# Phenomenology of an $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ model with lepton-flavour non-universality 

Sofiane M. Boucenna, ${ }^{a}$ Alejandro Celis, ${ }^{b}$ Javier Fuentes-Martín, ${ }^{c}$ Avelino Vicente ${ }^{c}$ and Javier Virto ${ }^{d}$<br>${ }^{a}$ Laboratori Nazionali di Frascati, INFN, Via Enrico Fermi 40, 100044 Frascati, Italy<br>${ }^{b}$ Arnold Sommerfeld Center for Theoretical Physics, Fakultät für Physik, Ludwig-Maximilians-Universität München, Theresienstrasse 37, 80333 München, Germany<br>${ }^{c}$ Instituto de Física Corpuscular, Universitat de València - CSIC, E-46071 València, Spain<br>${ }^{d}$ Albert Einstein Center for Fundamental Physics, Institute for Theoretical Physics, University of Bern, CH-3012 Bern, Switzerland

E-mail: boucenna@lnf.infn.it, Alejandro.Celis@physik.uni-muenchen.de, javier.fuentes@ific.uv.es, avelino.vicente@ific.uv.es, jvirto@mit.edu


#### Abstract

We investigate a gauge extension of the Standard Model in light of the observed hints of lepton universality violation in $b \rightarrow c \ell \nu$ and $b \rightarrow s \ell^{+} \ell^{-}$decays at BaBar, Belle and LHCb. The model consists of an extended gauge group $\mathrm{SU}(2)_{1} \times \mathrm{SU}(2)_{2} \times \mathrm{U}(1)_{Y}$ which breaks spontaneously around the TeV scale to the electroweak gauge group. Fermion mixing effects with vector-like fermions give rise to potentially large new physics contributions in flavour transitions mediated by $W^{\prime}$ and $Z^{\prime}$ bosons. This model can ease tensions in $B$-physics data while satisfying stringent bounds from flavour physics, and electroweak precision data. Possible ways to test the proposed new physics scenario with upcoming experimental measurements are discussed. Among other predictions, the ratios $R_{M}=\Gamma\left(B \rightarrow M \mu^{+} \mu^{-}\right) / \Gamma\left(B \rightarrow M e^{+} e^{-}\right)$, with $M=K^{*}, \phi$, are found to be reduced with respect to the Standard Model expectation $R_{M} \simeq 1$.


Keywords: Beyond Standard Model, Gauge Symmetry

ArXiv ePrint: 1608.01349

## Contents

1 Introduction ..... 2
2 Description of the model ..... 4
3 Gauge boson and fermion masses and interactions ..... 7
3.1 Fermion masses ..... 7
3.2 Vector boson masses and gauge mixing ..... 9
3.3 Gauge boson couplings to fermions ..... 10
4 Flavour constraints ..... 13
4.1 Leptonic tau decays ..... 13
$4.2 d \rightarrow u$ transitions ..... 15
$4.3 \quad s \rightarrow u$ transitions ..... 16
$4.4 \quad c \rightarrow s$ transitions ..... 16
$4.5 \quad b \rightarrow s$ transitions ..... 17
$4.6 \quad b \rightarrow c$ transitions ..... 19
4.7 Lepton flavour violation ..... 20
5 Global fit ..... 20
5.1 Fitting procedure ..... 20
5.2 Results of the fit ..... 21
6 Predictions ..... 24
6.1 Differential distributions in $B \rightarrow D^{(*)} \tau \nu$ decays ..... 24
6.2 Lepton universality tests in $R_{M}$ ..... 25
6.3 Lepton flavour violation ..... 25
6.4 Direct searches for new states at the LHC ..... 26
7 Conclusions ..... 28
A Details of the model ..... 29
A. 1 Tadpole equations ..... 29
A. 2 Scalar mass matrices ..... 30
B Pseudo-observables for $Z$ - and $W$-pole observables ..... 31

## 1 Introduction

The Standard Model (SM) of particle physics, based on the $\mathrm{SU}(3)_{C} \times \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{Y}$ gauge group, is an extremely successful theory that accounts for a wide range of high energy experiments at both the intensity and energy frontiers. It is nevertheless a theory that is widely considered to be incomplete, and manifestations of new physics (NP) are expected to show up around the TeV scale.

A large class of particularly attractive NP theories consider extensions of the SM where its gauge group is embedded into a larger one which breaks to the SM (directly or via various steps) at or above the TeV scale. In this view, the SM is seen as an effective model valid at low energies. These constructions include Grand Unified Theories (GUT), composite models and string-inspired models. Interestingly, when the last breaking of the extended gauge group occurs around the TeV scale, a plethora of observables are generally predicted. In particular, flavour physics observables constitute a powerful probe to test these models due to the impressive precision and reach of current experiments.

In this article we present a detailed phenomenological analysis focused on flavour observables of a minimal extension of the SM electroweak gauge group to $\mathrm{SU}(2)_{1} \times \mathrm{SU}(2)_{2} \times$ $\mathrm{U}(1)_{Y}$. We remain agnostic as to the origin of such a gauge group but assume it is broken around the TeV scale. Models based on an extra $\mathrm{SU}(2)$ factor have been considered since a long time and constitute some of the most studied NP theories as they are predicted by various well-motivated frameworks, such as $\mathrm{SO}(10)$ or $\mathrm{E}_{6}$ GUTs. Depending on how the $\mathrm{SU}(2)$ and $\mathrm{U}(1)$ factors are identified, we can have for instance Left-Right [1] and Ununified [2] schemes (for a general classification, cf. ref. [3]). The extra $\mathrm{SU}(2)$ factor implies the existence of new force carriers in the form of heavy partners of the SM $W$ and $Z$ bosons. In general, their couplings to matter are dictated by the choice of representations of the SM fields and the exotic new fields (if any). In any case, a rich phenomenology is predicted.

The model we will analyse was first presented in ref. [4]. While the construction of the model has been motivated mainly by recent anomalies in $B$ decays, we will carry out here a generic analysis of the model and impose the constraints arising from these hints only as a secondary step.

The salient features of our model are summarised as follows:

- The extended gauge symmetry $\mathrm{SU}(2)_{1} \times \mathrm{SU}(2)_{2} \times \mathrm{U}(1)_{Y}$ spontaneously breaks at the TeV scale to the SM electroweak group following the pattern

$$
\mathrm{SU}(2)_{1} \times \mathrm{SU}(2)_{2} \times \mathrm{U}(1)_{Y} \xrightarrow{\mathrm{TeV}} \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{Y} \xrightarrow{\mathrm{EW}} \mathrm{U}(1)_{\mathrm{em}} .
$$

- The SM fields are all charged under one of the $S U(2)$ 's only, with the same quantum numbers they have in the SM, whereas newly introduced vector-like fermions are charged similarly to the lepton and quark doublets but under the other $\mathrm{SU}(2)$ group.
- Fermion mixing effects (facilitated by the same scalar field which breaks the original group) between the exotic and SM fermions act as a source of flavour non-universal vector currents by modulating the couplings of the SM fermions to the new gauge bosons.


Figure 1. New physics contributions to $B \rightarrow K^{(*)} \mu^{+} \mu^{-}$and $B \rightarrow D^{(*)} \tau \nu$ from the tree-level exchange of massive vector bosons.

Let us now briefly summarise the current $B$ anomalies. Measurements of $b \rightarrow c\rangle \nu$ transitions for different final state leptons can be used to test lepton flavour universality to a great precision given the cancellation of many sources of theoretical uncertainties occurring in ratios such as

$$
R\left(D^{(*)}\right)=\frac{\Gamma\left(B \rightarrow D^{(*)} \tau \nu\right)}{\Gamma\left(B \rightarrow D^{(*)} \ell \nu\right)},
$$

with $\ell=e$ or $\mu$. The latest average of BaBar, Belle and LHCb measurements for these processes is $R(D)=0.397 \pm 0.049$ and $R\left(D^{*}\right)=0.316 \pm 0.019$, implying a combined deviation from the SM at the $4 \sigma$ level [5]. Additionally, a measurement of the ratio

$$
R_{K}=\frac{\Gamma\left(B \rightarrow K \mu^{+} \mu^{-}\right)}{\Gamma\left(B \rightarrow K e^{+} e^{-}\right)},
$$

performed by the LHCb collaboration in the low- $q^{2}$ region shows a $2.6 \sigma$ deviation from the $\mathrm{SM}, R_{K}=0.745_{-0.074}^{+0.090} \pm 0.036[6]$. This observable constitutes a clean probe of lepton nonuniversal new physics (NP) effects as many sources of uncertainty cancel in the ratio [79]. Intriguingly, departures from the SM have also been reported in $b \rightarrow s \mu^{+} \mu^{-}$decay observables such as branching fractions and angular distributions. Global fits to $b \rightarrow s \ell^{+} \ell^{-}$ data performed by different groups show a good overall agreement and obtain a consistent NP explanation of these departures from the SM with significances around the $4 \sigma$ level [1017]. While in the case of $b \rightarrow s \mu \mu$ observables the issue of hadronic uncertainties still raises some debate [18-23], it is clear that a common explanation to all anomalies is only possible in the presence of NP.

A considerable amount of efforts and model building activities have been devoted to these $B$-decay anomalies, though mainly focused on models that can accommodate only one of the anomalies: either $R\left(D^{(*)}\right)$ or $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$. The $R\left(D^{(*)}\right)$ anomalies have been explained with charged scalars [24-31], leptoquarks (or, equivalently, R-parity violating supersymmetry) [32-39], or a $W^{\prime}$ boson [40]. Effects due to the presence of light sterile neutrinos have also been explored in refs. [41, 42]. Models addressing the $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$ anomalies on the other hand involved mostly a $Z^{\prime}$ boson from an extended gauge group [4355], leptoquarks [56-66], or a massive resonance from a strong dynamics [67-69]. In contrast to these references, which rely on tree-level universality violation, ref. [70] systematically
explored renormalizable models that explain $R_{K}$ at the 1-loop level. The MSSM with Rparity was analysed in ref. [71], finding that it is difficult to address the $b \rightarrow s \mu \mu$ anomalies.

Unified explanations of both sets of anomalies are much more scarce. This is due to the difficulty of accounting for deviations of similar size in processes that take place in the SM at different orders: loop level for $R_{K}$ and tree-level for $R\left(D^{(*)}\right)$. Nevertheless, among the proposed models we find those based on leptoquarks [72-78], an extended perturbative gauge group [4], or strongly-interacting models [79]. An effective field theory approach has been adopted in refs. [72, 80-82] and some observations about the relevance of quantum effects have been given in ref. [83].

In our model, the massive gauge vector bosons arising from the breaking of the extended gauge group mediate flavour transitions at tree-level as shown in figure 1, providing a possible explanation to the deviations from the SM observed in $B$-meson decays [4].

The plan of the paper is as follows: in section 2 we present the model in detail. We derive the gauge boson and fermion masses and mixings, as well as the required textures in section 3. A detailed description of the flavour and electroweak observables included in the global fit is given in section 4 . Our global fit main results and predictions are presented in section 5 and section 6, respectively. Finally, in section 7 we provide our conclusions. Details of the model are provided in the appendices.

## 2 Description of the model

We consider a theory with the electroweak gauge group promoted to $\mathrm{SU}(2)_{1} \times \mathrm{SU}(2)_{2} \times$ $\mathrm{U}(1)_{Y}$. The factor $\mathrm{U}(1)_{Y}$ corresponds to the usual hypercharge while the $\mathrm{SM} \operatorname{SU}(2)_{L}$ is contained in the $\mathrm{SU}(2)$ product. The gauge bosons and gauge couplings of the extended electroweak group will be denoted as:

$$
\begin{array}{rll}
\mathrm{SU}(2)_{1}: & g_{1}, & W_{i}^{1}, \\
\mathrm{SU}(2)_{2}: & g_{2}, & W_{i}^{2},  \tag{2.1}\\
\mathrm{U}(1)_{Y}: & g^{\prime}, & B,
\end{array}
$$

where $i=1,2,3$ is the $\mathrm{SU}(2)$ index. All of the SM left-handed fermions transform exclusively under the second $\operatorname{SU}(2)$ factor, i.e.

$$
\begin{align*}
q_{L} & =(\mathbf{3}, \mathbf{1}, \mathbf{2})_{\frac{1}{6}}, & & \ell_{L}=(\mathbf{1}, \mathbf{1}, \mathbf{2})_{-\frac{1}{2}}, \\
u_{R} & =(\mathbf{3}, \mathbf{1}, \mathbf{1})_{\frac{2}{3}}, & & e_{R}=(\mathbf{1}, \mathbf{1}, \mathbf{1})_{-1},  \tag{2.2}\\
d_{R} & =(\mathbf{3}, \mathbf{1}, \mathbf{1})_{-\frac{1}{3}}, & &
\end{align*}
$$

where the representations refer to $\mathrm{SU}(3)_{C}, \mathrm{SU}(2)_{1}$ and $\mathrm{SU}(2)_{2}$, respectively, while the subscript denotes the hypercharge. The SM doublets $q_{L}$ and $\ell_{L}$ can be decomposed in $\mathrm{SU}(2)_{2}$ components in the usual way,

$$
\begin{equation*}
q_{L}=\binom{u}{d}_{L}, \quad \ell_{L}=\binom{\nu}{e}_{L} \tag{2.3}
\end{equation*}
$$

|  | generations | $\mathrm{SU}(3)_{C}$ | $\mathrm{SU}(2)_{1}$ | $\mathrm{SU}(2)_{2}$ | $\mathrm{U}(1)_{Y}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\phi$ | 1 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $1 / 2$ |
| $\Phi$ | 1 | $\mathbf{1}$ | $\mathbf{2}$ | $\overline{\mathbf{2}}$ | 0 |
| $\phi^{\prime}$ | 1 | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $1 / 2$ |
| $q_{L}$ | 3 | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{2}$ | $1 / 6$ |
| $u_{R}$ | 3 | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}$ | $2 / 3$ |
| $d_{R}$ | 3 | $\mathbf{3}$ | $\mathbf{1}$ | $\mathbf{1}$ | $-1 / 3$ |
| $\ell_{L}$ | 3 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $-1 / 2$ |
| $e_{R}$ | 3 | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | -1 |
| $Q_{L, R}$ | $n_{\mathrm{VL}}$ | $\mathbf{3}$ | $\mathbf{2}$ | $\mathbf{1}$ | $1 / 6$ |
| $L_{L, R}$ | $n_{\mathrm{VL}}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $-1 / 2$ |

Table 1. Particle content of the model.

In addition, we introduce $n_{\mathrm{VL}}$ generations of vector-like fermions transforming as

$$
\begin{equation*}
Q_{L, R} \equiv\binom{U}{D}_{L, R}=(\mathbf{3}, \mathbf{2}, \mathbf{1})_{\frac{1}{6}} ; \quad L_{L, R} \equiv\binom{N}{E}_{L, R}=(\mathbf{1}, \mathbf{2}, \mathbf{1})_{-\frac{1}{2}} \tag{2.4}
\end{equation*}
$$

For the moment we take the number of generations $n_{\mathrm{VL}}$ as a free parameter to be constrained by phenomenological requirements. Symmetry breaking is achieved via the following set of scalars: a self-dual bidoublet $\Phi$ (i.e., $\Phi=\sigma^{2} \Phi^{*} \sigma^{2}$, with $\sigma^{2}$ the usual Pauli matrix) and two doublets $\phi$ and $\phi^{\prime}$,

$$
\begin{equation*}
\phi=(\mathbf{1}, \mathbf{1}, \mathbf{2})_{\frac{1}{2}}, \quad \Phi=(\mathbf{1}, \mathbf{2}, \overline{\mathbf{2}})_{0}, \quad \phi^{\prime}=(\mathbf{1}, \mathbf{2}, \mathbf{1})_{\frac{1}{2}} \tag{2.5}
\end{equation*}
$$

which we decompose as:

$$
\phi=\binom{\varphi^{+}}{\varphi^{0}}, \quad \Phi=\frac{1}{\sqrt{2}}\left(\begin{array}{cc}
\Phi^{0} & \Phi^{+}  \tag{2.6}\\
-\Phi^{-} & \bar{\Phi}^{0}
\end{array}\right), \quad \quad \phi^{\prime}=\binom{\varphi^{\prime+}}{\varphi^{\prime 0}}
$$

with $\bar{\Phi}^{0}=\left(\Phi^{0}\right)^{*}$ and $\Phi^{-}=\left(\Phi^{+}\right)^{*}$. We summarise the particle content of the model in table 1.

