Therefore, we obtain the following equation:

$$Z_m(m,b) = \frac{Z_b(m,b)}{\eta}. \quad \textbf{(17)}$$

Market clearing requires that

$$(1-\mu)y^0 + \mu y^1 + Y = 0. \quad \textbf{(18)}$$

Symmetric stationary equilibrium. First, notice that equations (4) and (16) imply $\eta_+ = \rho$. If this were not true, an arbitrage opportunity would arise. For example, if $\eta_+ > \rho$, agents could buy an infinite amount of bonds in today's bonds market and resell them in tomorrow's settlement market to make an infinite profit. A similar arbitrage opportunity arises if $\eta_+ < \rho$. Furthermore, in a stationary equilibrium with a constant amount of bonds, we have $\phi\rho = \phi_+\rho_+$. It follows that

$$\gamma = \frac{\eta}{\eta_+} = \frac{\rho_+}{\rho}. \quad \textbf{(19)}$$

Also, combining stationarity, (3), (17), (13), and (14), we obtain our second equilibrium equation,

$$\frac{\gamma}{\beta} = \mu \upsilon^1 + (1-\mu)\,\upsilon^b, \tag{20}$$

where $\upsilon^1 \equiv (1-n)u'(q) + n$ and $\upsilon^b \equiv (\tilde{R} + \rho)/\eta$. On the left-hand side of (20) is the opportunity cost of holding money across periods, i.e., the gross inflation rate. On the right-hand side is the expected marginal benefit from an additional unit of money. It is composed of two parts. First, an agent that gets $\varepsilon = 1$ prefers to hold onto his or her money and get $\upsilon^1 \equiv (1-n)u'(q) + n$. With probability $1-n$, the agent gets to consume, and receives a marginal value $u'(q)$, and with probability n, the agent cannot consume and has to hold onto his or her money until the next settlement stage. Second, an agent with $\varepsilon = 0$ does not consume and can use money to buy bonds instead, with a return $\upsilon^b \equiv (\tilde{R} + \rho)/\eta$. Money buys $1/\eta$ units of bonds in the bonds market, each promising a return $(\tilde{R} + \rho)$ in the next settlement stage. Therefore, bonds are useful for two reasons: First, agents with $\varepsilon = 0$ (and $\varepsilon = 1$) can buy bonds to get some protection against inflation. Second, agents with $\varepsilon = 1$ can also sell bonds to relax their budget constraint in the goods market.

Next we determine the demand and supply of bonds in the bonds market. Clearly, agents with $\varepsilon = 0$ will demand as many bonds as they can (i.e., they will spend all their money on purchasing bonds, $y^0 = m/\eta$) if bonds dominate money in rate of return, or $\upsilon^b > 1$. In contrast, agents with $\varepsilon = 1$ may want to hold a mixed portfolio of money and bonds, as they can have opportunities to consume or produce. However, if buying bonds is a poor hedge against inflation, such as if $\gamma/\beta > \upsilon^b$, then agents with $\varepsilon = 1$ will sell as many bonds as they can (i.e., $y^1 = b = B - \bar{B}$) to relax their budget constraint.[17] Therefore, we obtain the following demands for bonds:

$$y^0 = m/\eta, \quad \text{if } \upsilon^b > 1, \tag{21}$$

$$y^1 = B - \bar{B}, \quad \text{if } \frac{\gamma}{\beta} > \upsilon^b, \tag{22}$$

$$y^1 = -\frac{(1-\mu)}{\mu} y^0 - \frac{1}{\mu} Y. \tag{23}$$

We can define an equilibrium with OMOs as follows:

DEFINITION 1. *Given the CB policy* $(\tau, B, Y/M, Y'/M)$, $\tau \geq 0$, *a symmetric stationary equilibrium is a list* $(\gamma, \eta, \upsilon^1, \upsilon^b)$ *that solves (20)–(23) and*

$$\gamma = 1 + \eta \frac{Y}{M} + \gamma\eta \frac{Y'}{M} + \frac{\tau}{M}.$$

In the Appendix, we characterize four types of equilibrium, depending on the agents' desire to hold bonds. Intuitively, as $u'(0) = +\infty$, money must have a higher return than bonds for those agents with $\varepsilon = 1$, in any equilibrium; i.e., $\upsilon^1 \geq \upsilon^b$. Conversely, the return on bonds for agents with $\varepsilon = 0$ must be higher than the return on money, if they are to hold bonds in equilibrium; i.e., $\upsilon^b \geq 1$. Therefore, in any equilibrium $\upsilon^1 \geq \upsilon^b \geq 1$, and this defines our four types of equilibrium.

We turn next to the welfare results.

3.2. Welfare with Open-Market Operations

Given an allocation q, we can show that welfare $\mathcal{W}$ is given by the discounted sum of the gains from trade,

$$(1-\beta)\,\mathcal{W} = (1-n)[u(q)-q].$$

The problem of the CB is to choose Y and Y' so as to maximize the welfare function, given the implied allocation is an equilibrium. Recall that q^* denotes the efficient allocation. In the Appendix, we show the following result.

PROPOSITION 2. *Suppose $M_0 \leq B_0\beta\tilde{R}/(1-\beta)$; then the efficient allocation q^* is an equilibrium allocation with OMOs.*

The equilibrium implementing the efficient allocation q^* requires a sufficiently large initial stock of bonds relative to the money supply, or $M_0 \leq B_0\beta\tilde{R}/(1-\beta)$. In this equilibrium, all agents, irrespective of their signal, are indifferent between holding money and bonds in the bonds market; $\upsilon^1 = \upsilon^b = 1$. In other words, bonds and cash have the same return for *all* agents in the bonds market. This implies that agents value money the same, irrespective of their shocks. Hence, agents with $\varepsilon = 1$ who are consumers in the goods market cannot be constrained, as otherwise they would value cash more than agents with $\varepsilon = 0$. Not surprisingly, this is the case whenever the Friedman rule holds, $\gamma = \beta$, as implied by (20). The CB implements this equilibrium by selling bonds to shrink the money supply at a rate γ. The CB can achieve this rate of money growth in several ways. The simplest way is to make no lump-sum transfers to agents, $\tau = 0$. Then, with no OMOs, the money stock would remain constant. To decrease the money stock at the right rate, the CB can just sell the right amounts of bonds in the settlement stage, making no intervention in the bonds market. Alternatively, the CB could buy bonds in the bonds market, and sell even more bonds in the next settlement stage. These policies require that the CB have enough bonds in hand, and this is why we need the condition $M_0 \leq B_0\beta\tilde{R}/(1-\beta)$. This can be restated as $M_0 \leq \rho B_0$, where $\rho = \beta\tilde{R}/(1-\beta)$. This condition says that the value of outstanding bonds in terms of money must be higher than the stock of money, when bonds are fairly priced. This condition ensures that the CB can sell enough bonds in the bonds market to implement the Friedman rule.

3.3. CB Rebates Profits to Government

Until now, we have assumed that the CB could retain the proceeds from selling bonds. Suppose this is no longer the case, so that the CB has to rebate the proceeds from its operations to the government, and the government makes a lump-sum transfer to agents. Given its policy (Y, Y'), the proceeds from the CB operations in a given period are $-(\eta Y + \rho Y')$: Going back to the evolution of the money supply (1), an operation that drains money requires that $\eta Y + \rho Y' < 0$, so that the CB carries a net sale of bonds, with proceeds $-(\eta Y + \rho Y') > 0$. When this is rebated to the government and then to agents, the money stock cannot decline, so that $\gamma \geq 1$. In other words, a CB that rebates its operational profit to the government has a limited ability to lower inflation. We show the following result in the Appendix.

PROPOSITION 3. *Suppose the CB rebates its profit; then welfare is maximized if* $\gamma = 1$ *and the best equilibrium allocation with OMOs satisfies*

$$\beta[(1-n)u'(q) + n] = 1. \tag{24}$$

Welfare is always decreasing with inflation: As inflation rises, money loses its value and producers are unwilling to produce as much. Welfare is maximized when the CB implements the lowest possible level of inflation, which is $\gamma = 1$.

Because there is inflation above the Friedman rule, agents with $\varepsilon = 0$ prefer to hold bonds, rather than money, so $\upsilon^b > 1$. Thus, the *best* equilibrium is one where agents with $\varepsilon = 1$ are indifferent between holding bonds and cash in the bonds market, or $\upsilon^1 = \upsilon^b$. When this is the case, these agents are the least constrained in the goods market, everything else being constant. Indeed, if they were slightly more constrained in the goods market, the return on money would be higher than the return on bonds, and they would want to sell all their bonds in the bonds market.

Given $\upsilon^1 = \upsilon^b$, agents with $\varepsilon = 0$ are insured against inflation, as (20) implies that $\upsilon^b = \gamma/\beta = 1/\beta$. Still, distortions in the goods market cannot be eliminated with monetary policy, because inflation reduces the value of money. Even if OMOs were conducted in the goods market, once trading shocks were known, inflation would still distort consumption.[18] An example of a policy that implements $\gamma = 1$ is $Y = -Y'$, which can be though of as exclusively repos. In the next section, we analyze how a channel system can reduce these distortions.

4. CHANNEL SYSTEM

In this section, we assume that the CB does not conduct OMOs. Instead, we study an implementation framework that relies exclusively on a lending and a deposit facility.

FIGURE 2. Timeline with standing facilities.

This framework is very similar to the model studied in BM. However, to allow closer comparison with the OMOs modeled in the preceding section, we modify BM's model in one important dimension. BM assume that agents can produce an asset bearing a real exogenous return, similar to a Lucas tree, at a cost. In contrast, we study the channel system under the assumption that agents must pledge bonds as collateral in order to borrow from the CB. In the channel system, agents can access the CB's lending and deposit facilities after they observe their trading shock on the goods market. The CB offers nominal loans ℓ, secured by bonds, at an interest rate i_ℓ, and promises to pay interest rate i_d on nominal deposits d, with $i_\ell \geq i_d$. All obligations contracted during the goods market are settled in the next settlement stage. The timeline is provided in Figure 2.

In a channel system with no OMOs, the stock of money M evolves endogenously as follows:

$$M_{+1} = M - i_\ell L + i_d D + \pi, \tag{25}$$

where $\pi \geq 0$ is a lump-sum transfer of money from the CB to agents. When aggregate borrowing at the lending facility is L, the money supply increases by this amount and becomes $M + L$. However, in the next settlement stage, agents have to repay L as well as any interests on this amount, $i_\ell L$. Therefore, the money supply becomes $M + L - (1 + i_\ell)L$, which simplifies to $M - i_\ell L$. Similarly, when agents use the deposit facility, the money supply shrinks by the aggregate amount deposited D, but agents will receive it back with interest in the next settlement stage. In this case, the money supply becomes $M - D + (1 + i_d)D$, which simplifies to $M + i_d D$. Combining the effects of the lending and deposit facilities, we obtain the evolution of the money stock (25). It is important to realize that the CB can only decrease the stock of money by inducing agents to borrow from the lending facility. Indeed, when the amount of aggregate loans is L, the money stock shrinks by the interest payments $i_\ell L$. However, this decline is limited by the interest payments that the CB makes on aggregate deposits, $i_d D$.

Finally, notice that the money stock is not affected by the amount of bonds outstanding. The reason is twofold: First, by assumption, the CB does not hold any bonds. Second, the return on bonds is financed via lump-sum taxes on agents. Therefore, the increase in the money stock that is due to the interest payments on bonds is undone by the lump-sum tax necessary to finance these payments.

4.1. Settlement Stage

At the settlement stage, the problem of an agent is

$$V(m, b, \ell, d) = \max_{h,m_+,b_+} -h + \beta Z(m_+, b_+),$$

$$\text{s.t.} \quad \phi m_+ + \phi\rho b_+ = h + \phi\,(m - T) + \phi(\tilde{R} + \rho)b$$
$$+\phi(1 + i_d)d - \phi(1 + i_\ell)\ell + \phi\pi.$$

Using the constraint to replace h in the objective function, the problem of an agent in the settlement stage becomes

$$V(m, b, \ell, d) = \max_{m_+,b_+} \phi\,(m - T) + \phi(\tilde{R} + \rho)b + \phi(1 + i_d)d - \phi(1 + i_\ell)\ell$$
$$+\phi\pi - \phi m_+ - \phi\rho b_+ + \beta Z\,(m_+, b_+)$$
$$= \phi\,(m - T) + \phi(\tilde{R} + \rho)b + \phi(1 + i_d)d - \phi(1 + i_\ell)\ell + \phi\pi$$
$$+ \max_{m_+,b_+} \{-\phi m_+ - \phi\rho b_+ + \beta Z\,(m_+, b_+)\}.$$

Again, notice that our assumption of a quasi-linear utility function implies that individual wealth can be pulled out of the agent's maximization problem. Therefore, individual wealth does not affect the portfolio choice. The first-order conditions for interior solutions are

$$\beta Z_{m_+} = \phi, \tag{26}$$

$$\beta Z_{b_+} = \phi\rho. \tag{27}$$

Because individual wealth does not affect the portfolio decision of agents, their decision is also unaffected by their individual history. Therefore, as implied by equations (26) and (27), all agents exit the settlement stage with the same portfolio of money and bonds (which can be zero). The envelope conditions are

$$V_m = \phi;\ V_b = \phi(\tilde{R} + \rho);\ V_\ell = -\phi\,(1 + i_\ell)\,;\ V_d = \phi(1 + i_d). \tag{28}$$

4.2. Goods Market

As in the preceding section, at the beginning of the goods market, agents experience trading shocks that determine whether they need money. We let q^ε and q_s^ε denote the quantities consumed by a buyer and produced by a seller in the goods market, respectively.

An agent with m units of money and b units of bonds at the beginning of the goods market may want to use the deposit or the lending facility. As agents with $\varepsilon = 0$ never consume, we define q as the consumption level of agents with $\varepsilon = 1$ who turn consumers. Also, only agents who need money will use the lending facility: We denote by ℓ the amount borrowed by consumers with $\varepsilon = 1$. Similarly, only agents who do not need money will use the deposit facility: We let

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d^1 denote the deposits made by producers with $\varepsilon = 1$, whereas d_c^0 and d^0 denote the deposits by consumers and producers with $\varepsilon = 0$, respectively. We denote by W^ε the expected lifetime utility of an agent with preference shock ε at the start of the goods market. Therefore, we obtain

$$W^1(m,b) = (1-n)[u(q) + V(m - pq + \ell, b, \ell, 0)]$$
$$+ n[-q_s + V(m + pq_s - d^1, b, 0, d^1)],$$

$$W^0(m,b) = (1-n)V\left(m - d_c^0, b, 0, d_c^0\right)$$
$$+ n\left[-q_s^\varepsilon + V\left(m + pq_s^\varepsilon - d^0, b, 0, d^0\right)\right],$$

where q_s^ε, q, ℓ, d_c^0, and d^ε solve the optimization problems described below.

The problem of producers is $\max_{q_s,d}[-q_s + V(m + pq_s - d, b, 0, d)]$ s.t. $m + pq_s - d \geq 0$. Using $V_m = \phi$ from (28), the two first-order conditions can be combined to obtain

$$p\phi(1 + i_d) = 1. \qquad \textbf{(29)}$$

Comparing (8) and (29), we can already observe one major difference between OMOs and a system relying on facilities. With a deposit facility, money is more valuable for producers: By depositing their money holdings, producers earn interest. Therefore, ceteris paribus, the price of goods p is lower with a deposit facility than with OMOs.[19]

Consumers with $\varepsilon = 0$ have no use for money and deposit their money holdings at the deposit facility. Hence $d_c^0 = m$. In contrast, consumers with $\varepsilon = 1$ solve the following maximization problem:

$$\max_{q,\ell} \quad u(q) + V(m - pq + \ell, b, \ell, 0),$$
$$\text{s.t.} \quad pq \leq m + \ell \text{ and } \ell \leq \bar{\ell},$$

where

$$\bar{\ell} \equiv (\tilde{R} + \rho)b/(1 + i_\ell). \qquad \textbf{(30)}$$

$\bar{\ell}$ is the maximum amount that a buyer can borrow from the CB. The amount of bonds pledged as collateral must cover the interest payment due, and b units of bonds yield $(\tilde{R} + \rho)b$ units of money at the beginning of the settlement market. Using (28) and (29), the consumers' first-order conditions can be combined to get

$$u'(q) = \frac{1 + i_\ell + \lambda_\ell}{1 + i_d}, \qquad \textbf{(31)}$$

where λ_ℓ is the multiplier on the borrowing constraint. If the borrowing constraint is not binding and the CB sets $i_\ell = i_d$, then trades are efficient. If the borrowing constraint is binding, then $u'(q) > 1$, which means that trades are inefficient even when $i_\ell = i_d$.

Using the envelope conditions and (29), the marginal value of money in the goods market for agents with $\varepsilon = 1$ and $\varepsilon = 0$ is given by, respectively,

$$W_m^1 = \phi(1 + i_d)[(1 - n)u'(q) + n], \quad (32)$$

$$W_m^0 = \phi(1 + i_d). \quad (33)$$

These expressions have the same economic interpretation as (14) and (13), with the difference that the value of money is increased by a factor $1 + i_d$. The deposit facility makes money more valuable, everything else kept constant, because agents can earn interests on their deposits. Now we can derive the marginal value of bonds for each type of agents using (31),

$$W_b^1 = \left[(1 - n)\frac{(1 + i_d)}{(1 + i_\ell)}u'(q) + n\right]\phi(\tilde{R} + \rho), \quad (34)$$

and

$$W_b^0 = \phi(\tilde{R} + \rho). \quad (35)$$

Comparing (12) with (34), we can see that the value of bonds is possibly higher in the bonds market when there is a lending facility, relative to the case with only OMOs, everything else constant. This is intuitive: Bonds are useless in the goods market when the CB does not offer a lending facility, but they can be exchanged for money when such a facility is available in the goods market. In this case, bonds carry a premium that is related to how severely agents are constrained, as summarized by $u'(q)$, and to how expensive it is to use the lending facility, as summarized by $1 + i_\ell$. The more constrained consumers are, the higher the premium. Also, the more expensive CB credit is, the lower the liquidity premium.

4.3. Bonds Market

The functioning of the bonds market is as before, with the difference that CB does not buy or sell bonds. Agents simply trade bonds among themselves. This implies that the first-order condition for an agent with signal ε is still given by (15). However, as agents can now obtain money from the facility in the goods market, an agent with signal $\varepsilon = 1$ may prefer to leave the bonds market with no money, and borrow at the lending facility if necessary. For similar reasons, an agent with signal $\varepsilon = 0$ may leave the market with only money, because it is now possible to earn interest by depositing the money with the CB. Therefore, we cannot yet conclude that some of the constraints are always slack, or $\lambda_m^1 = \lambda_b^0 = 0$. However, we will later show that this condition must hold in equilibrium. It follows that (16) and (17) still hold. Given the CB does not conduct OMOs, the market-clearing condition requires that

$$(1 - \mu)\, y^0 + \mu y^1 = 0. \quad (36)$$

4.4. Symmetric Stationary Equilibrium

First, we can show that the return from buying bonds in the bonds market, $\tilde{\upsilon}^b \equiv (\tilde{R}+\rho)/\eta$, is bounded above by $1+i_\ell$ and below by $1+i_d$, which validates $\lambda_m^1 = \lambda_b^0 = 0$ and so (16) and (17) as equilibrium equations in the channel system.[20]

LEMMA 4. *In any equilibrium*

$$1 + i_\ell \geq \tilde{\upsilon}^b \geq 1 + i_d. \tag{37}$$

Proof. Suppose $\tilde{\upsilon}^b/(1+i_\ell) > 1$. Then, agents with $\varepsilon = 1$ can purchase $1/\eta$ bonds with 1 unit of money. In turn, these bonds allow the agent to borrow $\tilde{\upsilon}^b/(1+i_\ell) > 1$ units of money from the CB. In addition, agents that do not need to borrow get $\tilde{\upsilon}^b \geq 1$ next period and are, therefore, willing to purchase bonds. But this implies that all agents want to buy bonds, which cannot be an equilibrium. Now suppose that $\tilde{\upsilon}^b/(1+i_d) < 1$. In that case, all agents would sell bonds to acquire money, because they could obtain at least $\eta(1+i_d)$ units of money by using the deposit facility, which is higher than the bond return $\tilde{R}+\rho$. This cannot be an equilibrium. ■

Second, (16) and (17) give us the bonds arbitrage condition

$$\gamma\eta = \rho. \tag{38}$$

Using (27), (16), (32), and (35), we also obtain an equilibrium condition for money holdings,

$$\frac{\gamma}{\beta} = \mu\tilde{\upsilon}^1 + (1-\mu)\,\tilde{\upsilon}^b. \tag{39}$$

where $\tilde{\upsilon}^1 \equiv [(1-n)u'(q)+n](1+i_d)$ and $\tilde{\upsilon}^b \equiv (\tilde{R}+\rho)/\eta$**.** This expression has an interpretation similar to (20): On the left-hand side is the cost of holding money, whereas on the right-hand side are the benefits. An agent's signal is $\varepsilon = 0$ with probability $1-\mu$, in which case he or she can trade money for bonds with the return $\tilde{\upsilon}^b$, whereas if the agent gets $\varepsilon = 1$ with probability μ, he or she can expect a benefit of $\tilde{\upsilon}^1 \equiv [(1-n)u'(q)+n](1+i_d)$ from holding onto his or her money. The main difference between (20) and (39) is that money is worth more in the channel system because agents benefit from the deposit facility offered by the CB. Hence, given a consumption level q, $\tilde{\upsilon}^1 \geq \upsilon^1$.

Next we determine the demand for and supply of bonds in the bonds market. First notice that the return on holding one unit of money is $1+i_d$, as agents can use the deposit facility. Clearly, agents with $\varepsilon = 0$ will demand as many bonds as they can, i.e., $y^0 = m/\eta$, if bonds dominate money in rate of return, or $\tilde{\upsilon}^b > 1+i_d$, and they are indifferent otherwise.[21] To determine the demand for bonds of agents with $\varepsilon = 1$, recall that bonds can now be used at the lending facility to borrow cash from the CB. By acquiring one more bond in the bonds market, a consumer can borrow $\tilde{\upsilon}^b/(1+i_\ell)$ units of cash at the lending facility, with additional value

$u'(q)(1+i_d)\tilde{v}^b/(1+i_\ell)$. If the consumer simply keeps this bond, he or she can obtain $\tilde{v}^b$. Hence, the lending facility gives bonds an additional premium that equals $\mu(1-n)\tilde{v}^b[u'(q)\frac{(1+i_d)}{(1+i_\ell)}-1]$.[22] This is another important difference from a system that relies on OMOs. If the return on bonds, including this additional liquidity premium, is less than inflation, then agents with $\varepsilon = 1$ will find it more profitable to hold only money, and therefore they sell all their bonds; i.e., $y^1 = -b$. And $b = \bar{B}$, because all agents hold the same quantity of bonds, whereas the CB does not hold any.

The liquidity premium on bonds is an equilibrium object and depends on whether consumers are constrained in the goods market. If they are constrained, agents are likely to bring bonds into the goods market in order to borrow at the lending facility, and bonds carry an additional liquidity premium. If consumers are not constrained, then agents do not borrow at the lending facility and bonds do not carry any liquidity premium. To summarize, we obtain the following demands for bonds:

$$y^0 = m/\eta, \quad \text{if } \tilde{v}^b > 1 + i_d, \tag{40}$$

$$y^1 = -\bar{B}, \quad \text{if } \frac{\gamma}{\beta} > \tilde{v}^b \left\{1 - \mu + \frac{\mu}{(1+i_\ell)}[\tilde{v}^1 + n\,(i_\ell - i_d)]\right\}, \tag{41}$$

$$y^1 = -\frac{(1-\mu)}{\mu} y^0, \tag{42}$$

where the last equation is the market-clearing condition given that the CB does not conduct any OMOs.