Yukawa interactions. The SM fermions couple to the SM Higgs-like $\phi$ doublet with the usual Yukawa terms,

$$
\begin{equation*}
-\mathcal{L}_{\phi}=\overline{q_{L}} y^{d} \phi d_{R}+\overline{q_{L}} y^{u} \tilde{\phi} u_{R}+\overline{\ell_{L}} y^{e} \phi e_{R}+\text { h.c. } \tag{2.7}
\end{equation*}
$$

with $\tilde{\phi} \equiv i \sigma^{2} \phi^{*}$. The $y^{u, d, e}$ Yukawa couplings represent $3 \times 3$ matrices in family space. The vector-like fermions, on the other hand, have gauge-invariant Dirac mass terms,

$$
\begin{equation*}
-\mathcal{L}_{M}=\overline{Q_{L}} M_{Q} Q_{R}+\overline{L_{L}} M_{L} L_{R}+\text { h.c. } \tag{2.8}
\end{equation*}
$$

and our choice of representations allows us to Yukawa-couple them to the SM fermions via

$$
\begin{equation*}
-\mathcal{L}_{\Phi}=\overline{Q_{R}} \lambda_{q}^{\dagger} \Phi q_{L}+\overline{L_{R}} \lambda_{\ell}^{\dagger} \Phi \ell_{L}+\text { h.c. }, \tag{2.9}
\end{equation*}
$$

and

$$
\begin{equation*}
-\mathcal{L}_{\phi^{\prime}}=\overline{Q_{L}} \widetilde{y}^{d} \phi^{\prime} d_{R}+\overline{Q_{L}} \widetilde{y}^{u} \tilde{\phi}^{\prime} u_{R}+\overline{L_{L}} \widetilde{y}^{e} \phi^{\prime} e_{R}+\text { h.c. }, \tag{2.10}
\end{equation*}
$$

where $\lambda_{q, \ell}$ and $\widetilde{y}^{u, d, e}$ are $3 \times n_{\mathrm{VL}}$ and $n_{\mathrm{VL}} \times 3$ Yukawa matrices, respectively. After spontaneous symmetry breaking, these couplings will induce mixings between the vector-like and SM chiral fermions. This is crucial for the phenomenology of the model, in particular in its flavour sector, as will be clear in the next sections.

Scalar potential and symmetry breaking. The scalar potential can be cast as follows:

$$
\begin{align*}
\mathcal{V}= & m_{\phi}^{2}|\phi|^{2}+\frac{\lambda_{1}}{2}|\phi|^{4}+m_{\phi^{\prime}}^{2}\left|\phi^{\prime}\right|^{2}+\frac{\lambda_{2}}{2}\left|\phi^{\prime}\right|^{4}+m_{\Phi}^{2} \operatorname{Tr}\left(\Phi^{\dagger} \Phi\right)+\frac{\lambda_{3}}{2}\left[\operatorname{Tr}\left(\Phi^{\dagger} \Phi\right)\right]^{2}  \tag{2.11}\\
& +\lambda_{4}\left(\phi^{\dagger} \phi\right)\left(\phi^{\prime \dagger} \phi^{\prime}\right)+\lambda_{5}\left(\phi^{\dagger} \phi\right) \operatorname{Tr}\left(\Phi^{\dagger} \Phi\right)+\lambda_{6}\left(\phi^{\prime \dagger} \phi^{\prime}\right) \operatorname{Tr}\left(\Phi^{\dagger} \Phi\right)+\left(\mu \phi^{\prime \dagger} \Phi \phi+\text { h.c. }\right) .
\end{align*}
$$

We will assume that the parameters in the scalar potential are such that the scalar fields develop vevs in the following directions:

$$
\langle\phi\rangle=\frac{1}{\sqrt{2}}\binom{0}{v_{\phi}}, \quad\left\langle\phi^{\prime}\right\rangle=\frac{1}{\sqrt{2}}\binom{0}{v_{\phi^{\prime}}}, \quad\langle\Phi\rangle=\frac{1}{2}\left(\begin{array}{ll}
u & 0  \tag{2.12}\\
0 & u
\end{array}\right) .
$$

Assuming $u \gg v_{\phi}, v_{\phi^{\prime}}$, the symmetry breaking proceeds via the following pattern:

$$
\begin{equation*}
\mathrm{SU}(2)_{1} \times \mathrm{SU}(2)_{2} \times \mathrm{U}(1)_{Y} \xrightarrow{u} \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{Y} \xrightarrow{v} \mathrm{U}(1)_{\mathrm{em}}, \tag{2.13}
\end{equation*}
$$

with the assumed vev hierarchy $u \sim \mathrm{TeV} \gg v \simeq 246 \mathrm{GeV}$. With this breaking chain, the charge of the unbroken $\mathrm{U}(1)_{\mathrm{em}}$ group is defined as

$$
\begin{equation*}
Q=\left(T_{3}^{1}+T_{3}^{2}\right)+Y=T_{3}^{L}+Y, \tag{2.14}
\end{equation*}
$$

with $T_{3}^{a}$ the diagonal generator of $\mathrm{SU}(2)_{a}$. In the first step, the original $\mathrm{SU}(2)_{1} \times \mathrm{SU}(2)_{2}$ group gets broken down to the diagonal $\mathrm{SU}(2)_{L}$. Under the diagonal sub-group, $\phi$ and $\phi^{\prime}$ transform as doublets and, as usual with two-Higgs doublet models (2HDM), we parametrize their vevs as

$$
\begin{align*}
v_{\phi} & =v \sin \beta,  \tag{2.15}\\
v_{\phi^{\prime}} & =v \cos \beta,
\end{align*}
$$

where $v^{2}=v_{\phi}^{2}+v_{\phi^{\prime}}^{2}$. Since the two doublets transformed originally in a 'mirror' way under the two original $\operatorname{SU}(2)$ factors, it is clear that the ratio between their vevs, $\tan \beta=$ $v_{\phi} / v_{\phi^{\prime}}$, controls the size of the gauge mixing effects. In particular, the limit $\tan \beta=g_{1} / g_{2}$ corresponds to the purely diagonal limit with no gauge mixing, see subsection 3.2 for more details.

The scalar fields $\left\{\phi, \Phi, \phi^{\prime}\right\}$ contain 12 real degrees of freedom, six of these become the longitudinal polarization components of the $W^{(\prime)}$ and $Z^{(\prime)}$ bosons. In the CP-conserving
limit the scalar spectrum is composed of three CP-even Higgs bosons, one CP-odd Higgs and one charged scalar, forming an effective (constrained) 2HDM plus CP-even singlet system. The scalar sector will present a decoupling behaviour, with a SM-like Higgs boson at the weak scale (to be associated with the 125 GeV boson) and the rest of the scalars at the scale $u \sim \mathrm{TeV} .{ }^{1}$ Further details of the scalar sector are given in appendix A.

## 3 Gauge boson and fermion masses and interactions

We now proceed to the analysis of the model presented in the previous section. Here we will derive the masses and mixing of the gauge bosons and fermions of the model, as well as the neutral and charged vectorial currents.

### 3.1 Fermion masses

We can combine the SM and the vector-like fermions as

$$
\begin{array}{rlrl}
\mathcal{U}_{L, R}^{I} & \equiv\left(u_{L, R}^{i}, U_{L, R}^{k}\right), & \mathcal{D}_{L, R}^{I} & \equiv\left(d_{L, R}^{i}, D_{L, R}^{k}\right), \\
\mathcal{N}_{L}^{I} & \equiv\left(\nu_{L}^{i}, N_{L}^{k}\right), & \mathcal{N}_{R}^{I} \equiv\left(0, N_{R}^{k}\right),  \tag{3.1}\\
\mathcal{E}_{L, R}^{I} & \equiv\left(e_{L, R}^{i}, E_{L, R}^{k}\right), & &
\end{array}
$$

where $i=1,2,3, k=1, \ldots, n_{\mathrm{VL}}$ and $I=1, \ldots, 3+n_{\mathrm{VL}}$. With this notation the fermion mass Lagrangian after symmetry breaking is given by

$$
\begin{equation*}
-\mathcal{L}_{m}^{f}=\overline{\mathcal{U}_{L}} \mathcal{M}_{\mathcal{U}} \mathcal{U}_{R}+\overline{\mathcal{D}_{L}} \mathcal{M}_{\mathcal{D}} \mathcal{D}_{R}+\overline{\mathcal{E}_{L}} \mathcal{M}_{\mathcal{E}} \mathcal{E}_{R}+\overline{\mathcal{N}_{L}} \mathcal{M}_{\mathcal{N}} \mathcal{N}_{R}+\text { h.c. } \tag{3.2}
\end{equation*}
$$

The mass matrices are given in terms of the Yukawa couplings, vector-like Dirac masses and vevs as

$$
\begin{align*}
& \mathcal{M}_{\mathcal{U}}=\left(\begin{array}{cc}
\frac{1}{\sqrt{2}} y_{u} v_{\phi} & \frac{1}{2} \lambda_{q} u \\
\frac{1}{\sqrt{2}} \widetilde{y}_{u} v_{\phi^{\prime}} & M_{Q}
\end{array}\right), \quad \mathcal{M}_{\mathcal{D}}=\left(\begin{array}{cc}
\frac{1}{\sqrt{2}} y_{d} v_{\phi} & \frac{1}{2} \lambda_{q} u \\
\frac{1}{\sqrt{2}} \widetilde{y}_{d} v_{\phi^{\prime}} & M_{Q}
\end{array}\right), \\
& \mathcal{M}_{\mathcal{E}}=\left(\begin{array}{cc}
\frac{1}{\sqrt{2}} y_{e} v_{\phi} & \frac{1}{2} \lambda_{\ell} u \\
\frac{1}{\sqrt{2}} \widetilde{y}_{e} v_{\phi^{\prime}} & M_{L}
\end{array}\right), \quad \mathcal{M}_{\mathcal{N}}=\left(\begin{array}{cc}
0 & \frac{1}{2} \lambda_{\ell} u \\
0 & M_{L}
\end{array}\right) . \tag{3.3}
\end{align*}
$$

Note that we did not include any mechanism to generate neutrino masses, and consequently $\mathcal{M}_{\mathcal{N}}$ leads to three massless neutrinos and $n_{\mathrm{VL}}$ heavy neutral Dirac fermions. It is nevertheless straightforward to account for neutrino masses without impacting our analysis and conclusions by including one of the usual mechanisms, such as the standard seesaw.

In order to have a manageable parameter space and simplify the analysis we will assume that the Yukawa couplings of $\phi^{\prime}$ can be neglected, $\widetilde{y}_{u, d, e} \simeq 0$. This can be justified by introducing a softly-broken discrete $\mathcal{Z}_{2}$ symmetry under which $\phi^{\prime}$ is odd and all the other fields are even. We take the Dirac masses of the vector-like fermions to be generically around the symmetry breaking scale $u \sim \mathrm{TeV}$.

[^0]The fermion mass matrices can be block-diagonalized perturbatively in the small ratio $\epsilon=v / u \ll 1$ by means of the following field transformations

$$
\begin{align*}
& \mathcal{U}_{L} \rightarrow V_{Q}^{\dagger} V_{u}^{\dagger} \mathcal{U}_{L}, \mathcal{U}_{R} \rightarrow W_{u}^{\dagger} \mathcal{U}_{R} \\
& \mathcal{D}_{L} \rightarrow V_{Q}^{\dagger} V_{d}^{\dagger} \mathcal{D}_{L},  \tag{3.4}\\
& \mathcal{D}_{R} \rightarrow W_{d}^{\dagger} \mathcal{D}_{R} \\
& \mathcal{E}_{L} \rightarrow V_{L}^{\dagger} V_{e}^{\dagger} \mathcal{E}_{L}, \mathcal{E}_{R} \rightarrow W_{e}^{\dagger} \mathcal{E}_{R} \\
& \mathcal{N}_{L} \rightarrow V_{L}^{\dagger} \mathcal{N}_{L}
\end{align*}
$$

defined in terms of the unitary matrices

$$
\begin{gather*}
V_{Q, L}=\left(\begin{array}{c|c}
\left.V_{Q, L}^{11}=\sqrt{\mathbb{1}-\frac{1}{4} \lambda_{q, \ell} \widetilde{M}_{Q, L}^{-2} \lambda_{q, \ell}^{\dagger}} \right\rvert\, V_{Q, L}^{12}=-\frac{u}{2} V_{Q, L}^{11} \lambda_{q, \ell} M_{Q, L}^{-1} \\
V_{Q, L}^{21}=\frac{1}{2} \widetilde{M}_{Q, L}^{-1} \lambda_{q, \ell}^{\dagger} & V_{Q, L}^{22}=\frac{1}{u} \widetilde{M}_{Q, L}^{-1} M_{Q, L}^{\dagger}
\end{array}\right)  \tag{3.5}\\
V_{f}=\mathbb{1}+i \epsilon^{2} H_{V}^{f}+\ldots ; \quad W_{f}=\mathbb{1}+i \epsilon H_{W}^{f}+\frac{(i \epsilon)^{2}}{2} H_{W}^{f} 2+\ldots \tag{3.6}
\end{gather*}
$$

Here the freedom in the definition of $V_{Q, L}^{11}$ is removed by choosing it to be hermitian. Furthermore, $u \widetilde{M}_{Q, L}$ is the physical vector-like mass at leading order in $\epsilon$,

$$
\begin{equation*}
\widetilde{M}_{Q, L}=\sqrt{\frac{M_{Q, L}^{\dagger} M_{Q, L}}{u^{2}}+\frac{\lambda_{q, \ell}^{\dagger} \lambda_{q, \ell}}{4}} \simeq \operatorname{diag}\left(\widetilde{M}_{Q_{1}, L_{1}}, \ldots, \widetilde{M}_{Q_{n} \mathrm{VL}}, L_{n}\right) \tag{3.7}
\end{equation*}
$$

and the matrices $H_{V}^{f}$ and $H_{W}^{f}$ are given by

$$
\begin{align*}
& H_{V}^{f}=\frac{i}{2}\left(\begin{array}{c|c}
0 & V_{F}^{11} y_{f} y_{f}^{\dagger} V_{F}^{21 \dagger} \widetilde{M}_{F}^{-2} \\
\hline-\widetilde{M}_{F}^{-2} V_{F}^{21} y_{f} y_{f}^{\dagger} V_{F}^{11} & 0
\end{array}\right)  \tag{3.8}\\
& H_{W}^{f}=\frac{i}{\sqrt{2}}\left(\begin{array}{c|c}
0 & y_{f}^{\dagger} V_{F}^{21 \dagger} \widetilde{M}_{F}^{-1} \\
\hline-\widetilde{M}_{F}^{-1} V_{F}^{21} y_{f} & 0
\end{array}\right) \tag{3.9}
\end{align*}
$$

with $F=Q, L$ and $f=u, d, e$. After the block-diagonalization, a further diagonalization of the SM fermion block can be done by means of the $3 \times 3$ unitary transformations

$$
\begin{align*}
u_{L} \rightarrow S_{u}^{\dagger} u_{L}, & u_{R} \rightarrow U_{u}^{\dagger} u_{R} \\
d_{L} \rightarrow S_{d}^{\dagger} d_{L}, & d_{R} \rightarrow U_{d}^{\dagger} d_{R}  \tag{3.10}\\
e_{L} \rightarrow S_{e}^{\dagger} e_{L}, & e_{R} \rightarrow U_{e}^{\dagger} e_{R}
\end{align*}
$$

As in the SM, only one combination of these transformations appears in the gauge couplings: the CKM matrix, $V_{\mathrm{CKM}}=S_{u} S_{d}^{\dagger}$.

### 3.2 Vector boson masses and gauge mixing

Neutral gauge bosons. The neutral gauge bosons mass matrix in the basis $\mathcal{V}^{0}=$ $\left(W_{3}^{1}, W_{3}^{2}, B\right)$ is given by:

$$
\mathcal{M}_{\mathcal{V}^{0}}^{2}=\frac{1}{4}\left(\begin{array}{ccc}
g_{1}^{2}\left(v_{\phi^{\prime}}^{2}+u^{2}\right) & -g_{1} g_{2} u^{2} & -g_{1} g^{\prime} v_{\phi^{\prime}}^{2}  \tag{3.11}\\
-g_{1} g_{2} u^{2} & g_{2}^{2}\left(v_{\phi}^{2}+u^{2}\right) & -g_{2} g^{\prime} v_{\phi}^{2} \\
-g_{1} g^{\prime} v_{\phi^{\prime}}^{2} & -g_{2} g^{\prime} v_{\phi}^{2} & g^{\prime 2}\left(v_{\phi}^{2}+v_{\phi^{\prime}}^{2}\right)
\end{array}\right)
$$

This matrix has one vanishing eigenvalue, corresponding to the photon and two massive eigenstates which are identified with the $Z$ and $Z^{\prime}$ bosons. Before fully diagonalizing this mass matrix we consider first the rotation from $\left(W_{3}^{1}, W_{3}^{2}\right)$ to $\left(Z_{h}, W_{3}\right)$, with $W_{3}$ the electrically neutral $\mathrm{SU}(2)_{L}$ gauge boson. In order to do this we have to study the first symmetry breaking step, i.e. $u \neq 0$ and $v=0$, diagonalize the top-left $2 \times 2$ block and identify the massless state with $W_{3}$ (the $\mathrm{SU}(2)_{L}$ group remains unbroken in the first step). As a result we get:

$$
\begin{equation*}
Z_{h}=\frac{1}{n_{1}}\left(g_{1} W_{3}^{1}-g_{2} W_{3}^{2}\right), \quad W_{3}=\frac{1}{n_{1}}\left(g_{2} W_{3}^{1}+g_{1} W_{3}^{2}\right) \tag{3.12}
\end{equation*}
$$

with $n_{1}=\sqrt{g_{1}^{2}+g_{2}^{2}}$ and the gauge coupling of $\mathrm{SU}(2)_{L}$ taking the value $g=g_{1} g_{2} / n_{1}$. In the $\left(Z_{h}, W_{3}, B\right)$ basis, the rotation from $\left(W_{3}, B\right)$ to $\left(Z_{l}, A\right)$ is just like in the SM and we obtain:

$$
\begin{equation*}
Z_{l}=\frac{1}{n_{2}}\left(g W_{3}-g^{\prime} B\right), \quad A=\frac{1}{n_{2}}\left(g^{\prime} W_{3}+g B\right) \tag{3.13}
\end{equation*}
$$

where $n_{2}=\sqrt{g^{2}+g^{\prime 2}}$ and the weak angle is defined as usual: $\hat{s}_{W}=g^{\prime} / n_{2}$ and $\hat{c}_{W}=g / n_{2}$. We are now in condition to write the neutral gauge boson mass matrix in the $\left(Z_{h}, Z_{l}, A\right)$ basis where it takes the form:

$$
\mathcal{M}_{\mathcal{V}^{0}}^{2}=\frac{1}{4}\left(\begin{array}{ccc}
\left(g_{1}^{2}+g_{2}^{2}\right) u^{2}+\frac{g^{2} g_{2}^{2}}{g_{1}^{2}} v^{2}\left(s_{\beta}^{2}+\frac{g_{1}^{4}}{g_{2}} c_{\beta}^{2}\right)-g n_{2} \frac{g_{2}}{g_{1}} v^{2}\left(s_{\beta}^{2}-\frac{g_{1}^{2}}{g_{2}^{2}} c_{\beta}^{2}\right) & 0  \tag{3.14}\\
-g n_{2} \frac{g_{2}}{g_{1}} v^{2}\left(s_{\beta}^{2}-\frac{g_{1}^{2}}{g_{2}^{2}} c_{\beta}^{2}\right) & \left(g^{2}+g^{\prime 2}\right) v^{2} & 0 \\
0 & 0 & 0
\end{array}\right) .
$$

We see from this mass matrix that in the particular limit $v=0$, only $Z_{h}$ gets a mass $M_{Z^{\prime}}=\frac{1}{4}\left(g_{1}^{2}+g_{2}^{2}\right) u^{2}$, which is expected since $\mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{Y}$ remains unbroken in that case. Moreover, we can extract the $Z_{l}-Z_{h}$ mixing. The mass eigenvectors $\left(Z^{\prime}, Z\right)$ are given, in terms of $\left(Z_{h}, Z_{l}\right)$, by:

$$
\begin{equation*}
Z^{\prime}=\cos \xi_{Z} Z_{h}-\sin \xi_{Z} Z_{l}, \quad Z=\sin \xi_{Z} Z_{h}+\cos \xi_{Z} Z_{l} \tag{3.15}
\end{equation*}
$$

with the mixing suppressed by the ratio $\epsilon \equiv v / u$,

$$
\begin{equation*}
\xi_{Z} \simeq \frac{g n_{2}}{n_{1}^{2}} \frac{g_{2}}{g_{1}} \epsilon^{2}\left(s_{\beta}^{2}-\frac{g_{1}^{2}}{g_{2}^{2}} c_{\beta}^{2}\right)=\frac{g}{n_{2}} \frac{g_{2}}{g_{1}} \frac{M_{Z}^{2}}{M_{Z^{\prime}}^{2}}\left(s_{\beta}^{2}-\frac{g_{1}^{2}}{g_{2}^{2}} c_{\beta}^{2}\right) \tag{3.16}
\end{equation*}
$$

We define the parameter controlling the mixing as

$$
\begin{equation*}
\zeta=s_{\beta}^{2}-\frac{g_{1}^{2}}{g_{2}^{2}} c_{\beta}^{2} \tag{3.17}
\end{equation*}
$$

In the limit $\zeta \rightarrow 0$, the $\mathrm{SU}(2)_{L}$ sub-group corresponds to the diagonal subgroup of the original $\mathrm{SU}(2)$ product and gauge mixing vanishes. As anticipated in section $2, \zeta \rightarrow 0$ corresponds to the limit $\tan \beta \rightarrow g_{1} / g_{2}$.

Finally, the masses of the neutral massive vector bosons are given by

$$
\begin{equation*}
M_{Z^{\prime}}^{2} \simeq \frac{1}{4}\left(g_{1}^{2}+g_{2}^{2}\right) u^{2}, \quad M_{Z}^{2} \simeq \frac{1}{4}\left(g^{2}+g^{\prime 2}\right) v^{2} . \tag{3.18}
\end{equation*}
$$

Charged gauge bosons. In the basis $\mathcal{V}^{+}=\left(W_{12}^{1}, W_{12}^{2}\right)$, with $W_{12}^{r}=\frac{1}{\sqrt{2}}\left(W_{1}^{r}-i W_{2}^{r}\right)$, the charged gauge boson mass matrix is given by

$$
\mathcal{M}_{\mathcal{V}^{+}}^{2}=\frac{1}{4}\left(\begin{array}{cc}
g_{1}^{2}\left(v_{\phi^{\prime}}^{2}+u^{2}\right) & -g_{1} g_{2} u^{2}  \tag{3.19}\\
-g_{1} g_{2} u^{2} & g_{2}^{2}\left(v_{\phi}^{2}+u^{2}\right)
\end{array}\right) .
$$

As before, it is convenient to work in the basis $\left(W_{h}, W_{l}\right)$ where the $\mathrm{SU}(2)_{L}$ gauge boson appears explicitly. To obtain this basis in terms of the original one, we set $v=0$, diagonalize the mass matrix and associate the null eigenvalue to $W_{l}\left(\mathrm{SU}(2)_{L}\right.$ remains unbroken in the first stage of symmetry breaking). We get:

$$
\begin{equation*}
W_{h}=\frac{1}{n_{1}}\left(g_{1} W^{1}-g_{2} W^{2}\right), \quad W_{l}=\frac{1}{n_{1}}\left(g_{2} W^{1}+g_{1} W^{2}\right) . \tag{3.20}
\end{equation*}
$$

In the basis $\left(W_{h}, W_{l}\right)$ the mass matrix reads:

$$
\mathcal{M}_{\mathcal{V}^{+}}^{2}=\frac{1}{4}\left(\begin{array}{cc}
\left(g_{1}^{2}+g_{2}^{2}\right) u^{2}+\frac{g^{2} g_{2}^{2}}{g_{1}^{2}} v^{2}\left(s_{\beta}^{2}+\frac{g_{1}^{4}}{g_{2}^{4}} c_{\beta}^{2}\right)-g^{2} \frac{g_{2}}{g_{1}} v^{2}\left(s_{\beta}^{2}-\frac{g_{1}^{2}}{g_{2}^{2}} c_{\beta}^{2}\right)  \tag{3.21}\\
-g^{2} \frac{g_{2}}{g_{1}} v^{2}\left(s_{\beta}^{2}-\frac{g_{1}^{1}}{g_{2}^{2}} c_{\beta}^{2}\right) & g^{2} v^{2}
\end{array}\right) .
$$

The $W_{l}-W_{h}$ mixing presents the same structure as in the neutral gauge boson sector and reads:

$$
\begin{equation*}
\xi_{W} \simeq \zeta \frac{g^{2}}{n_{1}^{2}} \frac{g_{2}}{g_{1}} \epsilon^{2}=\zeta \frac{g_{2}}{g_{1}} \frac{M_{W}^{2}}{M_{W^{\prime}}^{2}}, \tag{3.22}
\end{equation*}
$$

such that the physical eigenstates are given by:

$$
\begin{equation*}
W^{\prime}=\cos \xi_{W} W_{h}-\sin \xi_{W} W_{l}, \quad W=\sin \xi_{W} W_{h}+\cos \xi_{W} W_{l}, \tag{3.23}
\end{equation*}
$$

with masses

$$
\begin{equation*}
M_{W^{\prime}}^{2} \simeq M_{Z^{\prime}}^{2} \simeq \frac{1}{4}\left(g_{1}^{2}+g_{2}^{2}\right) u^{2}, \quad M_{W}^{2} \simeq \frac{1}{4} g^{2} v^{2} . \tag{3.24}
\end{equation*}
$$

### 3.3 Gauge boson couplings to fermions

Neutral currents. The neutral currents of the fermions are given by

$$
\begin{align*}
\mathcal{L}_{\mathrm{NC}} & =\bar{\psi} \gamma_{\mu}\left(g^{\prime} B^{\mu} Y+g_{1} W_{3}^{1 \mu} T_{3}^{1}+g_{2} W_{3}^{2 \mu} T_{3}^{2}\right) \psi \\
& =\bar{\psi} \gamma_{\mu}\left\{e Q_{\psi} A^{\mu}+\frac{g}{c_{W}} Z_{l}^{\mu}\left[\left(T_{3}^{1}+T_{3}^{2}\right)-s_{W}^{2} Q_{\psi}\right]+g Z_{h}^{\mu}\left[\frac{g_{1}}{g_{2}} T_{3}^{1}-\frac{g_{2}}{g_{1}} T_{3}^{2}\right]\right\} \psi, \tag{3.25}
\end{align*}
$$

with $\psi=\mathcal{U}, \mathcal{D}, \mathcal{E}, \mathcal{N}$, and $e=g g^{\prime} / n_{2}$ and $Q_{\psi}$ denoting the electric coupling and the electric charge of the fermions, respectively. Applying the transformations in eqs. (3.4) and (3.10) we can easily translate the above interactions to the fermion mass eigenbasis

$$
\begin{align*}
\mathcal{L}_{\mathrm{NC}} \rightarrow \mathcal{L}_{\mathrm{NC}}= & A^{\mu} \bar{\psi} \gamma_{\mu} e Q_{\psi} \psi+\frac{g}{c_{W}} Z_{l}^{\mu}\left\{\bar{\psi} \gamma_{\mu}\left[\left(T_{3}^{1}+T_{3}^{2}\right) P_{L}-s_{W}^{2} Q\right] \psi\right. \\
& \left.-\frac{1}{2}\left(\overline{\mathcal{D}_{R}} \gamma_{\mu} O_{R}^{d d} \mathcal{D}_{R}-\overline{\mathcal{U}_{R}} \gamma_{\mu} O_{R}^{u u} \mathcal{U}_{R}+\overline{\mathcal{E}_{R}} \gamma_{\mu} O_{R}^{e e} \mathcal{E}_{R}-\overline{N_{R}} \gamma_{\mu} N_{R}\right)\right\} \\
& +\frac{\hat{g}}{2} Z_{h}^{\mu}\left[\overline{\mathcal{D}_{L}} \gamma_{\mu} O_{L}^{Q} \mathcal{D}_{L}-\overline{\mathcal{U}_{L}} \gamma_{\mu} V O_{L}^{Q} V^{\dagger} \mathcal{U}_{L}+\overline{\mathcal{E}_{L}} \gamma_{\mu} O_{L}^{L} \mathcal{E}_{L}-\overline{\mathcal{N}_{L}} \gamma_{\mu} O_{L}^{L} \mathcal{N}_{L}\right. \\
& \left.-\frac{g_{1}^{2}}{g_{2}^{2}}\left(\overline{\mathcal{D}_{R}} \gamma_{\mu} O_{R}^{d d} \mathcal{D}_{R}-\overline{\mathcal{U}_{R}} \gamma_{\mu} O_{R}^{u u} \mathcal{U}_{R}+\overline{\mathcal{E}_{R}} \gamma_{\mu} O_{R}^{e e} \mathcal{E}_{R}-\overline{N_{R}} \gamma_{\mu} N_{R}\right)\right] \\
& +\mathcal{O}\left(\frac{m_{f}^{2}}{u^{2}}\right) . \tag{3.26}
\end{align*}
$$

Here $m_{f}$ denotes the mass of a SM fermion with $f=u, d, e$, and we introduced the following definitions:

$$
\begin{align*}
O_{L}^{Q, L} & \equiv\left(\begin{array}{cc}
\Delta^{q, \ell} & \Sigma \\
\Sigma^{\dagger} & \Omega^{Q, L}
\end{array}\right)=\mathbb{1}-\frac{g_{1}^{2}+g_{2}^{2}}{g_{2}^{2}}\binom{V_{Q, L}^{12}\left(V_{Q, L}^{12}\right)^{\dagger} V_{Q, L}^{12}\left(V_{Q, L}^{22}\right)^{\dagger}}{V_{Q, L}^{22}\left(V_{Q, L}^{12}\right)^{\dagger} V_{Q, L}^{22}\left(V_{Q, L}^{22}\right)^{\dagger}},  \tag{3.27}\\
O_{R}^{f f^{\prime}} & \equiv\left(\begin{array}{cc}
0 & \hat{\Sigma}^{f} \\
\left(\hat{\Sigma}^{f^{\prime}}\right)^{\dagger} & 1
\end{array}\right)=\left(\begin{array}{cc}
0 & -\frac{m_{f}}{u}\left(V_{F}^{11}\right)^{-1}\left(V_{F}^{21}\right)^{\dagger} \widetilde{M}_{F}^{-1} \\
-\widetilde{M}_{F}^{-1} V_{F}^{21}\left(V_{F}^{11}\right)^{-1} \frac{m_{f^{\prime}}}{u} & 1
\end{array}\right),  \tag{3.28}\\
V & =\left(\begin{array}{cc}
V_{\mathrm{CKM}} & 0 \\
0 & 1
\end{array}\right), \tag{3.29}
\end{align*}
$$

with $F=Q, L$, and finally $\hat{g} \equiv g g_{2} / g_{1}$.
Charged currents. Similarly, the charged currents take the following form

$$
\begin{align*}
\mathcal{L}_{\mathrm{CC}}= & \frac{g_{1}}{\sqrt{2}} W_{\mu}^{1}\left[\bar{U} \gamma^{\mu} D+\bar{N} \gamma^{\mu} E\right]+\frac{g_{2}}{\sqrt{2}} W_{\mu}^{2}\left[\bar{u} \gamma^{\mu} P_{L} d+\bar{\nu} \gamma^{\mu} P_{L} e\right]+\text { h.c. } \\
= & \frac{g}{\sqrt{2}} W_{l}^{\mu}\left[\overline{\mathcal{U}} \gamma_{\mu} P_{L} \mathcal{D}+\overline{\mathcal{N}} \gamma_{\mu} P_{L} \mathcal{E}+\bar{U} \gamma_{\mu} P_{R} D+\bar{N} \gamma_{\mu} P_{R} E\right]  \tag{3.30}\\
& -\frac{g}{\sqrt{2}} W_{h}^{\mu}\left[\frac{g_{2}}{g_{1}}\left(\bar{u} \gamma_{\mu} P_{L} d+\bar{\nu} \gamma_{\mu} P_{L} e\right)-\frac{g_{1}}{g_{2}}\left(\bar{U} \gamma_{\mu} D+\bar{N} \gamma_{\mu} E\right)\right]+\text { h.c. }
\end{align*}
$$

and, in the fermion mass eigenbasis (see eqs. (3.4) and (3.10)), we have

$$
\begin{align*}
\mathcal{L}_{\mathrm{CC}} \rightarrow \mathcal{L}_{\mathrm{CC}}= & \frac{g}{\sqrt{2}} W_{l}^{\mu}\left[\overline{\mathcal{U}_{L}} \gamma_{\mu} V \mathcal{D}_{L}+\overline{\mathcal{N}_{L}} \gamma_{\mu} \mathcal{E}_{L}+\overline{\mathcal{U}_{R}} \gamma_{\mu} O_{R}^{u d} \mathcal{D}_{R}+\overline{\mathcal{N}_{R}} \gamma_{\mu} O_{R}^{\nu e} \mathcal{E}_{R}\right] \\
& -\frac{\hat{g}}{\sqrt{2}} W_{h}^{\mu}\left[\overline{\mathcal{U}_{L}} \gamma_{\mu} V O_{L}^{Q} \mathcal{D}_{L}+\overline{\mathcal{N}_{L}} \gamma_{\mu} O_{L}^{L} \mathcal{E}_{L}-\frac{g_{1}^{2}}{g_{2}^{2}}\left(\overline{\mathcal{U}_{R}} \gamma_{\mu} O_{R}^{u d} \mathcal{D}_{R}+\overline{\mathcal{N}_{R}} \gamma_{\mu} O_{R}^{\nu e} \mathcal{E}_{R}\right)\right] \\
& + \text { h.c. }+\mathcal{O}\left(\frac{m_{f}^{2}}{u^{2}}\right) . \tag{3.31}
\end{align*}
$$

Flavour textures for the gauge interactions. In order to accommodate the hints of lepton universality violation from the recent anomalies in $B$ decays without being in tension with other bounds, we require negligible couplings of the new gauge bosons to the first family of SM-like leptons and a large universality violation among the other two. We now derive the conditions on the number of generations of the exotic fermions to accommodate such constraints.