The money supply evolves according to (25). To simplify this expression, notice that money flows from consumers with $\varepsilon = 1$ to producers in the goods market. So the entire supply of money is deposited at the CB facility. Hence, the entire money supply M earns the deposit interest rate i_d. Separately, note that only consumers with $\varepsilon = 1$ (a mass $\mu(1-n)$ of agents) borrow from the lending facility, so $L = \mu(1-n)\ell$. This amount is also used to buy goods, and then deposited at the deposit facility to earn interest i_d. Therefore, (25) can be simplified to

$$\gamma = 1 + i_d + (i_d - i_\ell)\,\mu\,(1-n)\,\frac{\ell}{M} + \frac{\pi}{M}, \tag{43}$$

where

$$\ell = pq - (M - \eta y^1) \leq b\eta\tilde{v}^b/(1+i_\ell). \tag{44}$$

We provide the explicit derivations in the Appendix.

DEFINITION 5. *A symmetric stationary equilibrium is a policy* (i_d, i_ℓ) *and* $\pi \geq 0$ *and a list* $(\gamma, \eta, \ell/M, \tilde{v}^1, \tilde{v}^b)$ *satisfying (38)–(44).*

In the Appendix, we show the following result.

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PROPOSITION 6. *The first-best q^* is not an equilibrium allocation of any channel system.*

This result has a simple intuition: at the efficient allocation, money has a lower value than bonds in the bonds market and, therefore, all agents prefer to hold bonds rather than money. To understand why this is the case, notice from (31) that q^* is an equilibrium allocation if consumers are unconstrained and $i_\ell = i_d$. However, in this case, equation (43) implies that there is inflation in this economy, $\gamma = 1 + i_d \geq 1$. Indeed, deflation requires $i_\ell > i_d$, so that interest payments drain money out of the economy.[23] By assumption, at q^*, consumers are unconstrained in the goods market, so bonds do not carry a liquidity premium and the return on cash is $1 + i_d$. Therefore, bonds are priced at their fair value in the settlement stage. However, this means that bonds are cheap and the return from buying bonds $\tilde{v}^b = (1 + i_d)/\beta$ is higher than that from keeping money and depositing it to earn $1 + i_d$. This cannot be an equilibrium, because all agents would then prefer to buy bonds than to hold money.

The inability of the channel system to implement the efficient allocation is not new, but it is worth repeating here: The efficient allocation requires deflation. In a channel system, this implies that agents must use the lending facility, as this is the only way to shrink the stock of money and create deflation. However, as consumers become less constrained, they borrow less from the CB, and the stock of money is reduced by a smaller amount. This creates an inconsistency between the way the channel system achieves a reduction in the money stock and the incentives of consumers to borrow from the CB.

Next we study which allocations the channel system can implement. To understand the mechanics of the main result, we present the case where consumers cannot borrow from the lending facility next.

4.5. Channel System with Deposit But No Lending Facility

To understand the equilibrium with both a lending and a deposit facility, it is instructive to contrast it with the benchmark case in which the CB does not offer any lending facility. In this section, we assume that the CB offers only a deposit facility and set $\pi = 0$. In such a case, the demand function for bonds on the bond market (40)–(42) simply becomes

$$y^0 = m/\eta, \quad \text{if } \tilde{v}^b > 1 + i_d, \tag{45}$$

$$y^1 = -\bar{B}, \quad \text{if } \frac{\gamma}{\beta} > \tilde{v}^b, \tag{46}$$

$$y^1 = -\frac{(1-\mu)}{\mu} y^0 \quad \text{otherwise.} \tag{47}$$

Notice that the absence of a lending facility eliminates the additional liquidity premium for bonds, because they cannot be pledged for money at the lending

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facility. Therefore, the absence of the lending facility reduces the value of holding bonds for agents with $\varepsilon = 1$. As a consequence, their short-selling constraint binds for lower values of η than in the case with a lending facility. Also, because $\ell = 0$, (25) becomes

$$\gamma = 1 + i_d. \tag{48}$$

The equilibrium conditions (38) and (39) remain unchanged.

We prove the following result in the Appendix.

PROPOSITION 7. *With no lending facility, the best feasible equilibrium allocation satisfies*

$$\beta[(1 - n)u'(q) + n] = 1. \tag{49}$$

Notice that in an economy where the CB does not offer a lending facility, the money supply evolves according to (25) with $L = 0$. Therefore, the CB has no possibility of reducing the money supply. Indeed, going back to (43) and setting $\ell = 0$, we obtain that $\gamma = 1 + i_d$, so that inflation is perfectly correlated with the interest rate on deposits. Interestingly, this implies that producers are perfectly insured against inflation, as they can deposit all their profit at the CB. Hence, whether the CB chooses $i_d > 0$ or sets $i_d = 0$ does not modify the goods market allocation q.

Nevertheless, when there is inflation, bonds are a better hedge against inflation for agents with $\varepsilon = 0$ who prefer to hold bonds rather than money: $\tilde{v}^b > 1 + i_d$. The second-best policy is to set returns so that agents with $\varepsilon = 1$ are indifferent between holding cash or buying bonds in the bonds market, $\tilde{v}^1 = \tilde{v}^b$, as was the case for OMOs. Under this scenario, agents with $\varepsilon = 1$ are indifferent between selling more bonds and acquiring more money, so that their short-selling constraint in the bonds market is not binding. We can show that such an equilibrium exists. Because $\tilde{v}^1 = \tilde{v}^b = [(1 - n)u'(q) + n](1 + i_d)$, and inflation is $\gamma = 1 + i_d$, we have $\tilde{v}^1 = [(1 - n)u'(q) + n]\gamma$. Using (39), we obtain that the consumption allocation is given by (49) and is independent of the inflation level. This allocation is the same as the one under OMOs, as defined by (24). Hence, a channel system with no lending facility can achieve the same allocations as a system with only OMOs when $\gamma = 1$. However, notice that for an arbitrary inflation level $\gamma > 1$, the channel system with a deposit facility will perform better than OMOs, as the deposit facility can perfectly insure producers against inflation.

This is informative about what a CB bank can and cannot do with OMOs. With OMOs, the CB can insure agents with $\varepsilon = 0$ against inflation, but not producers with $\varepsilon = 1$. With only a deposit facility, the CB can insure producers and agents with $\varepsilon = 0$ against inflation. However, without a lending facility the CB cannot insure agents with $\varepsilon = 1$ against the trading shock in the goods market. We turn to this aspect of a channel system next.

4.6. Channel System with Deposit and Lending Facilities

So far, we have shown that (1) a channel system cannot implement the first-best allocation and (2) a channel system cannot do better than OMOs when the CB uses only a deposit facility when $\gamma = 1$. In this section, we consider the case in which the CB offers a lending and a deposit facility but has to rebate any positive profits from operating the facility to the government. Profits are the difference between the gains of the CB from its lending operations, $i_\ell L$ and the losses made from paying interest on deposits, $i_d D$. The CB's profit is positive whenever $i_\ell L - i_d D \geq 0$. Looking back at the evolution of the money stock (25), it is easy to see that the money stock decreases whenever the CB makes a positive profit. However, when the CB rebates its profit to the government, which in turn transfers it lump sum to agents, the money stock cannot decrease. Therefore, we require that $M_+ \geq M$, or $\gamma \geq 1$.

We show the following result.

PROPOSITION 8. *Suppose $\bar{B}_0/M_0$ is high enough; then there is an equilibrium in a channel system with $i_\ell > i_d$ and where q is such that*

$$(1-n)\,u'\,(q) + n < 1/\beta.$$

The equilibrium that we consider is one in which agents with signal $\varepsilon = 1$ are indifferent between holding bonds and money in the bonds market, consumers borrow from the lending facility but they are not constrained by their bonds holdings, and inflation is lower than $1 + i_d$. Indeed, when agents borrow at the CB, $\gamma < 1 + i_d$, as interest on loans $i_\ell L$ reduces the stock of money in circulation. However, there is borrowing at the CB only if agents hold bonds that can be used as collateral. Hence, the only equilibrium where agents with $\varepsilon = 1$ access the lending facility is one where these agents exit the bonds market with bonds and money. But this implies that, for these agents, bonds and money must have the same rate of return in the bonds market; i.e., $\tilde{v}^1 = \tilde{v}^b$. Otherwise they would hold either one or the other, but not both. If $\tilde{v}^1 = \tilde{v}^b$, equation (39) implies that

$$\frac{\gamma}{\beta} = [(1-n)u'(q) + n](1 + i_d).$$

Now, because agents borrow at the lending facility, inflation is lower than $1 + i_d$. Hence, consumption has to be higher than in the case without a lending facility. Contrary to what one might think, the reason for this rise in consumption is not that the lending facility relaxes the budget constraint of consumers. Rather, consumption rises because aggregate borrowing from the CB lowers inflation and therefore makes money more valuable.

The lending facility makes money valuable for all agents: Absent the lending facility, the return on money in the goods market $\tilde{v}^0$ for those agents with $\varepsilon = 0$ was equal to the return of using the deposit facility: $1 + i_d$. However, when the CB offers a lending facility and agents use it, inflation is lower than $1 + i_d$.

This benefits all agents holding money, including agents with $\varepsilon = 0$ who do not consume. By choosing the correct combination of interest rates, the CB can insure agents against their preference and trading shocks, in the sense that the value of cash is the same for all; i.e., $\tilde{v}^1 = \tilde{v}^b = \tilde{v}^0$. The fact that the CB can increase $\tilde{v}^0$ by increasing the lending rate resembles a transfer from consumers to those agents that do not need money.

Because the argument relies on the fact that the collateral constraint at the lending facility does not bind, we need to require that the stock of bonds is high enough. Finally, notice that the equilibrium allocation is pinned down by $u'(q) = (1 + i_\ell)/(1 + i_d)$.[24] Therefore,

$$1 < \frac{(1 + i_\ell)}{(1 + i_d)} < \frac{1 - \beta n}{\beta (1 - n)},$$

and the lending rate cannot be too high; i.e., interestingly, it is optimal that the CB implements a narrow channel.

5. DISCUSSION

In this section, we compare the two implementation frameworks studied in the paper. We are interested in these systems' effectiveness in implementing the Friedman rule and in the welfare they yield for an exogenously fixed rate of growth of the supply of money.

5.1. Implementation of the Friedman Rule

The welfare-maximizing allocation, q^*, can be achieved when monetary policy is implemented with OMOs and the CB sells bonds so as to contract the money supply according to the Friedman rule. However, this allocation cannot be implemented in a channel system. Hence, the best allocation that can be implemented using OMOs is better than the best allocation that can be implemented using a channel system.

It is interesting to understand why the Friedman rule cannot be implemented in a channel system. Some insight can be gained by looking at the equations governing the evolution of the supply of money in both systems. When the CB uses OMOs, the supply of money evolves according to

$$M_+ = M - \rho Y - \eta Y'.$$

Hence, the CB can shrink the money supply by, for example, selling bonds in the bonds market or the settlement stage, or both ($Y \geq 0$ and $Y' \geq 0$). This is not the case, however, when the CB uses a channel system. In this case, the supply of money evolves according to

$$M_+ = M - i_\ell L + i_d D,$$

where L and D are total loans and deposits, respectively. Because deposits are positive, a necessary condition for the money supply to shrink is that agents borrow at the lending facility and repay interests on their loans; i.e., $i_\ell L > 0$. However, this is not enough. In addition, the interest of loans i_ℓ should be set sufficiently higher than i_d to shrink the supply of money. This obviously implies a cost on borrowers. However, money is costless to hold when the Friedman rule is in place. Therefore, if the Friedman rule is implemented, there will be no borrowing at the facility if the CB charges a positive interest rate and, as a consequence, the supply of money cannot shrink. This limits the ability of the CB to increase the rate of return on money, thereby making it impossible to implement the Friedman rule.

5.2. Welfare with a Given Inflation Rate

In this section, we consider which framework yields higher welfare for an exogenously given level of inflation. First, we define a feasible level of inflation under each system: γ is a feasible level of inflation under a given monetary policy implementation framework if there exists a monetary policy under that framework that can achieve γ. More precisely, γ is feasible with OMOs if there is a policy $(B, Y/M, Y'/M)$, such that the money growth rate is γ. Also, γ is feasible with facilities if there is a policy (i_ℓ, i_d) such that the money growth rate is γ. We offer the following result.

PROPOSITION 9. *If $\gamma < \bar{\gamma}$, then γ is feasible with OMOs, but it is not feasible with facilities. For all $\gamma \geq \bar{\gamma}$, welfare is higher when γ is implemented using facilities.*

If a channel system can implement a given level of inflation, then this system is preferable to OMOs. The intuition is straightforward: Producers are willing to produce more goods for a given amount of money because they can earn interest on their profit by using the deposit facility. However, as we showed in Sections 4.5 and 4.6, this is not enough for facilities to be better than OMOs. In addition to the deposit facility, agents should have access to a lending facility that imposes a higher interest rate $i_\ell > i_d$. By treating borrowers and depositors asymmetrically, a lending facility can perform a transfer from agents that need money to those that use the deposit facility. This transfer improves welfare, because it allows lower inflation (relative to the deposit rate) and additional production (because inflation is lower). With $\gamma = 1$, if there is no lending facility, or if the facilities treat agents symmetrically $i_\ell = i_d$, the gross interest rate paid on a deposit facility is equal to inflation and facilities cannot do better than OMOs.

OMOs necessarily treat agents identically as it cannot price discriminate between agents who need or do not need money. In a sense, OMOs are one instrument, whereas a facility with both a lending and a deposit facility offers two instruments to implement an allocation. The channel system is essentially equivalent to a system that would use two OMOs in the goods market: one liquidity providing before consumption takes place, and one liquidity-draining after consumption takes place.

5.3. Policy Implications

The above results suggest that an optimal system of implementation of monetary policy should include some elements of both of the pure systems we have studied in this paper. To achieve inflation rates that are sufficiently low, a CB operating a channel system may need to hold a portfolio of bonds and engage in OMOs to affect the evolution of the supply of money. Conversely, a CB implementing monetary policy using OMOs may want to pay interest on reserves to moderate some of the distortions that arise away from the Friedman rule.

This result is relevant for an important policy question in the United States. In October 2008, the Federal Reserve received the authority to pay interest on reserves. This was motivated in part by the financial crisis that started in 2007, which has greatly influenced monetary policy and its implementation. Once the crisis subsides, the Federal Reserve will have a new tool at its disposal. In the implementation framework of the Federal Reserve prior to the authorization to pay interest on reserves, banks were required to hold reserves against a fraction of their deposits and neither required nor excess reserves were remunerated. This led to a couple of potential distortions. On one hand, banks did expend resources in an effort to minimize their reserves requirement. One manifestation of such efforts was the creation of sweep accounts.[25] On the other hand, taking the requirement as given, banks tried to minimize the amount of reserves they held above their requirement because such reserves were costly at the margin.

As noted by Vice Chairman Kohn in testimony to Congress before the new law was passed, "The Board has long supported legislation that would authorize the Federal Reserve to pay depository institutions interest on the balances they hold at Reserve Banks. As we previously have testified, paying interest on required reserve balances would remove a substantial portion of the incentive for depositories to engage in reserve avoidance measures, and the resulting improvements in efficiency should eventually be passed through to bank borrowers and depositors. Having the authority also to pay interest on contractual clearing and excess reserve balances as well as required reserves would enhance the Federal Reserve's ability to efficiently conduct monetary policy."[26]

In our model, there are no reserve requirements, because all reserves held are excess reserves. An agent who is a producer is in a position similar to that of a bank holding excess reserves. Our analysis implies that this is suboptimal as it may distort the agent's incentives. In particular, our analysis provides support for the argument that it may be optimal for the Federal Reserve System to pay interest on both required and *excess* reserves.

6. CONCLUSION

This paper studies two stylized implementation frameworks for monetary policy. In one case, the CB only relies on OMOs, whereas in the other case, the CB operates facilities. In our environment, holding reserves is costly if the CB does not implement the Friedman rule. The implementation frameworks can reduce this cost in different ways.

If the CB can keep its profits, then OMOs can achieve the Friedman rule and thus the efficient allocation. However, this is not the case if the CB must rebate all its profits, for example, to a fiscal authority. On the contrary, facilities cannot implement the Friedman rule. To reach the Friedman rule, the CB must be able to shrink the money supply sufficiently. With only facilities, this can be achieved only if banks use the lending facility and the CB's lending rate is higher than the deposit rate. However, if the rate of growth of the money supply is low enough, agents will prefer to hold money, rather than the bonds. This is because the opportunity cost of holding money, in terms of foregone interest, becomes small compared with the cost of accessing the CB's lending facility. However, this implies that agents cannot pledge collateral at the CB, and, therefore, cannot borrow. When we compare the two frameworks at rates of inflation that both can implement the Friedman rule, we find that the framework using facilities achieves higher welfare. When $i_\ell > i_d$, facilities create a transfer from banks that access the lending facility to banks that use the deposit facility. Such a transfer is absent when the CB conducts monetary policy through OMOs, because the CB cannot discriminate between borrowers and depositors. Finally, our results highlight the benefits of using both OMOs and facilities to implement monetary policy. They also suggest that CBs should pay interest on both required and excess reserves.

NOTES

1. In December 2007, the Federal Reserve started to provide reserves through a term auction facility to alleviate some stress in financial markets. On January 27, 2010, the Federal Reserve announced that the final TAF auction would be conducted on March 8. More details are available at http://www.federalreserve.gov/monetarypolicy/taf.htm.

2. See, for example, Keister et al. (2008).

3. See Kocherlakota (2003) or Shi (2005) for a reason that bonds should or could be illiquid, respectively.

4. By the law of large numbers, μ is also the measure of banks with $\varepsilon = 1$.

5. We modify this assumption in the Appendix and show that our results are basically unchanged.

6. A significant share of interbank transactions are arranged by brokers that preserve the anonymity of the parties to the trade, so this is an option for banks to trade anonymously. Most transactions have a one-day term.

7. By essential we mean that the use of money expands the set of allocations [Kocherlakota (1998) and Wallace (2001)].

8. This condition eliminates the possibility for arbitrage in which agents borrow and subsequently make a deposit at interest $i_d > i_\ell$, increasing their money holdings at no cost.

9. The linear preferences in market 1, first introduced by Lagos and Wright (2005) to get a degenerate distribution of money holdings at the beginning of a period, allow us to interpret transactions that are taking place in the first market as settlement transactions, as in Koeppl et al. (2008).

10. See Edwards (1997) for details.

11. Conducting an OMO after the goods market closes is equivalent to opening a deposit facility. Indeed, after the goods market closes, no bank is interested in obtaining money from the CB. Banks with excess money are willing to sell the money to the CB at any price corresponding to a gross interest rate greater than or equal to 1.

12. For example, one could assume that the discount factor includes a default probability δ, such that the effective discount factor is $\beta = (1-\delta)\tilde{\beta}$ and $\tilde{\beta}$ is the time discount factor. See Chapman et al. (2009) for a model of the channel system with default.

13. This assumption is standard in the literature. See Freeman (1996), Martin (2004), or Koeppl et al. (2008), for example.

14. See Mattesini et al. (2009) for a model in which banks play a role beyond the provision of payments.

15. For notational simplicity we suppress the time index t and denote next period variables by the index $+$.

16. Notice that the budget constraint is expressed in nominal terms, whereas the objective function is in real (utility) terms. For the Lagrange problem to be well-defined, we need to express the budget constraint in real terms, multiplying both sides of it by ϕ. Then the Lagrange multiplier is a real number. Here we just renormalize the multiplier and keep the budget constraint in nominal terms.

17. This follows easily from (15).

18. In Section A.5, in the Appendix, we show that a CB that conducts OMOs in the goods market can provide some insurance against the trading shock, but cannot insure agents against inflation, as the best achievable allocation satisfies $u'(q) = \gamma/\beta = 1/\beta$.

19. Berentsen et al. (2007) obtain a similar result in a different environment.

20. The condition $\tilde{v}^b \geq 1 + i_d$ implies that agents with $\varepsilon = 0$ bring some bonds into the goods market. Hence, their short-selling constraint on bonds cannot be binding in the bonds market; i.e., $\lambda_b^0 = 0$. Similarly, $\tilde{v}^b \leq 1 + i_\ell$ implies that agents with $\varepsilon = 1$ bring money to the goods market. Hence, their short-selling constraint for money cannot be binding; i.e., $\lambda_m^1 = 0$. Therefore, the above argument validates (16) and (17) as equilibrium equations in the channel system.

21. Using (dV0 channel) and (dVb channel), we obtain the expression for λ_m^0, $\lambda_m^0 = \tilde{R} + \rho - \eta(1 + i_d)$.

22. Using (sigmak-), (dV1 channel), and (dVb1 channel), we can derive an expression for λ_b^1:

$$\mu\lambda_b^1 = \frac{\rho}{\beta} - (\tilde{R} + \rho)\left\{1 + \mu(1-n)\left[u'(q)\frac{(1+i_d)}{(1+i_\ell)} - 1\right]\right\}$$

$$= \frac{\rho}{\beta} - (\tilde{R} + \rho)\left\{1 - \mu + \frac{\mu}{(1+i_\ell)}[\tilde{v}^1 + n(i_\ell - i_d)]\right\}.$$

23. This condition is necessary but not sufficient for deflation. Deflation also require that $\ell > 0$ and $i_d D$ is low enough relative to $(i_\ell - i_d)L$.

24. To see why agents can consume the amount q such that $u'(q) = (1+i_\ell)/(1+i_d) > 1$, consider the following argument. When agents pledge one unit of bond at the lending facility, equation (30) implies that they obtain $(\tilde{R}+\rho)/(1+i_\ell)$ units of money. With it agents can purchase $1/p = \phi(1+i_d)$ units of goods, by equation (29). Hence, one additional unit of bond at the lending facility has a value $u'(q)\phi(1+i_d)[(\tilde{R}+\rho)/(1+i_\ell)]$. If agents do not borrow, then they get the return from the bonds $\phi(\tilde{R}+\rho)$. Consumers must be indifferent between the two options as they are unconstrained, so that $u'(q) = (1+i_\ell)/(1+i_d)$.

25. A sweep account transfers funds from deposit accounts, against which the banks would have to hold reserves, into another account against which no reserves need to be held at the end of each day. This allows the bank to minimize the amount of reserves it must hold.

26. The transcript of the testimony can be found at http://www.federalreserve.gov/newsevents/testimony/kohn20060301a.htm.