Using eqs. (3.27) and (3.5), the matrix $\Delta^{q, \ell}$, that parametrize NP contributions to the left-handed gauge interactions with SM fermions, can be readily written in the following form

$$
\begin{equation*}
\Delta^{q, \ell}=\mathbb{1}-\frac{g_{1}^{2}+g_{2}^{2}}{4 g_{2}^{2}} \lambda_{q, \ell} \widetilde{M}^{-2} \lambda_{q, \ell}^{\dagger}, \tag{3.32}
\end{equation*}
$$

where the second term is the source of lepton non-universality induced by the mixings between the SM and vector-like fermions generated by the $\lambda_{q, \ell}$ Yukawa couplings. On the other hand, right-handed couplings involving SM fermions, controlled by $O_{R}^{f f^{\prime}}$, are mass suppressed and they can be neglected for the interactions we are considering.

If we consider the minimal scenario with $n_{\mathrm{VL}}=1$, the Yukawa couplings $\lambda_{q, \ell}$ can be written generically as

$$
\lambda_{q, \ell}=\frac{2 g_{2}}{n_{1}} \widetilde{M}_{Q, L}\left(\begin{array}{c}
\Delta_{d, e}  \tag{3.33}\\
\Delta_{s, \mu} \\
\Delta_{b, \tau}
\end{array}\right) .
$$

Here $\Delta_{d, e}, \Delta_{s, b}$ and $\Delta_{\mu, \tau}$ are free real parameters, and without loss of generality we have chosen an appropriate normalization factor to simplify the expression of $\Delta^{q, \ell}$. We have also ignored possible complex phases in the couplings since we are not interested in CP violating observables. From eq. (3.32) it is then clear that, for $n_{\mathrm{VL}}=1$, NP contributions to the left-handed gauge couplings to SM fermions are given by

$$
\Delta_{n \mathrm{VL}}^{q, \ell}=\left(\begin{array}{ccc}
1-\left(\Delta_{d, e}\right)^{2} & \Delta_{d, e} \Delta_{s, \mu} & \Delta_{d, e} \Delta_{b, \tau}  \tag{3.34}\\
\Delta_{d, e} \Delta_{s, \mu} & 1-\left(\Delta_{s, \mu}\right)^{2} & \Delta_{s, \mu} \Delta_{b, \tau} \\
\Delta_{d, e} \Delta_{b, \tau} & \Delta_{s, \mu} \Delta_{b, \tau} & 1-\left(\Delta_{b, \tau}\right)^{2}
\end{array}\right) .
$$

As we can see, in the limit of no gauge boson mixing, NP contributions to the first family of SM fermions can only be suppressed if we fix $\Delta_{d, e} \simeq 1$ and $\Delta_{s, \mu}, \Delta_{b, \tau} \ll 1$ which then implies approximate universal couplings for the second and the third families. Hence, we need at least two generations of vector-like fermions in order to have enough freedom to accommodate the observed hints of lepton universality violation.

In the rest of this article we will take the minimal setup consisting of $n_{\mathrm{VL}}=2$ since there is no compeling reason to assume additional vector-like generations. Moreover, in order to reduce the number of free parameters in the analysis we choose the following texture for the Yukawa matrices $\lambda_{q, \ell}$ :

$$
\lambda_{q, \ell}=\frac{2 g_{2}}{n_{1}}\left(\begin{array}{cc}
\widetilde{M}_{Q_{1}, L_{1}} & 0  \tag{3.35}\\
0 & \widetilde{M}_{Q_{2}, L_{2}} \Delta_{s, \mu} \\
0 & \widetilde{M}_{Q_{2}, L_{2}} \Delta_{b, \tau}
\end{array}\right),
$$

where, again, $\Delta_{s, b}$ and $\Delta_{\mu, \tau}$ are free real parameters and the normalization factor is chosen for convenience. ${ }^{2}$ The left-handed currents, parametrized in terms of $O_{L}^{Q, L}$ (see eq. (3.27)) now read

$$
\begin{align*}
& \Delta^{q, \ell}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 1-\left(\Delta_{s, \mu}\right)^{2} & \Delta_{s, \mu} \Delta_{b, \tau} \\
0 & \Delta_{s, \mu} \Delta_{b, \tau} & 1-\left(\Delta_{b, \tau}\right)^{2}
\end{array}\right),  \tag{3.36}\\
& \Sigma=\left(\begin{array}{cc}
\frac{g_{1}}{g_{2}} & 0 \\
0 & \Delta_{s, \mu} \sqrt{\frac{n_{1}^{2}}{g_{2}^{2}}-\Delta_{s, \mu}^{2}-\Delta_{b, \tau}^{2}} \\
0 & \Delta_{b, \tau} \sqrt{\frac{n_{1}^{2}}{g_{2}^{2}}-\Delta_{s, \mu}^{2}-\Delta_{b, \tau}^{2}}
\end{array}\right),  \tag{3.37}\\
& \Omega^{Q, L}=\left(\begin{array}{cc}
1-\frac{g_{1}^{2}}{g_{2}^{2}} & 0 \\
0 & \Delta_{s, \mu}^{2}+\Delta_{b, \tau}^{2}-\frac{g_{1}^{2}}{g_{2}^{2}}
\end{array}\right), \tag{3.38}
\end{align*}
$$

which, by construction, provide the desired patterns for the NP contributions to accommodate the data.

## 4 Flavour constraints

We consider in our analysis flavour observables receiving new physics contributions at treelevel from the exchange of the massive vector bosons. Additionally, we consider bounds from electroweak precision measurements at the $Z$ and $W$ pole which are affected in our model due to gauge mixing effects.

Regarding electroweak precision observables at the $Z$ and $W$ pole, we use the fit to $Z$ and $W$-pole observables performed in ref. [84]. The fit includes the observables listed in tables 1 and 2 of [84], and provides mean values, standard deviations and the correlation matrix for the following parameters: the correction to the $W$ mass $(\delta m)$, anomalous $W$ and $Z$ couplings to leptons ( $\delta g_{L}^{W \ell_{i}}, \delta g_{L, R}^{Z \ell_{i}}$ ) and anomalous $Z$ couplings to quarks $\left(\delta g_{L, R}^{Z u_{i}}, \delta g_{L, R}^{Z d_{i}}\right)$. The results for these "pseudo-observables" can be found in eqs. (4.5-4.8) and appendix B of ref. [84]. The relevant expressions for these pseudo-observables within our model are given in appendix B.

We collect the list of flavour observables included in our analysis in table 2 and describe them in more detail in the following subsections.

### 4.1 Leptonic tau decays

Leptonic tau decays pose very stringent constraints on lepton flavour universality [85]. We consider the two decay rates $\Gamma(\tau \rightarrow\{e, \mu\} \nu \bar{\nu})$, normalized to the muon decay rate to cancel the dependence on $G_{F}$. We take the individual experimental branching ratios and lifetimes

[^1]| Leptonic $\tau$ decays |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Observable | Experiment | Correlation | SM | Theory |
| $\begin{aligned} & \Gamma_{\tau \rightarrow e \nu \bar{\nu}} / \Gamma_{\mu \rightarrow e \nu \bar{\nu}} \\ & \Gamma_{\tau \rightarrow \mu \nu \bar{\nu}} / \Gamma_{\mu \rightarrow e \nu \bar{\nu}} \end{aligned}$ | $1.350(4) \cdot 10^{6}$ | 0.45 | $1.3456(5) \cdot 10^{6}$ | Eq. (4.1) |
|  | $1.320(4) \cdot 10^{6}$ |  | $1.3087(5) \cdot 10^{6}$ | Eq. (4.2) |
| $d \rightarrow u$ transitions |  |  |  |  |
| Observable | Experiment | Correlation | SM | Theory |
| $\begin{aligned} & \Gamma_{\pi \rightarrow \mu \nu} / \Gamma_{\pi \rightarrow e \nu} \\ & \Gamma_{\tau \rightarrow \pi \nu} / \Gamma_{\pi \rightarrow e \nu} \end{aligned}$ | $8.13(3) \cdot 10^{3}$ | 0.49 | $8.096(1) \cdot 10^{3}$ | Eq. (4.4) |
|  | $7.90(5) \cdot 10^{7}$ |  | $7.91(1) \cdot 10^{7}$ | Eq. (4.5) |
| $s \rightarrow \boldsymbol{u}$ transitions |  |  |  |  |
| Observable | Experiment | Correlation | SM | Theory |
| $\begin{gathered} \Gamma_{K \rightarrow \mu \nu} / \Gamma_{K \rightarrow e \nu} \\ \Gamma_{\tau \rightarrow K \nu} / \Gamma_{K \rightarrow e \nu} \\ \Gamma_{K^{+} \rightarrow \pi \mu \nu} / \Gamma_{K^{+} \rightarrow \pi e \nu} \end{gathered}$ | 4.02(2) $\cdot 10^{4}$ | . | 4.037(2) $\cdot 10^{4}$ | Eq. (4.9) |
|  | $1.89(3) \cdot 10^{7}$ | 0.27 • . | $1.939(4) \cdot 10^{7}$ | Eq. (4.10) |
|  | 0.660(3) | [0.010.00 - | 0.663(2) | Eq. (4.11) |
| $c \rightarrow s$ transitions |  |  |  |  |
| Observable | Experiment |  | SM | Theory |
| $\begin{gathered} \Gamma_{D \rightarrow K \mu \nu} / \Gamma_{D \rightarrow K e \nu} \\ \Gamma_{D_{s} \rightarrow \tau \nu} / \Gamma_{D_{s} \rightarrow \mu \nu} \end{gathered}$ | 0.95(5) | ( $S=1.3$ ) | 0.921(1) | Eq. (4.13) |
|  | 10.0(6) |  | 9.6(1) | Eq. (4.14) |
| $b \rightarrow s$ transitions |  |  |  |  |
| Observable | Experiment |  | SM | Theory |
| $\Delta M_{s} / \Delta M_{d}$ | 35.13(15) |  | 31.2(1.8) | Eq. (4.15) |
| Coefficient Fit [16] | Correlation |  | SM | Theory |
| $\mathcal{C}_{9 \mu}^{\mathrm{NP}} \quad-1.1(0.2)$ |  |  | 0. | Eq. (4.19) |
| $\mathcal{C}_{10 \mu}^{\mathrm{NP}}+$ | 2) -0.08 | - | 0. | Eq. (4.19) |
| $\mathcal{C}_{\text {¢ }}{ }^{\text {NP }}$ | 7) $\quad 0.10$ | -0.10 . . | 0. | Eq. (4.19) |
| $\mathcal{C}_{10 e}^{\mathrm{NP}}++0$ | 6) $\quad 0.02$ | $0.02 \quad 0.87$ •] | 0. | Eq. (4.19) |
| $b \rightarrow \boldsymbol{c}$ transitions |  |  |  |  |
| Observable | Experiment | Correlation | SM | Theory |
| $\begin{gathered} \Gamma_{B \rightarrow D \mu \bar{\nu}} / \Gamma_{B \rightarrow D e \bar{\nu}} \\ \Gamma_{B \rightarrow D^{*} \mu \bar{\nu}} / \Gamma_{B \rightarrow D^{*} e \bar{\nu}} \end{gathered}$ | 0.95(09) | +0.51 | 0.995(1) | Eq. (4.20) |
|  | 0.97(08) |  | 0.996(1) | Eq. (4.20) |
| $R(D)$ $R\left(D^{*}\right)$ | 0.397(49) | -0.21 | 0.297(17) | Eq. (4.21) |
| $R\left(D^{*}\right)$ | 0.316(19) |  | 0.252(3) | Eq. (4.21) |
| $\Gamma_{B \rightarrow X_{c} \tau \nu} / \Gamma_{B \rightarrow X_{c} e \nu}$ | 0.222(22) |  | 0.223(5) | Eq. (4.22) |

Table 2. List of flavour observables used in the fit.
from the PDG [86]. For the branching ratios we take the result of the constrained fit, which gives a correlation of $14 \%$ between both measurements. Once normalized to the $\tau$ lifetime, the decay rates have a correlation of $45 \%$, while the normalization to the muon decay rate has a minor impact on the correlation of the ratios because its uncertainty is negligible. The experimental results are summarized in table 2.

In our model, we have:

$$
\begin{align*}
& \frac{\Gamma(\tau \rightarrow e \nu \bar{\nu})}{\Gamma(\mu \rightarrow e \nu \bar{\nu})}=\frac{\sum_{i, j}\left|\mathcal{C}_{i j}^{e \tau}\right|^{2}}{\sum_{i, j}\left|\mathcal{C}_{i j}^{e \mu}\right|^{2}} \times \frac{m_{\tau}^{5} f\left(x_{e \tau}\right)}{m_{\mu}^{5} f\left(x_{e \mu}\right)},  \tag{4.1}\\
& \frac{\Gamma(\tau \rightarrow \mu \nu \bar{\nu})}{\Gamma(\mu \rightarrow e \nu \bar{\nu})}=\frac{\sum_{i, j}\left|\mathcal{C}_{i j}^{\mu \tau}\right|^{2}}{\sum_{i, j}\left|\mathcal{C}_{i j}^{e \mu}\right|^{2}} \times \frac{m_{\tau}^{5} f\left(x_{\mu \tau}\right)}{m_{\mu}^{5} f\left(x_{e \mu}\right)}, \tag{4.2}
\end{align*}
$$

where $x_{\ell \ell^{\prime}}=m_{\ell}^{2} / m_{\ell^{\prime}}^{2}$ and $f(x)=1-8 x+8 x^{3}-x^{4}-12 x^{2} \ln x$. The Wilson coefficients $\mathcal{C}_{i j}^{\ell_{a} \ell_{b}}$ are given by

$$
\begin{equation*}
\mathcal{C}_{i j}^{\ell \ell_{a} \ell_{b}}=\frac{4 G_{F}}{\sqrt{2}} \delta_{a j} \delta_{i b}+\frac{\hat{g}^{2}}{4 M_{W^{\prime}}^{2}}\left[2 \Delta_{a j}^{\ell} \Delta_{i b}^{\ell}-\Delta_{a b}^{\ell} \Delta_{i j}^{\ell}+\zeta\left(\Delta_{a b}^{\ell} \delta_{i j}+2 \Delta_{a j}^{\ell} \delta_{i b}+2 \delta_{a j} \Delta_{i b}^{\ell}\right)\right] \tag{4.3}
\end{equation*}
$$

The resulting predictions in the SM can be found in table 2. Leading radiative corrections and $W$-boson propagator effects are included in the SM predictions [87-90].

## $4.2 \quad d \rightarrow u$ transitions

We consider the decay rates $\Gamma(\pi \rightarrow \mu \nu)$ and $\Gamma(\tau \rightarrow \pi \nu)$, normalized to $\Gamma(\pi \rightarrow e \nu)$ in order to cancel the dependence on the combination $G_{F}\left|V_{u d}\right| f_{\pi}$. These ratios constitute important constraints on flavour non-universality in $d \rightarrow u \ell \nu$ transitions.

We calculate the experimental values for these ratios taking the averages for branching fractions and lifetimes from the PDG [86], and imposing the constraint $\mathcal{B}(\pi \rightarrow e \nu)+\mathcal{B}(\pi \rightarrow$ $\mu \nu)=1$. We find a correlation of $49 \%$ between both ratios. The corresponding results are summarized in table 2.

The model predictions for these ratios are:

$$
\begin{align*}
& \frac{\Gamma(\pi \rightarrow \mu \bar{\nu})}{\Gamma(\pi \rightarrow e \bar{\nu})}=\frac{\sum_{j}\left|\mathcal{C}_{2 j}^{u d}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{1 j}^{u d}\right|^{2}} \times\left[\frac{\Gamma(\pi \rightarrow \mu \bar{\nu})}{\Gamma(\pi \rightarrow e \bar{\nu})}\right]_{\mathrm{SM}}  \tag{4.4}\\
& \frac{\Gamma(\tau \rightarrow \pi \nu)}{\Gamma(\pi \rightarrow e \bar{\nu})}=\frac{\sum_{j}\left|\mathcal{C}_{3 j}^{u d}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{1 j}^{u d}\right|^{2}} \times\left[\frac{\Gamma(\tau \rightarrow \pi \nu)}{\Gamma(\pi \rightarrow e \bar{\nu})}\right]_{\mathrm{SM}} \tag{4.5}
\end{align*}
$$

where the Wilson coefficients $\mathcal{C}_{a b}^{u_{i} d_{j}}$ are given by

$$
\begin{equation*}
\mathcal{C}_{a b}^{u_{i} d_{j}}=\frac{4 G_{F}}{\sqrt{2}} V_{i j} \delta_{a b}+\frac{\hat{g}^{2}}{2 M_{W^{\prime}}^{2}}\left[\left(V \Delta^{q}\right)_{i j} \Delta_{a b}^{\ell}-\zeta\left(V_{i j} \Delta_{a b}^{\ell}+\left(V \Delta^{q}\right)_{i j} \delta_{a b}\right)\right] \tag{4.6}
\end{equation*}
$$

For the SM contributions we follow ref. [85]. We have:

$$
\begin{align*}
& {\left[\frac{\Gamma(\pi \rightarrow e \bar{\nu})}{\Gamma(\pi \rightarrow \mu \bar{\nu})}\right]_{\mathrm{SM}}=\frac{m_{e}^{2}}{m_{\mu}^{2}}\left[\frac{1-m_{e}^{2} / m_{\pi}^{2}}{1-m_{\mu}^{2} / m_{\pi}^{2}}\right]^{2}\left(1+\delta R_{\pi \rightarrow e / \mu}\right)}  \tag{4.7}\\
& {\left[\frac{\Gamma(\tau \rightarrow \pi \nu)}{\Gamma(\pi \rightarrow \mu \nu)}\right]_{\mathrm{SM}}=\frac{m_{\tau}^{3}}{2 m_{\pi} m_{\mu}^{2}}\left[\frac{1-m_{\pi}^{2} / m_{\tau}^{2}}{1-m_{\mu}^{2} / m_{P}^{2}}\right]^{2}\left(1+\delta R_{\tau / \pi}\right) .} \tag{4.8}
\end{align*}
$$

The calculation of $\delta R_{\pi \rightarrow e / \mu}$ relies on Chiral Perturbation Theory to order $\mathcal{O}\left(e^{2} p^{2}\right)$ [91]. The radiative correction factor $\delta R_{\tau / \pi}$ can be found in ref. [92]. The SM predictions for both ratios are collected in table 2 .

## $4.3 s \rightarrow \boldsymbol{u}$ transitions

We consider the decay rates $\Gamma(K \rightarrow \mu \nu)$ and $\Gamma(\tau \rightarrow K \nu)$, normalized to $\Gamma(K \rightarrow e \nu)$ in order to cancel the dependence on the combination $G_{F}\left|V_{u s}\right| f_{K}$, as well as the semileptonic ( $K_{\ell 3}$ ) ratio $\Gamma\left(K^{+} \rightarrow \pi^{0} \mu^{+} \nu\right) / \Gamma\left(K^{+} \rightarrow \pi^{0} e^{+} \nu\right)$. These ratios pose also important constraints on flavour non-universality.

We take the experimental values for the decay rates $\Gamma\left(K^{+} \rightarrow \mu^{+} \nu\right), \Gamma\left(K^{+} \rightarrow \pi^{0} e^{+} \nu\right)$ and $\Gamma\left(K^{+} \rightarrow \pi^{0} \mu^{+} \nu\right)$ from the constrained fit to $K^{+}$decay data done by the PDG [86], including the correlation matrix. The correlation between $\Gamma\left(K^{+} \rightarrow \mu^{+} \nu\right)$ and $\Gamma\left(K^{+} \rightarrow\right.$ $e^{+} \nu$ ) is calculated comparing the averages for the individual rates with the ratio given by the PDG, resulting in a correlation of $60 \%$. Assuming no correlation between $\Gamma\left(K^{+} \rightarrow e^{+} \nu\right)$ and the semileptonic modes, and assuming that the $\tau$ mode is uncorrelated to the $K$ modes, we construct a $5 \times 5$ correlation matrix and calculate the three ratios of interest, including their $3 \times 3$ correlation matrix. These results are collected in table 2 .

The model predictions for these ratios are:

$$
\begin{align*}
\frac{\Gamma(K \rightarrow \mu \bar{\nu})}{\Gamma(K \rightarrow e \bar{\nu})} & =\frac{\sum_{j}\left|\mathcal{C}_{2 j}^{u s}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{1 j}^{\text {us }}\right|^{2}} \times\left[\frac{\Gamma(K \rightarrow \mu \bar{\nu})}{\Gamma(K \rightarrow e \bar{\nu})}\right]_{\mathrm{SM}}  \tag{4.9}\\
\frac{\Gamma(\tau \rightarrow K \nu)}{\Gamma(K \rightarrow e \bar{\nu})} & =\frac{\sum_{j}\left|\mathcal{C}_{3 j}^{u s}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{1 j}^{u s}\right|^{2}} \times\left[\frac{\Gamma(\tau \rightarrow K \nu)}{\Gamma(K \rightarrow e \bar{\nu})}\right]_{\mathrm{SM}},  \tag{4.10}\\
\frac{\Gamma\left(K^{+} \rightarrow \pi \mu \bar{\nu}\right)}{\Gamma\left(K^{+} \rightarrow \pi e \bar{\nu}\right)} & =\frac{\sum_{j}\left|\mathcal{C}_{2 j}^{u s}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{1 j}^{u s}\right|^{2}} \times\left[\frac{\Gamma\left(K^{+} \rightarrow \pi \mu \bar{\nu}\right)}{\Gamma\left(K^{+} \rightarrow \pi e \bar{\nu}\right)}\right]_{\mathrm{SM}} \tag{4.11}
\end{align*}
$$

with the Wilson coefficients $\mathcal{C}_{i j}^{u s}$ given in eq. (4.6). The SM contributions for the first two ratios are given by the analogous expressions to eqs. (4.7), (4.8) [91, 92]. The SM contributions to $K_{\ell 3}$ are given by [93, 94]

$$
\begin{equation*}
\frac{\Gamma\left(K^{+} \rightarrow \pi \mu \bar{\nu}\right)}{\Gamma\left(K^{+} \rightarrow \pi e \bar{\nu}\right)}=\frac{I_{K \mu}^{(0)}\left(\lambda_{i}\right)\left(1+\delta_{\mathrm{EM}}^{K \mu}+\delta_{\mathrm{SU}(2)}^{K \pi}\right)}{I_{K e}^{(0)}\left(\lambda_{i}\right)\left(1+\delta_{\mathrm{EM}}^{K e}+\delta_{\mathrm{SU}(2)}^{K \pi}\right)}, \tag{4.12}
\end{equation*}
$$

where quantities $I_{K \ell}^{(0)}\left(\lambda_{i}\right), \delta_{\mathrm{EM}}^{K \ell}, \delta_{\mathrm{SU}(2)}^{K \pi}$ encoding phase-space factors, electromagnetic and isospin corrections can be found in refs. [93-95]. The numerical results for the SM contributions are collected in table 2 .

## $4.4 \quad c \rightarrow s$ transitions

We consider the ratios $\Gamma(D \rightarrow K \mu \nu) / \Gamma(D \rightarrow K e \nu)$ and $\Gamma\left(D_{s} \rightarrow \tau \nu\right) / \Gamma\left(D_{s} \rightarrow \mu \nu\right)$, constraining respectively $\mu-e$ and $\tau-\mu$ non-universality.

For $D \rightarrow K \ell \nu$, we consider charged and neutral modes separately. For $D^{+} \rightarrow \bar{K}^{0} \ell^{+} \nu$ we take the separate branching ratios from the PDG assuming no correlation. For $D^{0} \rightarrow$
$K^{-} \ell^{+} \nu$ we take the results from the PDG constrained fit, including the $5 \%$ correlation. We construct the $D^{+}$and $D^{0}$ ratios separately, obtaining $\Gamma\left(D^{+} \rightarrow \bar{K}^{0} \mu^{+} \nu\right) / \Gamma\left(D^{+} \rightarrow\right.$ $\left.\bar{K}^{0} e^{+} \nu\right)=1.05(9)$ and $\Gamma\left(D^{0} \rightarrow K^{-} \mu^{+} \nu\right) / \Gamma\left(D^{0} \rightarrow K^{-} e^{+} \nu\right)=0.93(4)$. These two ratios, corresponding to the same theoretical quantity (isospin-breaking effects are neglected here), are combined according to the PDG averaging prescription. Since there is a $\sim 1 \sigma$ tension between both results, we rescale the error by the factor $S=1.3$.

For $D_{s} \rightarrow \ell \nu$ we take the individual branching fractions from the PDG, assuming no correlation. The resulting experimental numbers for both ratios are collected in table 2 .

The model predictions for these ratios are:

$$
\begin{align*}
\frac{\Gamma(D \rightarrow K \mu \bar{\nu})}{\Gamma(D \rightarrow K e \bar{\nu})} & =\frac{\sum_{j}\left|\mathcal{C}_{2 j}^{c s}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{1 j}^{c s}\right|^{2}} \times\left[\frac{\Gamma(D \rightarrow K \mu \bar{\nu})}{\Gamma(D \rightarrow K e \bar{\nu})}\right]_{\mathrm{SM}}  \tag{4.13}\\
\frac{\Gamma\left(D_{s} \rightarrow \tau \bar{\nu}\right)}{\Gamma\left(D_{s} \rightarrow \mu \bar{\nu}\right)} & =\frac{\sum_{j}\left|\mathcal{C}_{3 j}^{c s}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{2 j}^{c s}\right|^{2}} \times\left[\frac{\Gamma\left(D_{s} \rightarrow \tau \bar{\nu}\right)}{\Gamma\left(D_{s} \rightarrow \mu \bar{\nu}\right)}\right]_{\mathrm{SM}} \tag{4.14}
\end{align*}
$$

with the Wilson coefficients $\mathcal{C}_{i j}^{c s}$ given in eq. (4.6).
Our SM prediction for the leptonic decay modes includes electromagnetic corrections following [96]. For the SM prediction of the semileptonic modes we use the BESIII determination of the form factor parameters in the simple pole scheme as quoted in HFAG [5]. The resulting SM predictions are given in table 2 .

## $4.5 \quad b \rightarrow s$ transitions

We consider here $b \rightarrow s$ transitions that are loop-mediated in the SM but receive NP contributions at tree-level in our model (via $Z^{\prime}$ and $Z$ with anomalous couplings). To the level of precision we are working, the normalization factors in the SM amplitude ( $G_{F}$ and CKM elements) can be taken from tree-level determinations within the SM, and it is not necessary in this case to consider only ratios where these cancel out.

Mass difference in the $\boldsymbol{B}_{s}$ system. The observable $\Delta M_{s}$ constitutes a strong constraint on the $Z^{\prime} s b$ coupling, independent of the coupling to leptons. In order to minimize the uncertainty from hadronic matrix elements, we consider the ratio $\Delta M_{s} / \Delta M_{d}$. We note that within our model set-up, $\Delta M_{d}$ does not receive NP contributions at tree-level.

The experimental value for the ratio is obtained from the individual measurements for $\Delta M_{d, s}$, which are known to subpercent precision [5]. The result is given in table 2.

The theory prediction is given by:

$$
\begin{equation*}
\frac{\Delta M_{s}}{\Delta M_{d}}=\frac{M_{B_{s}}}{M_{B_{d}}} \xi^{2}\left|\frac{\mathcal{C}_{s b}}{\mathcal{C}_{d b}}\right|=\frac{M_{B_{s}}}{M_{B_{d}}} \xi^{2}\left|\frac{V_{t s}^{2}}{V_{t d}^{2}}+\frac{\mathcal{C}_{s b}^{\mathrm{NP}}}{\mathcal{C}_{d b}^{\mathrm{SM}}}\right|, \tag{4.15}
\end{equation*}
$$

where the Wilson coefficients $\mathcal{C}_{d_{i} b}=\mathcal{C}_{d_{i} b}^{\mathrm{SM}}+\mathcal{C}_{d_{i} b}^{\mathrm{NP}}$ are given by

$$
\begin{equation*}
\mathcal{C}_{d_{i} b}^{\mathrm{SM}}=\frac{G_{F}^{2} M_{W}^{2}}{4 \pi^{2}}\left(V_{t i} V_{t b}^{\star}\right)^{2} S_{0}\left(x_{t}\right), \quad \mathcal{C}_{d_{i} b}^{\mathrm{NP}}=\frac{\hat{g}^{2}}{8 M_{W^{\prime}}^{2}}\left(\Delta_{i 3}^{q}\right)^{2} . \tag{4.16}
\end{equation*}
$$

Here $S_{0}\left(x_{t}\right)=2.322 \pm 0.018$ is the loop function in the SM [97]. The parameter $\xi^{2}=$ $f_{B_{s}}^{2} B_{B_{s}}^{(1)} / f_{B_{d}}^{2} B_{B_{d}}^{(1)}$ is a ratio of decay constants and matrix elements determined from lattice

QCD. We consider the latest determination of the parameter $\xi$ from the FNAL/MILC collaborations [98]: $\xi=1.206(18)(6)$. The SM prediction is given by the first term in eq. (4.15) and results in $\left(\Delta M_{s} / \Delta M_{d}\right)_{\mathrm{SM}}=31.2(1.8)$.
$\boldsymbol{b} \rightarrow$ sौौ observables. We consider all $b \rightarrow$ s $\ell \ell$ observables used in the fit of ref. [16]:

- Branching ratios for $B \rightarrow X_{s} \mu^{+} \mu^{-}$and $B_{s} \rightarrow \mu^{+} \mu^{-}$[99-103].
- Branching ratios for $B \rightarrow K e^{+} e^{-}$(in the bin $[1,6] \mathrm{GeV}^{2}$ ) and $B \rightarrow K \mu^{+} \mu^{-}$(both at low and high $q^{2}$ ) $[6,104]$.
- Branching ratios, longitudinal polarization fractions and optimized angular observables [105-107] for $B \rightarrow K^{*} e^{+} e^{-}$(at very low $q^{2}$ ) and $B \rightarrow K^{*} \mu^{+} \mu^{-}, B_{s} \rightarrow \phi \mu^{+} \mu^{-}$ (both at low and high $q^{2}$ ) [108-114].
Definitions, theoretical expressions and discussions on theoretical uncertainties can be found in refs. [16, 107]. We follow the approach of ref. [19] for $B \rightarrow V$ form factors, and take into account the lifetime effect for $B_{s}$ measurements at hadronic machines [115] for $B_{s} \rightarrow \mu \mu[116]$ and $B_{s} \rightarrow \phi \mu \mu[117]$ decays.

We implement the fit in two different ways. First, we construct the full $\chi^{2}$ as a function of the model parameters, including all theoretical and experimental correlations, exactly as in ref. $[16] .^{3}$ Second, in order to provide simplified expressions to allow the reader to repeat the fit without too much work, we perform a global fit to the relevant coefficients of the effective weak Hamiltonian

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}} \supset-\frac{4 G_{F}}{\sqrt{2}} \frac{\alpha}{4 \pi} V_{t s}^{*} V_{t b} \sum_{i=9,10}\left[\mathcal{C}_{i \ell} Q_{i}^{\ell}+\mathcal{C}_{i \ell}^{\prime} Q_{i}^{\ell}\right], \tag{4.17}
\end{equation*}
$$

with

$$
\begin{align*}
Q_{9}^{\ell} & =\left(\bar{s} \gamma_{\alpha} P_{L} b\right)\left(\bar{\ell} \gamma^{\alpha} \ell\right), & Q_{9}^{\prime \ell} & =\left(\bar{s} \gamma_{\alpha} P_{R} b\right)\left(\bar{\ell} \gamma^{\alpha} \ell\right), \\
Q_{10}^{\ell} & =\left(\bar{s} \gamma_{\alpha} P_{L} b\right)\left(\bar{\ell} \gamma^{\alpha} \gamma_{5} \ell\right), & Q_{10}^{\prime \ell} & =\left(\bar{s} \gamma_{\alpha} P_{R} b\right)\left(\bar{\ell} \gamma^{\alpha} \gamma_{5} \ell\right) . \tag{4.18}
\end{align*}
$$

We consider those coefficients receiving non-negligible NP contributions within our model, i.e. $\left(\mathcal{C}_{9 \mu}, \mathcal{C}_{10 \mu}, \mathcal{C}_{9 e}, \mathcal{C}_{10 e}\right)$, and provide the best fit points, standard deviations and correlation matrix. ${ }^{4}$ These are collected in table 2. The NP contributions to the Wilson coefficients $\left(\mathcal{C}_{i \ell}=\mathcal{C}_{i \ell}^{\mathrm{SM}}+\mathcal{C}_{i \ell}^{\mathrm{NP}}\right)$ are

$$
\begin{align*}
& \mathcal{C}_{9 a}^{\mathrm{NP}}=-\frac{\sqrt{2}}{G_{F}} \frac{\pi}{\alpha} \frac{1}{V_{t b} V_{t s}^{*}} \frac{\hat{g}^{2}}{8 M_{W^{\prime}}^{2}}\left(\Delta^{q}\right)_{b s}\left[\left(\Delta^{\ell}\right)_{a a}+\zeta\left(4 s_{W}^{2}-1\right)\right],  \tag{4.19}\\
& \mathcal{C}_{10 a}^{\mathrm{NP}}=\frac{\sqrt{2}}{G_{F}} \frac{\pi}{\alpha} \frac{1}{V_{t b} V_{t s}^{*}} \frac{\hat{g}^{2}}{8 M_{W^{\prime}}^{2}}\left(\Delta^{q}\right)_{b s}\left[\left(\Delta^{\ell}\right)_{a a}-\zeta\right] .
\end{align*}
$$

Using these four coefficients as "pseudo observables" and constructing the $\chi^{2}$ function leads to a linearised approximation to the fit. We have checked that the result of such a fit is in reasonable agreement with the full fit.