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APPENDIX

A.1. DERIVATION OF EQUATION (43)

We know that

$$
\begin{aligned}
L &= \mu(1-n)[pq-(M-\eta y^1)],\\
D &= \mu n(M-\eta y^1+pq_s)\\
&\quad + (1-\mu)n(M-\eta y^0+pq_s)+(1-\mu)(1-n)(M-\eta y^0)\\
&= npq_s+nM+(1-\mu)(1-n)M-\mu n\eta y^1-(1-\mu)\eta y^0\\
&= \mu(1-n)pq+(1-\mu(1-n))M+\mu(1-n)\eta y^1\\
&= M+\mu(1-n)[pq-(M-\eta y^1)],
\end{aligned}
$$

$$M_{+1} = M - i_\ell(L) + i_d D + \pi \quad \textbf{(A.1)}$$

$$= M - i_\ell(\mu(1-n)[pq-(M-\eta y^1)])$$

$$+ i_d[M+\mu(1-n)[pq-(M-\eta y^1)]] + \pi \quad \textbf{(A.2)}$$

$$= M(1+i_d) + (i_d - i_\ell)\mu(1-n)[pq-(M-\eta y^1)] + \pi \quad \textbf{(A.3)}$$

$$= M(1+i_d) + (i_d - i_\ell)\mu(1-n)\ell + \pi. \quad \textbf{(A.4)}$$

Therefore,

$$\gamma = 1 + i_d + (i_d - i_\ell)\,\mu\,(1-n)\,\frac{\ell}{M} + \frac{\pi}{M},$$

where $\ell = pq - (M - \eta y^1)$.

A.2. EQUILIBRIUM WITH OMOS

Equilibrium and Proof of Proposition 2. *Case in which* $\lambda_b^1 = 0$, $\lambda_m^0 = 0$ (or $\upsilon^1 = \upsilon^b = \upsilon^0$). In this case,

$$\rho = \frac{\beta}{1-\beta}\tilde{R},$$

and because $\lambda_m^0 = 0$, we also have

$$\gamma = \beta,$$

so that

$$q = q^*.$$

Because this equilibrium has the first-best allocation as its outcome, let us analyze whether it exists. This equilibrium exists if there is a policy Y, Y' such that $\gamma = \beta$. Hence, using (1) with $\tau = 0$, we must have

$$\beta = 1 + \frac{\tilde{R}}{1-\beta}\frac{(Y+\beta Y')}{M},$$

so that

$$\frac{Y}{M} + \beta\frac{Y'}{M} = -\frac{(1-\beta)^2}{\tilde{R}}.$$

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Set $Y' < 0$ and $Y' = -Y - \theta$ (i.e., the CB buys bonds in the bonds market, but less than what it sells in the settlement stage, where θ is how much more it has to sell than it just bought). Thus,

$$\frac{Y}{M} + \beta\frac{(-\theta - Y)}{M} = -\frac{(1-\beta)^2}{\tilde{R}},$$

$$\beta\frac{\theta}{M} = \frac{(1-\beta)^2}{\tilde{R}} + \frac{Y}{M}(1-\beta),$$

because we consider a stationary equilibrium, $Y/M = Y_0/M_0$. Setting $Y_0/M_0 = 0$, we get that $Y' = -\theta$, where

$$\frac{\theta}{M} = \frac{(1-\beta)^2}{\beta\tilde{R}}.$$

This policy is feasible if and only if the CB has enough bonds, i.e., if

$$\sum_{t=0}^{\infty}\theta_t \leq B_0,$$

$$\frac{(1-\beta)^2}{\beta\tilde{R}}\sum_{t=0}^{\infty}M_t \leq B_0.$$

Because $M_{t+1} = \gamma M_t = \beta M_t$, this policy is feasible if and only if

$$\frac{(1-\beta)}{\beta\tilde{R}} \leq \frac{B_0}{M_0},$$

$$M_0 \leq \frac{\beta\tilde{R}}{(1-\beta)}B_0,$$

the money stock is less than the lifetime discounted value of the CB's stock of bonds.

Case in which $\lambda_b^1 > 0$, $\lambda_m^0 = 0$ (or $\upsilon^1 > \upsilon^b = \upsilon^0$). In this case, we have $\eta = \tilde{R} + \rho$, which we can write

$$\rho = \frac{\gamma\tilde{R}}{1-\gamma}.$$

Therefore, the return on bonds is as good as money for an agent with $\varepsilon = 0$ in the bonds market. By replacing the value for η and ρ in (1) and (20), we obtain

$$\gamma = 1 + \frac{\tilde{R}}{(1-\gamma)}\frac{(Y+\gamma Y')}{M}, \quad \textbf{(A.5)}$$

$$\frac{\gamma}{\beta} = \mu[(1-n)u'(q) + n] + (1-\mu). \quad \textbf{(A.6)}$$

We also need to check that $\lambda_b^1 > 0$, which gives us the restriction (if $\gamma < 1$)

$$\gamma > \beta.$$

In this case, notice that the first-best is only attainable for $\gamma \to \beta$, Because the equilibrium equations boil down to the one for the case in which $\lambda_b^1 = 0$ and $\lambda_m^0 = 0$.

Case in which $\lambda_b^1 = 0$, $\lambda_m^0 > 0$ (*or* $\upsilon^1 = \upsilon^b > \upsilon^0$). In this case,

$$\rho = \frac{\beta}{1-\beta}\tilde{R}.$$

Because $\lambda_m^0 > 0$, we have $\eta = \rho/\gamma < \tilde{R} + \rho$, so that holding bonds yield a higher return than holding money for those agents with $\varepsilon = 0$. Replacing the above expression for ρ, we obtain that this equilibrium exists only if

$$\gamma > \beta.$$

Thus, replacing the values for η and ρ in (1) and (20), we obtain

$$\gamma = 1 + \frac{\beta}{1-\beta}\tilde{R}\frac{(Y+\gamma Y')}{\gamma M},$$
$$\frac{\gamma}{\beta} = [(1-n)u'(q) + n].$$

Comparing (20) with the above equation, observe that $(\tilde{R} + \rho)/\eta = (1-n)u'(q) + n$, so that the bond return equals the yield on money for agents with $\varepsilon = 1$.

Given Y'/M and Y/M, the first equation gives us γ, and then we get q from the second. An equilibrium with $\lambda_b^1 = 0$ exists for feasible Y'/M and Y/M such that the above equation holds if $\gamma > \beta$. The first-best is clearly not attainable in this case, and the best equilibrium allocation is achieved when $\gamma \to \beta$.

Case in which $\lambda_b^1 > 0$, $\lambda_m^0 > 0$ (*or* $\upsilon^1 > \upsilon^b > \upsilon^0$). In this case, both borrowing short-selling constraints are binding, so that $y^1 = -b$ and $y^0 = M/\eta$. By replacing these values in the market-clearing condition in the bonds market, we obtain

$$\eta = \frac{(1-\mu)\,M}{(\mu b - Y)}.$$

Notice that this implies that $\mu > Y/b$. By replacing the value for η and ρ in (1) and (20), we obtain

$$\gamma = 1 + \frac{(1-\mu)\,M}{(\mu b - Y)}\frac{(Y+\gamma Y')}{M},$$
$$\gamma = \frac{\mu\,(b-Y)}{\mu\,(b+Y') - Y - Y'},$$
$$\gamma = \frac{\mu\,(1 - Y_0/b_0)}{\mu(1 + Y_0'/b_0) - (Y_0 + Y_0')/b_0}.$$

Also,

$$\frac{\gamma}{\beta}[1 - \beta(1-\mu)] = \mu[(1-n)u'(q) + n] + \frac{\tilde{R}(\mu b - Y)}{M}. \quad \textbf{(A.7)}$$

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Given the initial conditions M_0, B_0 and policies, these two equations define an equilibrium. We also need to check that $\lambda_m^0 > 0$, which gives us the restriction

$$\gamma > 1 - \frac{\tilde{R}\,(\mu b - Y)}{(1-\mu)\,M}.$$

Finally, we need to check that $\lambda_b^1 > 0$, which gives us the restriction

$$\gamma > \frac{\beta}{(1-\beta)}\frac{\tilde{R}\,(\mu b - Y)}{(1-\mu)\,M}.$$

Notice that q^* is not attainable here. At q^*, we must have from (A.7) that

$$\frac{\beta}{(1-\beta)}\frac{\tilde{R}(\mu b - Y)}{M} = 1.$$

Thus the above restriction implies $\gamma > 1/(1-\mu)$. Because $\upsilon^1 > \upsilon^b$ and $\gamma/\beta = \mu\upsilon^1 + (1-\mu)\upsilon^b$, it must be that $\upsilon^1 > 1$, which is a contradiction.

Proof of Proposition 3. Note that welfare is decreasing in γ in all four equilibria. Therefore, the CB should seek to implement $\gamma = 1$. It is easy to see that $\gamma = 1$ cannot be an equilibrium in the case in which $\upsilon^1 = \upsilon^b = \upsilon^0$ and $\upsilon^1 > \upsilon^b = \upsilon^0$. Let us now consider the case in which $\upsilon^1 > \upsilon^b > \upsilon^0$. In this case, we have $\gamma/\beta = 1/\beta = \mu\upsilon^1 + (1-\mu)\upsilon^b$. So that $\upsilon^1 > 1/\beta$. Finally, in the case in which $\upsilon^1 = \upsilon^b > \upsilon^0$, we have $\upsilon^1 = 1/\beta$. Therefore, this is the case the CB should aim for. It is easy to see that the policy $Y = -Y'$ (pure repos) implements $\gamma = 1$. The rest of the proposition follows from the definition of the equilibrium.

A.3. CHANNEL SYSTEM WITH NO LENDING FACILITY: EQUILIBRIUM

Proof of Proposition 6. Suppose there is a channel system that achieves the first-best. Then because $u'(q^*) = 1$, (31) implies that $\lambda_\ell = 0$, and $i_\ell = i_d$. In turn, this implies that $\gamma = 1 + i_d$. From (41) we have

$$\frac{1}{\beta} \geq \frac{\tilde{R} + \rho}{\rho}, \qquad \textbf{(A.8)}$$

as the term in square brackets equals 1. Then replacing $\gamma = 1 + i_d$ and $u'(q^*) = 1$ in (39), we get

$$\frac{1}{\beta} = \mu + (1-\mu)\,\frac{\tilde{R} + \rho}{\rho},$$

which contradicts (A.8).

Proof of Proposition 7. It is easy to see that only $\lambda_m^0 > 0$ is an equilibrium, so that $y^0 = M/\eta$. Indeed, because $\gamma\eta = \rho$, we obtain $\eta(1 + i_d) = \rho$, because $\gamma = 1 + i_d$. Hence, we always get that $\tilde{R} + \rho > \eta(1 + i_d) = \rho$. Therefore we need to consider only two types of equilibrium, one where $\rho > \beta\tilde{R}/(1-\beta)$ and one where $\rho = \beta\tilde{R}/(1-\beta)$.

Case in which $\lambda_m^0 > 0$, $\lambda_b^1 = 0$ $(\upsilon^1 = \upsilon^b > \upsilon^0)$. In this case,

$$\rho = \frac{\beta}{1-\beta}\tilde{R},$$

and replacing this value and the expression for η and $\gamma = 1 + i_d$ in (39), we obtain

$$\frac{1}{\beta} = (1-n)\,u'(q) + n. \tag{A.9}$$

This is an equilibrium only if $y^1 > -b$, because $\lambda_b^1 = 0$. Using the bonds market clearing condition together with $y^0 = M/\eta$, we obtain

$$b > \frac{(1-\mu)}{\mu}\frac{M}{\eta}.$$

Notice that $b = \bar{B}$, because the CB does not hold any bonds. Replacing the value for η, we obtain that $\lambda_m^0 > 0$, $\lambda_b^1 = 0$ is an equilibrium if and only if

$$\frac{\beta\tilde{R}}{(1-\beta)}\bar{B} = \rho\bar{B} > \frac{1-\mu}{\mu}M(1+i_d).$$

(Multiplying both sides by ϕ, we have an expression linking the real value of bonds on the left-hand side to the real value of money multiplied by a constant on the right-hand side.) Hence, for this to be an equilibrium, $1 + i_d$ should be low enough.