[^2]
## $4.6 \quad b \rightarrow c$ transitions

We consider the exclusive ratios $R\left(D^{(*)}\right) \equiv \Gamma\left(B \rightarrow D^{(*)} \tau \bar{\nu}\right) / \Gamma\left(B \rightarrow D^{(*)} \ell \bar{\nu}\right)$, and the inclusive ratio $R\left(X_{c}\right) \equiv \Gamma\left(B \rightarrow X_{c} \tau \bar{\nu}\right) / \Gamma\left(B \rightarrow X_{c} \ell \bar{\nu}\right)$ as measures of flavour non-universality between the $\tau$ and the light leptons, as well as the ratios $\Gamma\left(B \rightarrow D^{(*)} \mu \bar{\nu}\right) / \Gamma\left(B \rightarrow D^{(*)} e \bar{\nu}\right)$ constraining $e-\mu$ non-universality.

The experimental value for the inclusive ratio $R\left(X_{c}\right)$ is obtained from the PDG averages for $\operatorname{Br}\left(\bar{b} \rightarrow X \tau^{+} \nu\right)$ and $\operatorname{Br}\left(\bar{b} \rightarrow X e^{+} \nu\right)$. The allowed size of lepton flavour universality violating effects in $b \rightarrow c \ell \nu(\ell=e, \mu)$ transitions is not trivial to account for given that experimental analyses tend to present combined results for the electron and muon data samples. This aspect was also stressed in ref. [81]. Experimental results are however reported separately for the $e$ and $\mu$ samples in an analysis performed by the BaBar collaboration [118]. We use the values of $\operatorname{Br}\left(B \rightarrow D^{(*)} \ell \bar{\nu}\right)$ reported in table IV of ref. [118] to extract the ratios $\Gamma\left(B \rightarrow D^{(*)} \mu \bar{\nu}\right) / \Gamma\left(B \rightarrow D^{(*)} e \bar{\nu}\right)$. The correlation between the two ratios is estimated from the information provided in [118], adding the covariance for the systematic and statistical errors. For the experimental values of $R(D)$ and $R\left(D^{*}\right)$ we consider the latest HFAG average [5] . The latter includes $R(D)$ and $R\left(D^{*}\right)$ measurements performed by BaBar and Belle [119, 120], the LHCb measurement of $R\left(D^{*}\right)$ [121], and the independent Belle measurement of $R\left(D^{*}\right)$ using a semileptonic tagging method [122]. ${ }^{5}$ The results are summarized in table 2 .

The model expressions for these ratios are:

$$
\begin{align*}
\frac{\Gamma\left(B^{-} \rightarrow D^{(*)} \mu \bar{\nu}\right)}{\Gamma\left(B^{-} \rightarrow D^{(*)} e \bar{\nu}\right)} & =\frac{\sum_{j}\left|\mathcal{C}_{2 j}^{c b}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{1 j}^{c b}\right|^{2}} \times\left[\frac{\Gamma\left(B^{-} \rightarrow D^{(*)} \mu \bar{\nu}\right)}{\Gamma\left(B^{-} \rightarrow D^{(*)} e \bar{\nu}\right)}\right]_{\mathrm{SM}}  \tag{4.20}\\
R\left(D^{(*)}\right) & =\frac{2\left(\sum_{j}\left|\mathcal{C}_{3 j}^{c b}\right|^{2}\right)}{\sum_{j}\left(\left|\mathcal{C}_{1 j}^{c b}\right|^{2}+\left|\mathcal{C}_{2 j}^{c b}\right|^{2}\right)} \times R\left(D^{(*)}\right)^{\mathrm{SM}}  \tag{4.21}\\
R\left(X_{c}\right) & =\frac{\sum_{j}\left|\mathcal{C}_{3 j}^{c b}\right|^{2}}{\sum_{j}\left|\mathcal{C}_{1 j}^{c b}\right|^{2}} \times R\left(X_{c}\right)^{\mathrm{SM}} \tag{4.22}
\end{align*}
$$

where the Wilson coefficients $\mathcal{C}_{i j}^{c b}$ are given in eq. (4.6). We use the SM predictions of $R(D)$ and $R\left(D^{*}\right)$ obtained in refs. [124, 125]. Note that recent determinations of $R(D)$ in Lattice QCD are compatible with the one used here [126, 127]. For $R\left(X_{c}\right)$ we use the SM prediction reported in ref. [128]. For the ratios $\Gamma\left(B^{-} \rightarrow D^{(*)} \mu \bar{\nu}\right) / \Gamma\left(B^{-} \rightarrow D^{(*)} e \bar{\nu}\right)$ we derive the SM predictions using the Caprini-Lellouch-Neubert parametrization of the form factors [129], with the relevant parameters taken from HFAG [5]. The resulting SM predictions are given in table 2.

[^3]
### 4.7 Lepton flavour violation

We consider current limits on the lepton flavour violating decays $\tau \rightarrow 3 \mu$ and $Z \rightarrow \tau \mu$. The decay $Z \rightarrow \tau \mu$ occurs due to gauge mixing effects. The decay rate for $Z \rightarrow \tau \mu \equiv$ $\left(\tau^{+} \mu^{-}+\tau^{-} \mu^{+}\right)$is

$$
\begin{equation*}
\Gamma(Z \rightarrow \tau \mu)=\frac{M_{Z}}{48 \pi}\left(\zeta n_{2} \frac{g_{2}^{4}}{n_{1}^{4}} \Delta_{\mu} \Delta_{\tau} \epsilon^{2}\right)^{2} \tag{4.23}
\end{equation*}
$$

We use the limit $\operatorname{Br}(Z \rightarrow \tau \mu)<1.2 \times 10^{-5}$ [86].
The decay $\tau \rightarrow 3 \mu$ receives tree-level contributions from $Z^{(\prime)}$ exchange, the decay rate is given by

$$
\begin{equation*}
\Gamma(\tau \rightarrow 3 \mu)=\frac{\left[2\left(\mathcal{C}_{L L}^{\tau \mu}\right)^{2}+\left(\mathcal{C}_{L R}^{\tau \mu}\right)^{2}\right] m_{\tau}^{5}}{1536 \pi^{3}} \tag{4.24}
\end{equation*}
$$

where the Wilson coefficients $\mathcal{C}_{L L}^{\tau \mu}$ and $\mathcal{C}_{L R}^{\tau \mu}$ are given by

$$
\begin{align*}
\mathcal{C}_{L L}^{\ell_{a} \ell_{b}} & =\frac{\hat{g}^{2}}{4 M_{W^{\prime}}^{2}} \Delta_{a b}^{\ell}\left[\Delta_{b b}^{\ell}+\zeta\left(2 s_{W}^{2}-1\right)\right] \\
\mathcal{C}_{L R}^{\ell} \ell_{a} \ell_{b} & =\frac{\hat{g}^{2}}{2 M_{W^{\prime}}^{2}} \zeta \Delta_{a b}^{\ell} s_{W}^{2} \tag{4.25}
\end{align*}
$$

We use the HFAG limit $\operatorname{Br}(\tau \rightarrow 3 \mu)<1.2 \times 10^{-8}[5]$.

## 5 Global fit

### 5.1 Fitting procedure

We first fix the values of $g, g^{\prime}$ and the electroweak vev $v$ with the values of $\left\{G_{F}, \alpha, M_{Z}\right\}$ reported in table 3. The $\mathrm{SU}(2)_{1}$ gauge coupling $g_{1}$ is then determined as a function of $g_{2}$. The observables considered will depend on seven model parameters:
$M_{Z^{\prime}}$ : The $Z^{\prime}$ mass, note that $M_{W^{\prime}} \simeq M_{Z^{\prime}}$,
$g_{2}$ : The $\mathrm{SU}(2)_{2}$ gauge coupling,
$\zeta:$ Controls the size of gauge mixing effects, see eq. (3.17),
$\Delta_{s}, \Delta_{b}, \Delta_{\mu}, \Delta_{\tau}$ : Determine the gauge couplings to fermions, see eq. (3.36).
The observables will also depend on the CKM inputs $\{\lambda, A, \bar{\rho}, \bar{\eta}\}$. We construct a global $\chi^{2}$ function that includes information from electroweak precision data at the $Z$ and $W$ poles together with flavour data. It reads

$$
\begin{equation*}
\chi^{2} \equiv\left(O-O_{\exp }\right)^{T} \bar{\Sigma}^{-1}\left(O-O_{\exp }\right)+\sum_{x=\lambda, A, \bar{\rho}, \bar{\eta}} \frac{(x-\hat{x})^{2}}{\sigma_{ \pm}^{2}} \tag{5.1}
\end{equation*}
$$

with $\bar{\Sigma}$ being the covariance matrix, $O$ denoting the observables included in the analysis and $O_{\exp }$ the corresponding experimental mean values. These are described in section 4. The CKM inputs $\{\lambda, A, \bar{\rho}, \bar{\eta}\}$ are included as pseudo-observables in the fit taking into account

| $\lambda=0.22541\left({ }_{-21}^{+30}\right)$ | $[130]$ | $A=0.8212\left({ }_{-338}^{+66}\right)$ | $[130]$ |
| :---: | :---: | :---: | :---: |
| $\bar{\rho}=0.132\left({ }_{-21}^{+21}\right)$ | $[130]$ | $\bar{\eta}=0.383\left({ }_{-22}^{+22}\right)$ | $[130]$ |
| $G_{F}=1.16638(1) \times 10^{-5}$ | $\mathrm{GeV}^{-2}$ | $[86]$ | $M_{Z}=91.1876(21) \mathrm{GeV}$ |
| $\alpha=1 / 137.036$ | $[86]$ |  |  |

Table 3. Electroweak and CKM inputs.
the values in table $3 .{ }^{6}$ The latter are reported in the form $\hat{x}_{-\sigma_{-}}^{+\sigma_{+}}$. In the $\chi^{2}$ we introduce the asymmetric error: $\sigma_{ \pm}=\sigma_{+}($for $x>\hat{x})$ and $\sigma_{ \pm}=\sigma_{-}($for $x<\hat{x})$.

The global fit takes into account then seven model parameters $\left\{M_{Z^{\prime}}, g_{2}, \Delta_{s}, \Delta_{b}, \Delta_{\mu}, \Delta_{\tau}, \zeta\right\}$ and four CKM quantities $\{\lambda, A, \bar{\rho}, \bar{\eta}\}$. To sample the 11-dimensional parameter space we use the affine invariant Markov chain Monte Carlo ensemble sampler emcee [131].

### 5.2 Results of the fit

We restrict the parameter space to $500 \mathrm{GeV} \leq M_{Z^{\prime}} \leq 3000 \mathrm{GeV}, g<g_{2}<\sqrt{4 \pi},\left|\Delta_{a}\right| \leq 3$ and $0 \leq \zeta \leq 1$. The minimum of the $\chi^{2}$ is found to be at

$$
\begin{equation*}
\left\{M_{Z^{\prime}}[\mathrm{GeV}], g_{2}, \Delta_{s}, \Delta_{b},\left|\Delta_{\mu}\right|,\left|\Delta_{\tau}\right|, \zeta\right\}=\{1436,1.04,-1.14,0.016,0.39,0.075,0.14\} \tag{5.2}
\end{equation*}
$$

with the CKM values $\{\lambda, A, \bar{\rho}, \bar{\eta}\}$ within the $1 \sigma$ range in table 3 . It is enlightening to characterise the best-fit point in terms of the couplings appearing in the Lagrangian. We find that the corresponding Yukawas are, up to a global sign,

$$
\lambda_{\ell} \simeq\left(\begin{array}{cc}
-1.2 & 0  \tag{5.3}\\
0 & -0.3 \\
0 & -0.06
\end{array}\right) \times \frac{M_{L}}{\mathrm{TeV}}, \quad \lambda_{q} \simeq\left(\begin{array}{cc}
-1.2 & 0 \\
0 & 1.8 \\
0 & -0.03
\end{array}\right) \times \frac{M_{Q}}{\mathrm{TeV}}
$$

At the best-fit point we obtain $\chi_{\text {min }}^{2}=54.8$, to be compared with the corresponding value in the SM-limit $\chi_{\mathrm{SM}}^{2}=93.7$. We derive contours of $\Delta \chi^{2} \equiv \chi^{2}-\chi_{\min }^{2}$ in twodimensional planes after profiling over all the other parameters, taking $\Delta \chi^{2}=2.3$ for $68 \%$ confidence level (CL) and $\Delta \chi^{2}=6.18$ for $95 \%$ CL. Allowed regions for the model parameters obtained in this way are shown in figure 2.

There is a four-fold degeneracy of the $\chi^{2}$ minimum with the sign of $\Delta_{\mu, \tau}$ as no observable in the fit is sensitive to the relative sign between $\Delta_{\mu}$ and $\Delta_{\tau}$. The allowed values of $\Delta_{\mu, \tau}$ lie in the region $\left|\Delta_{\mu, \tau}\right| \lesssim 1$. While $\Delta_{b}$ is bounded to be very small $\sim 10^{-2}$, the allowed values for $\Delta_{s}$ are around -1 . The negative sign obtained for the combination $\Delta_{s} \Delta_{b}$ is related to the preference for negative values of $\mathcal{C}_{9 \mu}^{\mathrm{NP}}$ by $b \rightarrow s \ell^{+} \ell^{-}$data. The allowed regions for the Wilson coefficients of $b \rightarrow s \ell^{+} \ell^{-}$transitions from the global fit are shown in figure 3. Note that with the assumed flavour structure we have the correlation $\mathcal{C}_{10 e}^{\mathrm{NP}}=\left(4 s_{W}^{2}-1\right) \mathcal{C}_{9 e}^{\mathrm{NP}}$. The relation $\mathcal{C}_{9 \mu}^{\mathrm{NP}}=-\mathcal{C}_{10 \mu}^{\mathrm{NP}}$ on the other hand holds in our model

[^4]

Figure 2. Allowed regions for the model parameters at $68 \%$ and $95 \%$ CL from the global fit. The best fit point is illustrated with a star.


Figure 3. Allowed regions at $68 \%$ and $95 \%$ CL from the global fit for the Wilson coefficients of $b \rightarrow s \ell^{+} \ell^{-}$transitions. The best fit point is illustrated with a star. The red line on the left plot illustrates the correlation $\mathcal{C}_{9 \mu}^{\mathrm{NP}}=-\mathcal{C}_{10 \mu}^{\mathrm{NP}}$.
only in the absence of gauge mixing effects. Departures from this correlation are possible as gauge mixing effects can be sizeable, see figure 3 (left).


Figure 4. Allowed regions at $68 \%$ and $95 \%$ CL from the global fit. Experimental values for these observables are also shown at $1 \sigma$ (dark-band) and $2 \sigma$ (light-band). The best fit point is illustrated with a star.

Allowed values at $68 \%$ and $95 \%$ CL for $R_{K}$ and $R\left(D^{*}\right)$ are shown in figure 4. The best fit point presents a sizeable deviation from the SM in $R_{K}$ in the direction of the LHCb measurement while the ratios $R\left(D^{*}\right)$ are SM-like. Note that the NP scaling of $R(D)$ is the same as for $R\left(D^{*}\right)$ because the $W^{\prime}$ couplings are mostly left-handed, with the right-handed couplings suppressed by $m_{f}^{2} / u^{2}$. A significant enhancement of $R\left(D^{(*)}\right)$ is possible within the allowed parameter region. The model presents a positive correlation between $R_{K}$ and $R\left(D^{(*)}\right)$ so that $R_{K}$ is above its best-fit value whenever $R\left(D^{(*)}\right)$ gets enhanced. The ratios $\Gamma\left(B \rightarrow D^{(*)} \mu \nu\right) / \Gamma\left(B \rightarrow D^{(*)} e \nu\right)$ are found to be SM-like with possible deviations only at the $\sim 1 \%$ level. As expected, $R\left(X_{c}\right)$ and $R\left(D^{(*)}\right)$ show a strong correlation, in the region of the parameter space where $R\left(D^{(*)}\right)$ accommodates the current experimental values one obtains a slight tension in $R\left(X_{c}\right)$ with experiment. The flavour observables with light-mesons and leptonic $\tau$-decays are found to be in good agreement with the SM and experiment, we show the resulting allowed values for $K \rightarrow \mu \nu / K \rightarrow e \nu$ and $\tau \rightarrow \mu \nu \bar{\nu} / \mu \rightarrow e \nu \bar{\nu}$ as an example in figure 4.

As noted in ref. [4], gauge mixing effects play a crucial role in the possible enhancement of $R\left(D^{(*)}\right)$ in this model. In figure 4 we also show the results of the global fit for $R\left(D^{*}\right)$ as a function of the parameter controlling the size of gauge mixing effects $\zeta$. Having an
enhancement of $R\left(D^{*}\right)$ of order $\sim 20 \%$ as suggested by the experimental measurements is only possible for $\zeta \ll 1$. The situation is very different for $R_{K}$, with the parameter $\zeta$ playing no major role in this case as shown in figure 4 . We find that the allowed points from the global fit accommodating both $R_{K}$ and $R\left(D^{(*)}\right)$ within $2 \sigma$ lie within a very restricted region:

$$
\begin{array}{lrl}
M_{Z^{\prime}} \in[500,1710] \mathrm{GeV}, & g_{2} \in[1.2,3.5], & \Delta_{s} \in[-1.16,-0.97], \quad \Delta_{b} \in[0.003,0.007] \\
\left|\Delta_{\mu}\right| \in[0.94,0.99], & \left|\Delta_{\tau}\right| \in[0,0.11], & \zeta \in[0,0.02] . \tag{5.4}
\end{array}
$$

The $Z^{\prime}$ mass and the $\mathrm{SU}(2)_{2}$ gauge coupling $g_{2}$ are positively correlated, going from $g_{2} \sim 1$ for $M_{Z^{\prime}} \sim 500 \mathrm{GeV}$ up to the perturbativity limit $g_{2} \leq \sqrt{4 \pi}$ for $M_{Z^{\prime}} \sim 1700 \mathrm{GeV}$. A limit on $\tan \beta$ can be derived in this region using eq. (3.17), we get $\tan \beta \in[0.2,0.65]$. Similarly, in this region the $\mathrm{SU}(2)_{1}$ gauge coupling satisfies $0.66 \leq g_{1} \leq 0.78$ and the combination $\hat{g}=g g_{2} / g_{1}$ is found to be within $1 \leq \hat{g} \leq 3.4$. Note that the $Z^{\prime}$ and $W^{\prime}$ interactions with the SM fermions are proportional to $1-\Delta_{a}^{2}$, see eq. (3.36). In the parameter space region where both $R_{K}$ and $R\left(D^{(*)}\right)$ are accommodated within $2 \sigma$, the massive gauge bosons, $Z^{\prime}$ and $W^{\prime}$, couple predominantly to the third fermion generation.

## 6 Predictions

In the following we take the current measured values of $R_{K}$ and $R\left(D^{(*)}\right)$ at face value, focusing on the parameter space region described in eq. (5.4). We are interested in possible signatures that can be used to test or falsify this scenario with upcoming measurements at the LHC and flavour factories.

### 6.1 Differential distributions in $B \rightarrow D^{(*)} \tau \nu$ decays

Due to the gauge structure of the model, new physics contributions to the $B \rightarrow D^{(*)} \ell \nu$ decay amplitudes have the same Dirac structure as the SM contribution to a good approximation. This gives rise to a clean prediction

$$
\begin{equation*}
\frac{R(D)}{R\left(D^{*}\right)}=\left[\frac{R(D)}{R\left(D^{*}\right)}\right]_{\mathrm{SM}} \tag{6.1}
\end{equation*}
$$

which is compatible with current data [5]. The inclusive ratio $R\left(X_{c}\right)$ can provide an additional handle to test the proposed scenario. The model gives rise to an enhancement in $R\left(X_{c}\right)$ within the parameter space region considered, we obtain $0.24 \leq R\left(X_{c}\right) \leq 0.29$. The Dirac structure of the new physics contributions can also be tested by using information from the $q^{2} \equiv\left(p_{B}-p_{D^{(*)}}\right)^{2}$ spectra and by measuring additional observables that exploit the rich kinematics and spin of the final state particles. The differential decay rate for $B \rightarrow D^{(*)} \tau \nu$ is affected in the model with a global rescaling factor, implying that forwardbackward asymmetries as well as the $\tau$ and $D^{*}$ polarization fractions are expected to be as in the SM. For recent studies of differential distributions in $b \rightarrow c \tau \nu$ decays see refs. [25, $125,132-143]$. Future measurements of $b \rightarrow c \tau \nu$ transitions at the Belle-II experiment will be crucial to disentangle possible new physics contributions in these decays [144].

### 6.2 Lepton universality tests in $\boldsymbol{R}_{M}$

Confirming the violation of lepton flavour universality in other $b \rightarrow s$ observables would be definite evidence in favour of new physics at work. Examples of such additional observables are $R_{M}$, with $M=K^{*}, \phi[145,146]$, defined analogously to $R_{K}$,

$$
\begin{equation*}
R_{M}\left[q_{1}^{2}, q_{2}^{2}\right]=\frac{\int_{q_{1}^{2}}^{q_{2}^{2}} d q^{2} d \Gamma\left(B_{q} \rightarrow M \mu^{+} \mu^{-}\right) / d q^{2}}{\int_{q_{1}^{2}}^{q_{2}^{2}} d q^{2} d \Gamma\left(B_{q} \rightarrow M e^{+} e^{-}\right) / d q^{2}} \tag{6.2}
\end{equation*}
$$

with $q=d, s$ for $M=K^{*}, \phi{ }^{7}$
The expected values for $R_{K}, R_{K^{*}}$ and $R_{\phi}$ within each bin are strongly correlated, except for the fact that hadronic uncertainties are mostly independent (but small). From the results of the fit, we find the following expected ranges for the different ratios:

$$
\begin{align*}
& R_{K}[1,6] \in[0.62,0.91] \text { at } 68 \% \mathrm{CL}, \quad R_{K}[1,6] \in[0.57,0.95] \text { at } 95 \% \mathrm{CL}, \\
& R_{K^{*}}[1.1,6] \in[0.66,0.91] \text { at } 68 \% \mathrm{CL}, \quad R_{K^{*}}[1.1,6] \in[0.62,0.95] \text { at } 95 \% \mathrm{CL}, \\
& R_{K^{*}}[15,19] \in[0.61,0.90] \text { at } 68 \% \mathrm{CL}, \quad R_{K^{*}}[15,19] \in[0.56,0.94] \text { at } 95 \% \mathrm{CL} \text {, }  \tag{6.3}\\
& R_{\phi}[1.1,6] \in[0.64,0.91] \text { at } 68 \% \mathrm{CL}, \quad R_{\phi}[1.1,6] \in[0.60,0.94] \text { at } 95 \% \mathrm{CL}, \\
& R_{\phi}[15,19] \in[0.61,0.90] \text { at } 68 \% \mathrm{CL}, \quad R_{\phi}[15,19] \in[0.56,0.94] \text { at } 95 \% \mathrm{CL},
\end{align*}
$$

where it is understood that a strong (positive) correlation exists among all the predictions, lower values of one observable corresponding to lower values of another and viceversa.

### 6.3 Lepton flavour violation

One of the first generic consequences of the violation of lepton flavour universality is lepton flavour violation [148], as explored in connection to the $B$-meson anomalies in refs. [55, 82, 140, 149-156]. In our model, the branching fraction for $\tau \rightarrow 3 \mu$ is proportional to $\Delta_{\tau}^{2}$ and is therefore suppressed for $\left|\Delta_{\tau}\right| \simeq 0$. When $\left|\Delta_{\tau}\right|$ is near its upper bound, $\left|\Delta_{\tau}\right| \simeq$ 0.1, we obtain values for $\operatorname{Br}(\tau \rightarrow 3 \mu)$ that saturate the current experimental limit $1.2 \times 10^{-8}$. Semileptonic decays of the tau lepton into a muon and a pseudo-scalar meson also receive tree-level contributions from $Z^{(\prime)}$ exchange, these will also be proportional to $\Delta_{\tau}^{2}$ so that the largest rates possible will be obtained for $\left|\Delta_{\tau}\right| \simeq 0.1$. In our model the decays $\tau \rightarrow \mu \eta^{(\prime)}$ receive important new physics contributions through the axial-vector strange-quark current. Following [157] we obtain $\operatorname{Br}\left(\tau \rightarrow \mu \eta^{\prime}\right) \leq 3.9 \times 10^{-8}$ and $\operatorname{Br}(\tau \rightarrow \mu \eta) \leq 4.2 \times 10^{-8}$, very close to the current experimental limits $\operatorname{Br}\left(\tau \rightarrow \mu \eta^{\prime}\right)_{\exp } \leq 1.3 \times 10^{-7}$ and $\operatorname{Br}(\tau \rightarrow \mu \eta)_{\exp } \leq$ $6.5 \times 10^{-8}$ [158]. The observation of lepton flavour violating tau decays decays might therefore lie within the reach of future machines such as Belle-II, where an improvement of the current experimental bounds by an order of magnitude can be expected [144]. On the other hand, due to the suppression of gauge mixing effects $(\zeta \ll 1)$ the decay $Z \rightarrow \tau \mu$ lies well-below the current experimental limit, for which we obtain $\operatorname{Br}(Z \rightarrow \mu \tau) \leq 1.2 \times 10^{-9}$.

[^5]
### 6.4 Direct searches for new states at the LHC

In this model we expect a plethora of new states lying at the TeV scale: scalar bosons (in the CP-conserving limit we would have two CP-even Higgs bosons, one CP-odd Higgs and one charged scalar, cf. section 2), heavy fermions and the massive vector bosons $W^{\prime}, Z^{\prime}$.

The heavy vector-like leptons will be pair-produced at the LHC via Drell-Yan processes due to their coupling to the massive electroweak gauge bosons. These will decay into gauge bosons and charged leptons or neutrinos. Though no dedicated searches for vectorlike leptons have been performed at the LHC, one can obtain limits on their mass and production cross-section by recasting existing multilepton searches [159]. It was found that current limits for a heavy lepton doublet decaying to $\ell=e, \mu$ flavours are around 450 GeV while for decays into $\tau$-leptons the limits are around 270 GeV [159]. Searches for pair production of heavy vector-like quarks at the LHC focus primarily into final states with a third generation fermion and bosonic states, setting upper limits on the vector-like quark masses ranging from $\sim 700 \mathrm{GeV}$ up to $\sim 1 \mathrm{TeV}$ [160-165].

The massive vector bosons $W^{\prime}, Z^{\prime}$ couple predominantly to the third fermion generation. The LHC phenomenology of this type of states has been discussed in ref. [81]. The $Z^{\prime}$ coupling to muons is found to be at most $\sim 12 \%$ of its coupling to $\tau$-leptons. In the quark sector, the $Z^{\prime}$ coupling to the second quark generation is found to be at most $\sim 36 \%$ of the coupling to third generation quarks. The $Z^{\prime}$ boson would be produced at the LHC via Drell-Yan processes due to its coupling to $b$-quarks and $s / c$-quarks.

The total $Z^{\prime}$ width normalized by the $Z^{\prime}$ mass $\left(\Gamma_{Z^{\prime}} / M_{Z^{\prime}}\right)$ is found to grow with $M_{Z^{\prime}}$, since $\hat{g}$ and $M_{Z^{\prime}}$ are positively correlated. Assuming that the $Z^{\prime}$ can only decay into the SM fermions we have

$$
\begin{equation*}
\frac{\Gamma_{Z^{\prime}}}{M_{Z^{\prime}}} \simeq \frac{\hat{g}^{2}}{48 \pi}\left[3 \sum_{q=s, b}\left(1-\Delta_{q}^{2}\right)^{2}+\sum_{\ell=\mu, \tau}\left(1-\Delta_{\ell}^{2}\right)^{2}\right] \tag{6.4}
\end{equation*}
$$

where we have neglected fermion mass effects. We obtain that $\Gamma_{Z^{\prime}} / M_{Z^{\prime}}$ is between $2 \%$ and $31 \%$, with $\Gamma_{Z^{\prime}} / M_{Z^{\prime}} \gtrsim 10 \%$ for $M_{Z^{\prime}} \gtrsim 1 \mathrm{TeV}$.

If kinematically open, additional decay channels of the $Z^{\prime}$ boson would reduce the branching fractions to SM particles by enhancing the total $Z^{\prime}$ width, making the $Z^{\prime}$ resonance broader. The latter scenario will generically be the case provided the vector-like fermions are light enough, opening decay channels of the $Z^{\prime}$ boson into a heavy vector-like fermion and a SM-like fermion or into a vector-like fermion pair. The decay rate for these processes is given by:

$$
\begin{align*}
& \Gamma\left(Z^{\prime} \rightarrow F_{i} \bar{f}_{j}\right) \simeq \frac{\lambda^{1 / 2}\left(1, x_{i}, x_{j}\right) \hat{g}^{2} N_{C} M_{Z^{\prime}}}{192 \pi}\left[2-x_{i}-x_{j}-\left(x_{i}-x_{j}\right)^{2}\right]\left(\Sigma_{i j}\right)^{2} \\
& \Gamma\left(Z^{\prime} \rightarrow F_{i} \bar{F}_{i}\right) \simeq \frac{\lambda^{1 / 2}\left(1, x_{i}, x_{i}\right) \hat{g}^{2} N_{C} M_{Z^{\prime}}}{96 \pi}\left\{\left(1-x_{i}\right)\left(\left(\Omega_{i i}^{Q, L}\right)^{2}+\frac{g_{1}^{4}}{g_{2}^{4}}\right)-6 \frac{g_{1}^{2}}{g_{2}^{2}} x_{i} \Omega_{i i}^{Q, L}\right\} \tag{6.5}
\end{align*}
$$

Here $\lambda(x, y, z)=x^{2}+y^{2}+z^{2}-2(x y+y z+x z), N_{C}=3(1)$ for (un)coloured fermions and $x_{i}=m_{i}^{2} / M_{Z^{\prime}}^{2}$. We have denoted by $F_{i}$ a generic heavy fermion and by $f_{j}$ one of the SM-like fermions. The matrices $\Sigma$ and $\Omega^{Q, L}$ have been defined in eqs. (3.37) and (3.38).

The $Z^{\prime}$ decays into a heavy fermion and a SM-like fermion are accidentally suppressed due to the small entries of the $\Sigma$ matrix within the parameter region of interest. These decays therefore give small contributions to the total width in general. The decays into a pair of heavy fermions, on the other hand, can give a significant contribution to the total $Z^{\prime}$ width when kinematically allowed. For instance, if the masses of the heavy leptons lie around 450 GeV we obtain a contribution to $\Gamma_{Z^{\prime}} / M_{Z^{\prime}}$ from the decays $Z^{\prime} \rightarrow E_{i} \bar{E}_{i}, N_{i} \bar{N}_{i}(i=1,2)$ of about $20 \%$ for $M_{Z^{\prime}} \sim 1.2 \mathrm{TeV}$, making the $Z^{\prime}$ boson a very wide resonance in this case: $\Gamma_{Z^{\prime}} / M_{Z^{\prime}} \sim 30 \%-50 \%$.