Case in which $\lambda_m^0 > 0$, $\lambda_b^1 > 0$ $(\upsilon^1 > \upsilon^b > \upsilon^0)$. In this case, $y^0 = M/\eta$ and $y^1 = -b = -\bar{B}$, so that η is given by the market clearing condition on the bonds market,

$$\eta = \frac{1-\mu}{\mu}\frac{M}{\bar{B}}.$$

Hence,

$$\rho = \frac{1-\mu}{\mu}\frac{M}{\bar{B}}(1+i_d).$$

Replacing these values in (39), we obtain

$$\frac{1}{\beta} = \mu[(1-n)u'(q) + n] + (1-\mu)\frac{\tilde{R}+\rho}{\rho}. \tag{A.10}$$

This is an equilibrium if and only if $\rho > \beta\tilde{R}/(1-\beta)$ and $(\tilde{R}+\rho)/\eta > 1 + i_d$. We know the second condition is always satisfied. Turning to the first condition, this requires

$$\frac{1-\mu}{\mu}\frac{M}{\bar{B}}(1+i_d) > \frac{\beta}{1-\beta}\tilde{R}.$$

Hence, this equilibrium exists if $1 + i_d$ is large enough.

Notice that

$$\frac{\tilde{R}+\rho}{\rho} = \frac{\tilde{R}}{\rho} + 1 = \tilde{R}\frac{\mu}{(1-\mu)}\frac{\bar{B}}{M(1+i_d)} + 1 < \frac{1}{\beta}.$$

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Therefore, using (A.10), we have

$$\frac{1}{\beta} < [(1-n)u'(q) + n].$$

Comparing this with (A.9), we find that welfare is highest in the first equilibrium (where i_d is relatively small). When i_d is too large, bonds are relatively unattractive and agents cannot get much money for their bonds on the bonds market.

A.4. CHANNEL SYSTEM WITH A LENDING FACILITY ($\lambda_\ell > 0$): EQUILIBRIUM

Suppose now $\lambda_\ell > 0$. In words, agents are constrained when they borrow from the CB. Then

$$\ell = \frac{(\tilde{R} + \rho)}{(1 + i_\ell)}(\bar{B} + y^1).$$

Because agents borrow from the CB, it must be that $\lambda_b^1 = 0$, so that they do not sell all their bonds on the bonds market, and hence (41) gives

$$\frac{1}{\beta} = \frac{(\tilde{R} + \rho)}{\rho} \left\{ 1 + \mu (1 - n) \left[u'(q) \frac{(1 + i_d)}{(1 + i_\ell)} - 1 \right] \right\}. \qquad \textbf{(A.11)}$$

Consider now the case in which $\lambda_m^0 > 0$; then (42) implies that

$$y^1 = -\frac{1 - \mu}{\mu} \frac{M}{\eta}.$$

The budget constraint on the goods market gives us

$$pq = M - \eta y^1 + \ell,$$

or

$$\phi \ell = q \frac{1}{(1 + i_d)} - \frac{1}{\mu} \phi M.$$

Replacing y^1 in the expression for ℓ above, we obtain

$$\phi \ell = \frac{(\tilde{R} + \rho)}{\rho (1 + i_\ell)} \left(\phi \rho \bar{B} - \frac{1 - \mu}{\mu} \phi M \gamma \right).$$

Hence, combining the last two equations, we get

$$q \mu \frac{(1 + i_\ell)}{(1 + i_d)} - \phi M (1 + i_\ell) = \frac{(\tilde{R} + \rho)}{\rho} (\mu \phi \rho \bar{B} - (1 - \mu) \phi M \gamma).$$

Replacing this value in (43), we have an expression for γ (with $\pi = 0$):

$$\gamma = (1 - n)(1 + i_\ell) + n(1 + i_d) - (i_\ell - i_d)\mu(1 - n) q \frac{1}{\phi M (1 + i_d)}.$$

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Also, we have from (39)

$$\frac{1}{\beta} = \mu(1-n)u'(q)\frac{1}{\gamma} + (1-\mu)\frac{\tilde{R}+\rho}{\rho} + \mu n\frac{(1+i_d)}{\gamma}. \qquad \textbf{(A.12)}$$

and arranging (A.11),

$$\frac{1}{\beta} = \mu(1-n)u'(q)\frac{(1+i_d)}{(1+i_\ell)}\frac{(\tilde{R}+\rho)}{\rho} + (1-\mu)\frac{(\tilde{R}+\rho)}{\rho} + \mu n\frac{(\tilde{R}+\rho)}{\rho}.$$

Combining both of the above equations, we obtain the equilibrium expression for the market rate,

$$(1-n)u'(q)\frac{1}{\gamma} + n\frac{(1+i_d)}{\gamma} = (1-n)u'(q)\frac{(1+i_d)}{(1+i_\ell)}\frac{(\tilde{R}+\rho)}{\rho} + n\frac{(\tilde{R}+\rho)}{\rho},$$

$$\frac{(\tilde{R}+\rho)}{\eta} = (1+i_\ell)\frac{(1-n)u'(q)+n(1+i_d)}{(1-n)u'(q)(1+i_d)+n(1+i_\ell)}.$$

Notice that $(\tilde{R}+\rho)/\eta < 1+i_\ell$, as required in equilibrium. Also, $(\tilde{R}+\rho)/\eta > 1+i_d$ if and only if $1+i_\ell > (1+i_d)^2$. Thus, the equilibrium in this case is defined by ϕ_0, ρ, q, and γ. It satisfies

$$\frac{(\tilde{R}+\rho)}{\rho}\gamma = (1+i_\ell)\,\frac{(1-n)\,u'\,(q)+n\,(1+i_d)}{(1-n)\,u'\,(q)\,(1+i_d)+n\,(1+i_\ell)},$$

$$\gamma = (1-n)(1+i_\ell)+n(1+i_d) - \frac{(i_\ell-i_d)}{(1+i_\ell)}(1-n)\frac{q}{\phi M}\mu\frac{(1+i_\ell)}{(1+i_d)},$$

$$\frac{\gamma}{\beta} = \mu(1-n)u'(q) + (1-\mu)\,\frac{\tilde{R}+\rho}{\rho}\gamma + \mu n\,(1+i_d)\,,$$

$$\frac{q}{\phi M}\mu\frac{(1+i_\ell)}{(1+i_d)} - (1+i_\ell) = \mu\frac{\phi\rho\bar{B}}{\phi M}\frac{(\tilde{R}+\rho)}{\rho} - (1-\mu)\,\frac{(\tilde{R}+\rho)}{\rho}\gamma.$$

Combining the second and fourth equations, we get

$$\gamma = \frac{1+i_d - \dfrac{(i_\ell-i_d)}{(1+i_\ell)}\,(1-n)\,\mu\dfrac{\phi\rho\bar{B}}{\phi M}\dfrac{(\tilde{R}+\rho)}{\rho}}{1-\dfrac{(i_\ell-i_d)}{(1+i_\ell)}\,(1-n)\,(1-\mu)\,\dfrac{(\tilde{R}+\rho)}{\rho}}.$$

Consider now the case in which $\lambda_m^0 = 0$. Then

$$\frac{(\tilde{R}+\rho)}{\eta} = 1+i_d,$$

and because $\lambda_b^1 = 0$, we have

$$\frac{\gamma}{\beta} = (1+i_d)\left\{1+\mu\,(1-n)\left[u'\,(q)\,\frac{(1+i_d)}{(1+i_\ell)} - 1\right]\right\}, \qquad \textbf{(A.13)}$$

$$\gamma = (1-n)\,(1+i_\ell)+n\,(1+i_d) - (i_\ell-i_d)\,\mu\,(1-n)\,q\,\frac{1}{\phi M\,(1+i_d)}.$$

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Proof of Proposition 8. Suppose $i_\ell > i_d$, and consider the following candidate allocation:

$$u'(q) = \frac{1+i_\ell}{1+i_d}, \tag{A.14}$$

such that from (31) $\lambda_\ell = 0$. In other words, agents are not constrained when they borrow from the CB. Also, consider an equilibrium in which agents borrow from the CB. Then must they must still hold bonds in the goods market, so that $y^1 > -b$. Therefore, any equilibrium in which agents borrow from the CB has $\lambda_b^1 = 0$. Combining (A.14) and (41), $\lambda_b^1 = 0$ if and only if $\rho = \beta \tilde{R}/(1-\beta)$, or

$$\frac{\tilde{R}+\rho}{\rho} = \frac{1}{\beta}. \tag{A.15}$$

Now consider the case in which $\lambda_m^0 > 0$. (40) implies that

$$\frac{\tilde{R}+\rho}{\eta} = \frac{\tilde{R}+\rho}{\rho}\gamma > 1+i_d.$$

Combining both of the above equations, we obtain

$$1+i_d > \gamma > \beta(1+i_d), \tag{A.16}$$

where the first inequality follows from the fact that $i_\ell > i_d$ and $\ell > 0$. Now replacing (A.15) in (39), we get

$$\frac{\gamma}{\beta} = [(1-n)u'(q)+n](1+i_d). \tag{A.17}$$

Notice that (A.16) and (A.17) give us the desired result.

We now check that this equilibrium exists. Replacing $u'(q)$ using (A.14),

$$\frac{\gamma}{\beta} = (1-n)(1+i_\ell)+n(1+i_d). \tag{A.18}$$

Using (43) and (A.18) we get

$$1+i_d+(i_d-i_\ell)\,\mu\,(1-n)\,\frac{\ell}{M} = \beta\,(1+i_d)+(1-n)\,\beta(i_\ell-i_d),$$

$$(1+i_d)\,(1-\beta)-(1-n)\,\beta\,(i_\ell-i_d) = \beta\,(i_\ell-i_d)\,\mu\,(1-n)\,\frac{\ell}{M},$$

$$1+\mu\frac{\ell}{M} = \frac{(1-\beta)}{\beta}\,\frac{(1+i_d)}{(i_\ell-i_d)\,(1-n)}.$$

Finally, to check that this is indeed an equilibrium, we need to verify that (using the budget constraint on the goods market)

$$0 < \frac{\ell}{M} \leq \frac{(\bar{B}+y^1)}{M}\,\frac{(\tilde{R}+\rho)}{(1+i_\ell)}, \tag{A.19}$$

where

$$y^1 = -\frac{(1-\mu)}{\mu}y^0 = -\frac{(1-\mu)}{\mu}M/\eta.$$

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Focusing on the first inequality and replacing the value for y^1 and p, this requires that

$$\frac{(1-\beta)}{\beta}\frac{(1+i_d)}{(i_\ell - i_d)(1-n)} > 1,$$

or

$$\frac{1-\beta n}{\beta(1-n)} > \frac{1+i_\ell}{1+i_d},$$

and focusing on the second inequality, this requires that

$$\frac{(1-\beta)}{\beta}\frac{(1+i_d)}{\mu(i_\ell - i_d)(1-n)} - \frac{1}{\mu} \le \left(\frac{\phi\rho\bar{B}}{\phi M} - \frac{(1-\mu)}{\mu}\gamma\right)\frac{1}{\beta(1+i_\ell)}, \qquad \textbf{(A.20)}$$

where $\phi\rho\bar{B}$ is the real value of the stock of bonds (a constant) and ϕM is the real value of the stock of money, also a constant. Notice that for all $\bar{B}$ and M, this inequality puts a limit on how close i_ℓ can be to i_d, i.e., on the size of the interest rate channel. In other words, if the lending rate is too close to the deposit rate, borrowing is relatively cheap and consumers will want to borrow too much from the CB. To reduce borrowing, the CB must increase i_ℓ.

Replacing ℓ in the budget constraint on the goods market and $p\phi(1+i_d) = 1$ gives us ϕM, as

$$pq = M - y^1\eta + \ell,$$

or

$$\phi M = \frac{\beta}{(1-\beta)}\mu(1-n)\frac{(i_\ell - i_d)}{(1+i_d)^2}q.$$

Setting

$$M_0 = \frac{\beta}{(1-\beta)}\mu(1-n)\frac{(i_\ell - i_d)}{(1+i_d)^2}q,$$

we know that $\phi_0 = 1$, and because $\rho_0 = \beta\tilde{R}_0/(1-\beta)$, we can pick $\bar{B}_0$ high enough so that (A.20) is satisfied.

A.5. OPEN-MARKET OPERATIONS IN THE GOODS MARKET

In this section, we consider the model in which the bonds market is open in the goods market, once preference and trading shocks are realized but before trades actually take place.

In the settlement stage, the problem of an agent with portfolio (m, b) is now

$$V(m,b) = \max_{h,m_+,b_+} -h + \beta W(m_+, b_+),$$

$$\text{s.t.} \quad \phi m_+ + \phi\rho b_+ = h + \phi(m-T) + \phi(\tilde{R}+\rho)b + \phi\pi.$$

Notice that the value of the portfolio (m_+, b_+) is now directly evaluated in the goods market: Because the bonds market is open in the goods market, we can simply assume that agents wait for the realization of their shock and ignore their signals before trading their

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bonds. Using the budget constraint to eliminate h in the objective function, one obtains the first-order conditions

$$\beta W_{m_+} = \phi, \tag{A.21}$$

$$\beta W_{b_+} \leq \phi\rho(= \text{ if } b_+ > 0). \tag{A.22}$$

The envelope conditions are

$$V_m = \phi \quad \text{and} \quad V_b = \phi(\tilde{R} + \rho). \tag{A.23}$$

When they enter the goods market, agents can be either consumers or producers. An agent who received shock $\varepsilon = 1$ needs money with probability $1 - n$ and does not need any with probability n. An agent who received shock $\varepsilon = 0$ does not need money. In general, the expected payoff of an agent with shock ε and portfolio (m, b) is

$$W^{\varepsilon}(m, b) = (1 - n)W^{\varepsilon,c}(m, b) + nW^{\varepsilon,p}(m, b). \tag{A.24}$$

Let q and q_p^{ε} denote the quantities consumed and produced in the goods market by agents who received shock ε, respectively (only those agents with $\varepsilon = 1$ consume, and q denotes their consumption level). Producers (whether they get $\varepsilon = 0$ or $\varepsilon = 1$) solve the following problem:

$$W^{\varepsilon,p}(m, b) = \max_{q_p^{\varepsilon}, y_p^{\varepsilon}} \left[-q_p^{\varepsilon} + V\left(m - \eta y_p^{\varepsilon} + pq_p^{\varepsilon}, b + y_p^{\varepsilon}\right)\right],$$

$$\text{s.t.} \quad -b \leq y_p^{\varepsilon} \leq \frac{m}{\eta}.$$

Because bonds have to be exchanged before goods are traded, the sales pq_p^{ε} do not enter the short-selling constraints. Because producers have no need for money, their bonds' short-selling constraint does not bind, $\phi\lambda_b = 0$, and the first-order conditions are

$$p\phi = 1,$$

$$-\eta V_m + V_b = \phi\lambda_m^{\varepsilon},$$

where $\phi\lambda_m$ is the real multiplier on the money short-selling constraint. Using the envelope conditions (A.23), we obtain

$$(\tilde{R} + \rho) - \eta = \lambda_m^{\varepsilon}.$$

Therefore, $\lambda_m^{\varepsilon} > 0$, so that $y_p^{\varepsilon} = m/\eta$ if $\tilde{R} + \rho > \eta$. In other words, agents who do not need money buy as many bonds as possible if they are cheap relative to their return. The envelope conditions are then

$$W_m^{\varepsilon,p} = V_m + \phi\lambda_m^{\varepsilon}/\eta = \phi(\tilde{R} + \rho)/\eta,$$

$$W_b^{\varepsilon,p} = V_b = \phi(\tilde{R} + \rho).$$

For those agents who do not need money, the incremental value of money is the value of purchasing bonds, whereas the incremental value of bonds is just their real return at the next settlement stage.

Agents with $\varepsilon = 0$ who cannot produce solve

$$W^{0,c}(m, b) = \max_{y_c^0} V\left(m - \eta y_c^0, b + y_c^0\right),$$
$$\text{s.t.} \quad -b \leq y_c^0 \leq \frac{m}{\eta}.$$

As these agents do not need money, the short-selling constraint on bonds will not bind. The first-order and envelope conditions then give

$$(\tilde{R} + \rho) - \eta = \lambda_m^{0,c},$$
$$W_m^{0,c} = \phi(\tilde{R} + \rho)/\eta,$$
$$W_b^{0,c} = \phi(\tilde{R} + \rho).$$

Finally, consumers solve

$$W^{1,c}(m, b) = \max_{q, y_c^1} u(q) + V(m - \eta y_c^1 - pq, b + y_c^1),$$
$$\text{s.t.} \quad -b \leq y_c^1 \leq \frac{m}{\eta},$$
$$pq \leq m - \eta y_c^1.$$

Because these agents need money, the short-selling constraint on money will never bind and the first-order conditions are

$$u'(q) - pV_m - \phi p\lambda = 0,$$
$$-\eta V_m + V_b + \phi\lambda_b^1 - \eta\phi\lambda = 0.$$

Using the envelope conditions on the settlement stage and $p\phi = 1$, we obtain

$$u'(q) = (\tilde{R} + \rho)/\eta + \lambda_b^1/\eta,$$
$$\lambda = (\tilde{R} + \rho)/\eta - 1 + \lambda_b^1/\eta.$$

Consumers equate the marginal utility of money to the marginal value of a bond. Whenever their short-selling constraint is binding, money has more value than a bond alone. This is the case (i.e., $y_c^1 = -b$) if $u'(q) > (\tilde{R} + \rho)/\eta$. The envelope conditions for these agents are

$$W_m^{1,c} = V_m + \phi\lambda = \phi(\tilde{R} + \rho)/\eta + \phi\lambda_b^1/\eta,$$
$$W_b^{1,c} = V_b + \phi\lambda_b^1 = \phi(\tilde{R} + \rho) + \phi\lambda_b^1.$$

The incremental value of money is again the value of purchasing bonds. However, bonds are more valuable for $\varepsilon = 1$ agents, because they can be sold to relax their budget constraint.

Independent of the shock received, the value of a portfolio (m, b) for an agent that enters the goods market is therefore

$$W(m, b) = \mu[nW^{1,p}(m, b) + (1 - n)W^{1,c}(m, b)]$$
$$+ (1 - \mu)[nW^{0,p}(m, b) + (1 - n)W^{0,c}(m, b)],$$

so that we obtain the envelope conditions

$$W_m = \phi \frac{(\tilde{R} + \rho)}{\eta} + \mu (1 - n) \phi \left[u'(q) - \frac{(\tilde{R} + \rho)}{\eta} \right],$$

$$W_b = \phi(\tilde{R} + \rho) + \mu (1 - n) \phi \eta \left[u'(q) - \frac{(\tilde{R} + \rho)}{\eta} \right].$$

Combining them with (A.21) and (A.22), we get

$$\frac{\gamma}{\beta} - \frac{(\tilde{R} + \rho)}{\eta} = \mu (1 - n) \left[u'(q) - \frac{(\tilde{R} + \rho)}{\eta} \right]$$

and the no-arbitrage condition $\rho \geq \eta_+$ with equality if $b > 0$. Using our stationarity assumptions $\phi\rho = \phi_- \rho_-$, we get

$$\rho \geq \gamma\eta \quad \text{with } = \text{ if } b > 0.$$

We will still assume that $q_p^1 = q_p^0 = q_p$, where market clearing requires that $q_p = (1 - n)\mu q/n$. The other equilibrium condition (obtained from combining (1) and (6)) is still valid and is determined as before. Therefore, we can define an equilibrium with OMOs in the goods stage as follows.

DEFINITION 10. *Given the CB policy $(\tau, B, Y/M, Y'/M)$, a symmetric stationary equilibrium is a list (γ, q, η, ρ) that solves*

$$\gamma = 1 + \eta \frac{Y}{M} + \rho \frac{Y'}{M} + \frac{\tau}{M},$$

$$\frac{\gamma}{\beta} = \frac{(\tilde{R} + \rho)}{\eta} + \mu (1 - n) \left[u'(q) - \frac{(\tilde{R} + \rho)}{\eta} \right],$$

$$\rho = \gamma\eta,$$

and

$$y_p^0 = y_p^1 = y_p = \frac{m}{\eta} \quad \text{if } \tilde{R} + \rho > \eta,$$

$$y_c^1 = \begin{cases} -b & \text{if } u'(q) > (\tilde{R} + \rho)/\eta \\ -\dfrac{(1 - \mu) + \mu n}{\mu (1 - n)} y_p - \dfrac{1}{\mu (1 - n)} Y & \text{otherwise.} \end{cases}$$

Let us suppose once again that the CB has to redistribute its profit from OMOs, or $\gamma \geq 1$. Then the best-case scenario is that consumers are not constrained on the goods market, so that $u'(q) = (\tilde{R} + \rho)/\eta$. In this case, the best equilibrium is $\gamma = 1$, so that $\rho = \eta$. Then bonds are priced at their fair value, because we have

$$(\tilde{R} + \rho)/\rho = 1/\beta,$$

$$\rho = \beta\tilde{R}/(1 - \beta).$$

Notice that, in this case, $u'(q) = 1/\beta$. This is the best allocation that can be achieved when the CB conducts OMOs in the goods market.

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PROPOSITION 11. *Suppose the CB rebates its profit; then* $\gamma = 1$ *and the best equilibrium allocation with OMOs in the goods market satisfies*

$$u'(q) = 1/\beta.$$

In this equilibrium, notice that OMOs can perfectly insure agents against their liquidity shock, because they all value money at $1/\beta$. However, OMOs have little control over the inflation rate, which remains at $\gamma = 1$.

We now derive the equilibrium conditions when the CB does not conduct OMOs but offers facilities instead. The bonds market is still open in the goods market.

When they enter the goods market, agents realize whether they need money or not. An agent who received shock $\varepsilon = 1$ needs money with probability $1 - n$ and does not need any with probability n. An agent who received shock $\varepsilon = 0$ does not need money. The expected payoff of an agent ε and portfolio (m, b) is

$$W^{\varepsilon}(m, b) = (1 - n)W^{\varepsilon,c}(m, b) + nW^{\varepsilon,p}(m, b).$$

Let q and q_p^{ε} denote the quantities consumed and produced in the goods market by agents who received shock ε, respectively (only those agents with $\varepsilon = 1$ consume, and q denotes their consumption level). Producers (whether they get $\varepsilon = 0$ or $\varepsilon = 1$) solve the following problem:

$$W^{\varepsilon,p}(m, b) = \max_{q_p^{\varepsilon}, y_p^{\varepsilon}, d} \left[-q_p^{\varepsilon} + V\left(m - \eta y_p^{\varepsilon} + p q_p^{\varepsilon} - d, b + y_p^{\varepsilon}, 0, d\right)\right],$$

$$\text{s.t.} \quad -b \le y_p^{\varepsilon} \le \frac{m}{\eta},$$

$$d \le m - \eta y_p^{\varepsilon} + p q_p^{\varepsilon},$$

where we have already taken into account that producers do not borrow at the CB. As producers have no need for money, their short-selling constraint on bonds does not bind, $\phi\lambda_b = 0$, and the first-order conditions are

$$-1 + pV_m + p\phi\lambda_d^{\varepsilon} = 0,$$

$$-\eta V_m + V_b - \eta\phi\lambda_d^{\varepsilon} = \phi\lambda_m^{\varepsilon},$$

$$-V_m + V_d - \phi\lambda_d^{\varepsilon} = 0,$$

where $\phi\lambda_m^{\varepsilon}$ is the multiplier on the money short-selling constraint and $\phi\lambda_d^{\varepsilon}$ is the multiplier on the deposit constraint. Using the envelope conditions on the settlement stage, $V_d = \phi(1 + i_d)$, we obtain

$$p\phi\,(1 + i_d) = 1,$$

$$(\tilde{R} + \rho) - \eta\,(1 + i_d) = \lambda_m^{\varepsilon},$$

$$\lambda_d^{\varepsilon} = i_d.$$

Therefore, $\lambda_m^{\varepsilon} > 0$, so that $y_p^{\varepsilon} = m/\eta$ if $\tilde{R} + \rho > \eta(1 + i_d)$. In other words, agents who do not need money buy as many bonds as possible if their return is higher than the deposit rate. Also, the shadow price of money, λ_d^ε, is naturally the deposit rate, i_d. The envelope conditions are then

$$\begin{aligned} W_m^{\varepsilon,p} &= V_m + \phi\lambda_m^\varepsilon/\eta + \phi\lambda_d^\varepsilon = \phi(\tilde{R}+\rho)/\eta, \\ W_b^{\varepsilon,p} &= V_b = \phi(\tilde{R}+\rho). \end{aligned}$$

Agents with $\varepsilon = 0$ who cannot produce solve

$$\begin{aligned} W^{0,c}(m,b) = \max_{y_c^0} V\big(m - \eta y_c^0 - d, b + y_c^0, 0, d\big), \\ \text{s.t.} \quad -b \le y_c^0 \le \frac{m}{\eta}, \\ d \le m - \eta y_c^0. \end{aligned}$$

As these agents do not need money, the short-selling constraint on bonds will not bind. The first-order and envelope conditions then give

$$\begin{aligned} (\tilde{R}+\rho) - \eta\,(1+i_d) &= \lambda_m^{0,c}, \\ \lambda_d^0 &= i_d, \\ W_m^{0,c} &= \phi(\tilde{R}+\rho)/\eta, \\ W_b^{0,c} &= \phi(\tilde{R}+\rho). \end{aligned}$$

Hence, once again $y_c^0 = m/\eta$ if $\tilde{R}+\rho > \eta(1+i_d)$. Finally, consumers solve

$$\begin{aligned} W^{1,c}(m,b) = \max_{q,y_c^1,\ell} u\,(q) + V\big(m - \eta y_c^1 + \ell - pq, b + y_c^1, \ell, 0\big), \\ \text{s.t.} \quad -b \le y_c^1 \le \frac{m}{\eta}, \\ pq \le m + \ell - \eta y_c^1, \\ \ell \le \big(b + y_c^1\big)\,(\tilde{R}+\rho)/\,(1+i_\ell), \end{aligned}$$

where we have taken into account that at equilibrium, these agents do not use the deposit facility. Because these agents need money, their short-selling constraint will never bind and the first-order conditions are

$$\begin{aligned} u'\,(q) - pV_m - \phi p\lambda &= 0, \\ -\eta V_m + V_b + \phi\lambda_b^1 - \eta\phi\lambda + \phi\lambda_\ell(\tilde{R}+\rho)/\,(1+i_\ell) &= 0, \\ V_m + V_\ell + \phi\lambda - \phi\lambda_\ell &= 0. \end{aligned}$$

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Using the envelope conditions on the settlement stage and $p\phi = 1/(1+i_d)$, we obtain

$$u'(q) = \frac{1+i_\ell}{1+i_d} + \frac{\lambda_\ell}{1+i_d},$$
$$\phi\lambda_b^1 = \phi u'(q)(1+i_d)\left[\eta - \frac{(\tilde{R}+\rho)}{(1+i_\ell)}\right],$$
$$\lambda = \lambda_\ell + i_\ell.$$

Agents who need money equate the marginal utility of money to the marginal value of a bond. Whenever their short-selling constraint is binding ($\lambda_b^1 > 0$), they will have no more bonds in the goods stage, and, therefore, they will not be able to borrow at the CB. In this case $\lambda_\ell > 0$. This is the case ($y_c^1 = -b$) if $1 + i_\ell > (\tilde{R}+\rho)/\eta$; i.e., borrowing at the CB is more expensive than "borrowing" on the bonds market. The envelope conditions for these agents are

$$W_m^{1,c} = V_m + \phi\lambda = \phi(\tilde{R}+\rho)/\eta + \phi\lambda_b^1/\eta,$$
$$W_b^{1,c} = V_b + \phi\lambda_b^1 + \phi\lambda_\ell(\tilde{R}+\rho)/(1+i_\ell)$$
$$= \eta V_m + \eta\phi\lambda = \phi(\tilde{R}+\rho) + \phi\lambda_b^1.$$

The value of a portfolio (m, b) for an agent that enters the goods market is, therefore,

$$W(m,b) = \mu[nW^{1,p}(m,b) + (1-n)W^{1,c}(m,b)]$$
$$+ (1-\mu)[nW^{0,p}(m,b) + (1-n)W^{0,c}(m,b)],$$

so that we obtain the envelope conditions

$$W_m = \phi\frac{(\tilde{R}+\rho)}{\eta} + \mu(1-n)\phi u'(q)\frac{(1+i_d)}{(1+i_\ell)}\left[1+i_\ell - \frac{(\tilde{R}+\rho)}{\eta}\right],$$
$$W_b = \phi(\tilde{R}+\rho) + \mu(1-n)\phi u'(q)\frac{(1+i_d)}{(1+i_\ell)}\left[\eta(1+i_\ell) - (\tilde{R}+\rho)\right].$$

Combining them with (A.21) and (A.22), we get

$$\frac{\gamma}{\beta} - \frac{(\tilde{R}+\rho)}{\eta} = \mu(1-n)u'(q)\frac{(1+i_d)}{(1+i_\ell)}\left[1+i_\ell - \frac{(\tilde{R}+\rho)}{\eta}\right],$$

as well as the no-arbitrage condition $\rho \geq \eta_+$ with equality if $b > 0$. Using the stationarity assumptions $\phi\rho = \phi_-\rho_-$, we get

$$\rho \geq \gamma\eta \quad \text{with } = \text{ if } b > 0.$$

We will still assume that $q_p^1 = q_p^0 = q_p$, where market clearing requires that $q_p = (1-n)\mu q/n$.

The money supply still evolves according to

$$\gamma = 1 + i_d + (i_d - i_\ell)\mu(1-n)\ell/M.$$

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An equilibrium with a facility is, therefore, characterized by the following equations:

$$1 + i_\ell \geq (\tilde{R} + \rho)/\eta \geq 1 + i_d, \quad \textbf{(A.25)}$$

$$\frac{\gamma}{\beta} - \frac{(\tilde{R} + \rho)}{\eta} = \mu (1 - n) u' (q) \frac{(1 + i_d)}{(1 + i_\ell)} \left[1 + i_\ell - \frac{(\tilde{R} + \rho)}{\eta} \right], \quad \textbf{(A.26)}$$

$$\gamma = 1 + i_d + (i_d - i_\ell) \mu (1 - n) \ell / M, \quad \textbf{(A.27)}$$

$$\rho = \gamma \eta, \quad \textbf{(A.28)}$$

$$\phi \ell = q / (1 + i_d) - \phi \left(M - \eta y_c^1 \right), \quad \textbf{(A.29)}$$

and

$$y_p^\varepsilon = y_c^0 = y^0 = \frac{m}{\eta} \quad \text{if} \quad (\tilde{R} + \rho)/\eta > 1 + i_d,$$

$$y_c^1 = -b \quad \text{and} \quad \ell = 0 \quad \text{if} \quad 1 + i_\ell > (\tilde{R} + \rho)/\eta,$$

$$\mu (1 - n) y_c^1 = - [(1 - \mu) + \mu n] y^0.$$

PROPOSITION 12. *There is an equilibrium with a facility such that the allocation q^s satisfies $u'(q^s) < 1/\beta$.*

Proof. The best equilibrium with a facility occurs, when consumers are not constrained at the lending facility, i.e., $\lambda_\ell = 0$, although still borrowing at the CB. This requires that they still have bonds to pledge as collateral, so that $y_c^1 > -b$, or $\lambda_b^1 = 0$. These two requirements impose

$$u' (q) = \frac{1 + i_\ell}{1 + i_d} \quad \textbf{(A.30)}$$

and

$$\frac{(\tilde{R} + \rho)}{\eta} = 1 + i_\ell.$$

Then (A.26) implies that for money to be valued, its return has to be equal to that on bonds, or

$$\frac{\gamma}{\beta} = \frac{(\tilde{R} + \rho)}{\eta} = 1 + i_\ell.$$

The inflation rate is, therefore, $\gamma = \beta(1 + i_\ell)$. We obtain from (A.27) that $\gamma \leq 1 + i_d$ (with strict inequality whenever $\ell > 0$), so that in this equilibrium

$$\frac{1 + i_\ell}{1 + i_d} \leq \frac{1}{\beta}, \quad \textbf{(A.31)}$$

with strict inequality if there is borrowing at the CB. Combining (A.30) and (A.31), we obtain the result whenever $\ell > 0$ and $i_\ell > i_d$. We now show that such an equilibrium exists.

Because $(\tilde{R} + \rho)/\eta > 1 + i_d$, we get $y_p^\varepsilon = y_c^0 = y^0 = m/\eta$, so that

$$y_c^1 = - \frac{[(1 - \mu) + \mu n]}{\mu (1 - n)} M/\eta.$$

Because all the money in the economy is in the hands of consumers, this implies that they borrow at the CB an amount

$$\phi\ell = \frac{q}{(1+i_d)} - \frac{\phi M}{\mu(1-n)}. \tag{A.32}$$

Therefore, we need

$$\phi\ell = \frac{q}{(1+i_d)} - \frac{\phi M}{\mu(1-n)} \leq \phi\left(b + y_c^1\right)\frac{(\tilde{R}+\rho)}{(1+i_\ell)},$$

$$\frac{q}{(1+i_d)} - \frac{\phi M}{\mu(1-n)} \leq b\phi\eta - \frac{[(1-\mu)+\mu n]}{\mu(1-n)}\phi M,$$

where we have used $(\tilde{R}+\rho)/(1+i_\ell) = \eta$. This implies that agents are unconstrained at the facility if

$$\frac{q}{(1+i_d)} - \phi M \leq \bar{B}\phi\eta,$$

which implies a high enough deposit rate. ϕM is given by (A.27) once we replace for $\phi\ell$

$$\gamma = 1 + i_d + (i_d - i_\ell)\,\mu(1-n)\,\frac{\dfrac{q}{(1+i_d)} - \dfrac{\phi M}{\mu(1-n)}}{\phi M},$$

$$\frac{\beta\dfrac{1+i_\ell}{1+i_d} - 1}{1 - \dfrac{1+i_\ell}{1+i_d}}\phi M = \frac{\mu(1-n)\,q}{(1+i_d)} - \phi M,$$

$$\phi M \Delta\left[\frac{1-\beta}{\Delta - 1}\right] = \frac{\mu(1-n)\,q}{(1+i_d)}, \tag{A.33}$$

where $\Delta = (1+i_\ell)/(1+i_d)$ and where q is given by (A.30). Because $\Delta > 1$, we get that $\phi M > 0$. Finally, we need to check that $\ell > 0$. Using (A.32), this imposes

$$\frac{q}{(1+i_d)} > \frac{\phi M}{\mu(1-n)}, \tag{A.34}$$

and replacing ϕM using (A.33), this requires

$$\frac{1}{\beta} > \frac{1+i_\ell}{1+i_d},$$

which is satisfied. This completes the proof. ■

Notice that, in the equilibrium in the proof, all agents value money in the same way, so that once again the channel policy insures agents against their trading shock. Because consumers are not constrained, $\lambda_b^1 = 0$, and from the envelope condition, all agents' money valuation is $(\tilde{R}+\rho)/\eta$. And as before, the difference between the deposit and the lending rates explain why the CB can achieve the allocation q^s. If we now combine (11) and (12), we obtain that even if OMOs take place once the shocks are realized, it is better to use the channel system than OMOs, given a level of inflation that is implementable using both systems. Therefore, the result in the main body of the paper is robust to a change in the timing of OMOs.

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