The ATLAS and CMS collaborations have searched for a resonance in the $\tau^{+} \tau^{-}$channel at $\sqrt{s}=8 \mathrm{TeV}$ [166-169]. Among these, the strongest limits are those coming from ATLAS and they place important bounds on the model. We have evaluated the $Z^{\prime}$ production crosssection at the LHC using MadGraph (MG5_aMC_2.4.2) [170]. We find that it is possible to exclude the low-mass region where the $Z^{\prime}$ resonance remains reasonably narrow and there is not much room for additional decay channels giving large contributions to the total width. The latter would require having very light exotic fermions, entering in conflict with direct searches for these states at colliders. In the heavy $Z^{\prime}$ mass region ( $\gtrsim 1 \mathrm{TeV}$ ) the $Z^{\prime}$ resonance becomes wide ( $\Gamma_{Z^{\prime}} / M_{Z^{\prime}} \gtrsim 10 \%$ ) and the interpretation of the current experimental results based on the search of a relatively narrow resonance is not valid anymore. Dedicated searches at the LHC for a broad resonance in the $\tau^{+} \tau^{-}$channel within the mass range $\sim 1-1.7 \mathrm{TeV}$ would then be needed in order to test this scenario. ${ }^{8}$

The proposed scenario also gives some predictions in the scalar sector relevant for collider searches. Neglecting mixing between the scalar bidoublet $\Phi$ and the Higgs doublets $\phi^{(1)}$, the scalar spectrum will contain a heavy CP-even neutral scalar transforming as an $\mathrm{SU}(2)_{L}$ singlet originating from $\Phi$. We will denote this state by $h_{2}$. The mass of this scalar is expected to be around the symmetry breaking scale $u \sim \mathrm{TeV}$. The dominant interactions of $h_{2}$ are with the heavy fermions and the heavy gauge vector bosons, these are described by

$$
\begin{equation*}
\mathcal{L} \supset 2\left(M_{W^{\prime}}^{2} W_{\mu}^{\prime+} W^{\prime-\mu}+\frac{1}{2} M_{Z^{\prime}}^{2} Z_{\mu}^{\prime} Z^{\prime \mu}\right) \frac{h_{2}}{u}-\left(y_{Q}\right)_{i i} \bar{Q}_{i} Q_{i} h_{2}-\left(y_{L}\right)_{i i} \bar{L}_{i} L_{i} h_{2}, \tag{6.6}
\end{equation*}
$$

with $Q_{i}^{T}=\left(U_{i}, D_{i}\right), L_{i}^{T}=\left(N_{i}, E_{i}\right)(i=1,2)$ and

$$
y_{Q}=\frac{g_{2}^{2}}{n_{1}^{2}}\left(\begin{array}{cc}
\widetilde{M}_{Q_{1}} & 0  \tag{6.7}\\
0 & \widetilde{M}_{Q_{2}}\left(\Delta_{s}^{2}+\Delta_{b}^{2}\right)
\end{array}\right), \quad y_{L}=\frac{g_{2}^{2}}{n_{1}^{2}}\left(\begin{array}{cc}
\widetilde{M}_{L_{1}} & 0 \\
0 & \widetilde{M}_{L_{2}}\left(\Delta_{\mu}^{2}+\Delta_{\tau}^{2}\right)
\end{array}\right) .
$$

The production of $h_{2}$ at the LHC is dominated by gluon fusion mediated by the heavy quarks and is determined by the same parameters entering in the low-energy global fit. At the centre-of mass energy $\sqrt{s}$ the production cross-section reads

$$
\begin{equation*}
\sigma\left(p p \rightarrow h_{2}\right) \simeq \frac{c_{g g} \Gamma\left(h_{2} \rightarrow g g\right)}{M_{h_{2}} s}, \quad \Gamma\left(h_{2} \rightarrow g g\right) \simeq \frac{\alpha_{s}^{2} M_{h_{2}}^{3}}{18 \pi^{3}}\left|\sum_{i=1}^{2} \frac{\left(y_{Q}\right)_{i i}}{u \bar{M}_{Q_{i}}}\right|^{2} . \tag{6.8}
\end{equation*}
$$

Here $c_{g g}$ represents a dimensionless partonic integral which we estimate using the set of parton distribution functions MSTW2008NLO [171] evaluated at the scale $\mu=M_{h_{2}}$.

[^6]In writing the decay rate for $h_{2} \rightarrow g g$ we have taken the local approximation for the fermionic loops. For $M_{h_{2}} \sim 1 \mathrm{TeV}$, and restricting the rest of the parameters to the region described in eq. (5.4), we obtain $\sigma\left(p p \rightarrow h_{2}\right) \simeq 110-290 \mathrm{fb}$ at $\sqrt{s}=13 \mathrm{TeV}$ centre-of-mass energy. For $M_{Z^{\prime}} \sim 1.7 \mathrm{TeV}$ (and $M_{h_{2}} \sim 1 \mathrm{TeV}$ ) the production cross-section converges towards $\sim 110 \mathrm{fb}$. The interactions of $h_{2}$ in eq. (6.6) will induce loop-mediated decays into gluons (which will hadronize into jets) and electroweak gauge bosons $W^{+} W^{-}, Z Z$, $\gamma \gamma, Z \gamma$. Assuming negligible tree-level decays, the $h_{2}$ boson will manifest in this case as a very narrow resonance decaying mainly into a pair of jets. The current experimental sensitivity for dijet-resonances at the LHC around this mass range ( $M_{h_{2}} \sim 1 \mathrm{TeV}$ ) is at the level of $10^{3} \mathrm{fb}[172,173]$. The decays into electroweak gauge bosons are found to be subdominant and for $M_{Z^{\prime}} \in[1,1.7] \mathrm{TeV}$ we have: $\operatorname{Br}\left(h_{2} \rightarrow W W\right) \sim 10^{-2}$, $\operatorname{Br}\left(h_{2} \rightarrow Z Z, Z \gamma\right) / \operatorname{Br}\left(h_{2} \rightarrow W W\right) \sim 25 \%, \operatorname{Br}\left(h_{2} \rightarrow \gamma \gamma\right) / \operatorname{Br}\left(h_{2} \rightarrow W W\right) \sim 1 \%$. Note however that in the case where some of the heavy fermions are below the threshold $M_{h_{2}} / 2$, tree-level decay of $h_{2}$ into these fermions becomes kinematically open and will generically dominate over the loop-induced decays commented above.

## 7 Conclusions

We have performed a phenomenological analysis of a renormalizable and perturbative gauge extension of the Standard Model. We took into account flavour observables sensitive to treelevel new physics contributions as well as bounds from electroweak precision measurements at the $Z$ and $W$ pole. More specifically, we have analysed the model in light of the current hints of new physics in $b \rightarrow c \ell \nu$ and $b \rightarrow s \ell^{+} \ell^{-}$semileptonic decays, finding that the flavour anomalies can be accommodated within the allowed regions of the parameter space.

As derived from the phenomenological analysis, strong hierarchies in the flavour structure of the Yukawa couplings are required in order to accommodate both $b \rightarrow s \ell^{+} \ell^{-}$and $b \rightarrow c \ell \nu$ anomalies. We have taken a phenomenologically oriented approach in this work, not invoking any flavour symmetry behind such structure. One interesting question would be the exploration of possible flavour symmetries accommodating the observed flavour structure. We confirm the conclusions of ref. [4] regarding the importance of suppressing gauge bosons mixing. This translates in a tuning of $\tan \beta$. Such accidental tuning would be more satisfactory if there was a dynamical or symmetry-based explanation behind. These last points also bring us to the question of the validity of our analysis, based on tree-level new physics effects, once quantum corrections are considered. These corrections might alter the flavour structure of the theory, remove accidental tunings which hold at the classical level as well as introduce new constraints from loop-induced processes such as $b \rightarrow s \gamma$. Though such analysis lies beyond the scope of our work, it would be relevant in order to establish the viability of the proposed framework if the present deviations in $b \rightarrow s \ell^{+} \ell^{-}$ and $b \rightarrow c \ell \nu$ are confirmed in the future.

From the model building point of view, there are many open questions which we have not addressed in this work and would deserve further investigation, one of them being the implementation of a mechanism for the generation of the observed neutrino masses and lepton mixing angles. Our model also lacks a dark matter candidate, motivating the
extension of our framework. It would be interesting to pursue the investigation of possible embeddings of the model within a larger gauge group, where the mass of the heavy fermions arise from spontaneous symmetry breaking.

## Acknowledgments

We thank Sébastien Descotes-Genon, Thorsten Feldmann, Martin Jung, Admir Greljo, and the participants of the June 2016 BaBar Collaboration Meeting at SLAC for helpful discussions. We also thank Adam Falkowski for providing numerical results used in our analysis of electroweak precision data. S.M.B. acknowledges support of the MIUR grant for the Research Projects of National Interest PRIN 2012 No.2012CP-PYP7 Astroparticle Physics, of INFN I.S. TASP2014, and of MultiDark CSD2009-00064. The work of A.C. is supported by the Alexander von Humboldt Foundation. The work of J.F. is supported in part by the Spanish Government, ERDF from the EU Commission and the Spanish Centro de Excelencia Severo Ochoa Programme [Grants No. FPA2011-23778, FPA2014-53631-C2-1-P, PROMETEOII/2013/007 and SEV-2014-0398]. J.F. is also supported by an "Atracció de Talent" scholarship from VLC-CAMPUS. A.V. acknowledges financial support from the "Juan de la Cierva" program (27-13-463B-731) funded by the Spanish MINECO as well as from the Spanish grants FPA2014-58183-P, Multidark CSD2009-00064, SEV-2014-0398 and PROMETEOII/ 2014/084 (Generalitat Valenciana). J.V. is funded by the Swiss National Science Foundation. J.V. acknowledges support from Explora project FPA2014-61478-EXP. We have used SARAH [174] and SPheno [175, 176] to cross-check some of our results, and matplotlib to produce most of the plots in the paper [177].

## A Details of the model

## A. 1 Tadpole equations

The vev configuration introduced in section 2 leads to three minimization conditions or tadpole equations. In the following we will consider all the parameters in the scalar potential to be real. Defining

$$
\begin{equation*}
t_{i}=\frac{\partial \mathcal{V}}{\partial v_{i}}=0 \tag{A.1}
\end{equation*}
$$

these are

$$
\begin{align*}
t_{\phi} & =m_{\phi}^{2} v_{\phi}+\frac{1}{2} v_{\phi}\left(\lambda_{4} v_{\phi^{\prime}}^{2}+\lambda_{5} u^{2}\right)+\frac{1}{2} v_{\phi^{\prime}} u \mu+\frac{1}{2} \lambda_{1} v_{\phi}^{3} \\
t_{\phi^{\prime}} & =m_{\phi^{\prime}}^{2} v_{\phi^{\prime}}+\frac{1}{2} v_{\phi^{\prime}}\left(\lambda_{4} v_{\phi}^{2}+\lambda_{6} u^{2}\right)+\frac{1}{2} v_{\phi} u \mu+\frac{1}{2} \lambda_{2} v_{\phi^{\prime}}^{3},  \tag{A.2}\\
t_{u} & =m_{\Phi}^{2} u+\frac{1}{2} u\left(\lambda_{5} v_{\phi}^{2}+\lambda_{6} v_{\phi^{\prime}}^{2}\right)+\frac{1}{2} v_{\phi} v_{\phi^{\prime}} \mu+\frac{1}{2} \lambda_{3} u^{3} .
\end{align*}
$$

These three conditions can be solved for the mass squared parameters $m_{\phi}^{2}, m_{\phi^{\prime}}^{2}$ and $m_{\Phi}^{2}$.

## A. 2 Scalar mass matrices

The neutral scalar fields can be decomposed as

$$
\begin{align*}
\varphi^{0} & =\frac{1}{\sqrt{2}}\left(v_{\phi}+S_{\phi}+i A_{\phi}\right) \\
\varphi^{\prime 0} & =\frac{1}{\sqrt{2}}\left(v_{\phi^{\prime}}+S_{\phi^{\prime}}+i A_{\phi^{\prime}}\right)  \tag{A.3}\\
\Phi^{0} & =\frac{1}{\sqrt{2}}\left(u+S_{\Phi}+i A_{\Phi}\right)
\end{align*}
$$

Since we assume that CP is conserved in the scalar sector, the CP-even and CP-odd states do not mix. In this case, one can define the bases

$$
\begin{align*}
\mathcal{S}^{T} & \equiv\left(S_{\phi}, S_{\phi^{\prime}}, S_{\Phi}\right), & \mathcal{P}^{T} & \equiv\left(A_{\phi}, A_{\phi^{\prime}}, A_{\Phi}\right)  \tag{A.4}\\
\left(\mathcal{H}^{-}\right)^{T} & \equiv\left(\left(\varphi^{+}\right)^{*},\left(\varphi^{\prime+}\right)^{*},\left(\Phi^{+}\right)^{*}\right), & \left(\mathcal{H}^{+}\right)^{T} & \equiv\left(\varphi^{+}, \varphi^{\prime+}, \Phi^{+}\right)
\end{align*}
$$

which allow us to obtain the scalar mass Lagrangian

$$
\begin{equation*}
-\mathcal{L}_{m}^{s}=\frac{1}{2} \mathcal{S}^{T} \mathcal{M}_{\mathcal{S}}^{2} \mathcal{S}+\frac{1}{2} \mathcal{P}^{T} \mathcal{M}_{\mathcal{P}}^{2} \mathcal{P}+\left(\mathcal{H}^{-}\right)^{T} \mathcal{M}_{\mathcal{H}^{ \pm}}^{2} \mathcal{H}^{+} \tag{A.5}
\end{equation*}
$$

The mass matrix for the CP-even scalars is given by

$$
\mathcal{M}_{\mathcal{S}}^{2}=\left(\begin{array}{ccc}
\mathcal{M}_{S_{\phi} S_{\phi}}^{2} & \mathcal{M}_{S_{\phi} S_{\phi^{\prime}}}^{2} & \mathcal{M}_{S_{\phi} S_{\Phi}}^{2}  \tag{A.6}\\
\mathcal{M}_{S_{\phi} S_{\phi^{\prime}}}^{2} & \mathcal{M}_{S_{\phi^{\prime}} S_{\phi^{\prime}}}^{2} & \mathcal{M}_{S_{\phi^{\prime}} S_{\Phi}}^{2} \\
\mathcal{M}_{S_{\phi} S_{\Phi}}^{2} & \mathcal{M}_{S_{\phi^{\prime}} S_{\Phi}}^{2} & \mathcal{M}_{S_{\Phi} S_{\Phi}}^{2}
\end{array}\right)
$$

with

$$
\begin{align*}
\mathcal{M}_{S_{\phi} S_{\phi}}^{2} & =m_{\phi}^{2}+\frac{1}{2}\left(3 v_{\phi}^{2} \lambda_{1}+v_{\phi^{\prime}}^{2} \lambda_{4}+u^{2} \lambda_{5}\right) \\
\mathcal{M}_{S_{\phi} S_{\phi^{\prime}}}^{2} & =v_{\phi} v_{\phi^{\prime}} \lambda_{4}+\frac{1}{2} u \mu \\
\mathcal{M}_{S_{\phi} S_{\Phi}}^{2} & =v_{\phi} u \lambda_{5}+\frac{1}{2} v_{\phi^{\prime}} \mu \\
\mathcal{M}_{S_{\phi^{\prime}} S_{\phi^{\prime}}}^{2} & =m_{\phi^{\prime}}^{2}+\frac{1}{2}\left(3 v_{\phi^{\prime}}^{2} \lambda_{2}+v_{\phi}^{2} \lambda_{4}+u^{2} \lambda_{6}\right)  \tag{A.7}\\
\mathcal{M}_{S_{\phi^{\prime}} S_{\Phi}}^{2} & =v_{\phi^{\prime}} u \lambda_{6}+\frac{1}{2} v_{\phi} \mu \\
\mathcal{M}_{S_{\Phi} S_{\Phi}}^{2} & =m_{\Phi}^{2}+\frac{1}{2}\left(v_{\phi}^{2} \lambda_{5}+v_{\phi^{\prime}}^{2} \lambda_{6}+3 u^{2} \lambda_{3}\right)
\end{align*}
$$

The lightest CP-even state, $\mathcal{S}_{1} \equiv h$, is identified with the recently discovered SM-like Higgs boson with a mass $\sim 125 \mathrm{GeV}$. Similarly, in the Landau gauge $(\xi=0)$, the mass matrix for the CP-odd scalars is given by

$$
\mathcal{M}_{\mathcal{P}}^{2}=\left(\begin{array}{ccc}
\mathcal{M}_{A_{\phi} A_{\phi}}^{2} & \mathcal{M}_{A_{\phi} A_{\phi^{\prime}}}^{2} & \mathcal{M}_{A_{\phi} A_{\Phi}}^{2}  \tag{A.8}\\
\mathcal{M}_{A_{\phi} A_{\phi^{\prime}}}^{2} & \mathcal{M}_{A_{\phi^{\prime}} A_{\phi^{\prime}}}^{2} & \mathcal{M}_{A_{\phi^{\prime}} A_{\Phi}}^{2} \\
\mathcal{M}_{A_{\phi} A_{\Phi}}^{2} & \mathcal{M}_{A_{\phi^{\prime}} A_{\Phi}}^{2} & \mathcal{M}_{A_{\Phi} A_{\Phi}}^{2}
\end{array}\right)
$$

with

$$
\begin{align*}
\mathcal{M}_{A_{\phi} A_{\phi}}^{2} & =m_{\phi}^{2}+\frac{1}{2}\left(v_{\phi}^{2} \lambda_{1}+v_{\phi^{\prime}}^{2} \lambda_{4}+u^{2} \lambda_{5}\right) \\
\mathcal{M}_{A_{\phi} A_{\phi^{\prime}}}^{2} & =\frac{1}{2} u \mu \\
\mathcal{M}_{A_{\phi} A_{\Phi}}^{2} & =\frac{1}{2} v_{\phi^{\prime}} \mu \\
\mathcal{M}_{A_{\phi^{\prime}} A_{\phi^{\prime}}}^{2} & =m_{\phi^{\prime}}^{2}+\frac{1}{2}\left(v_{\phi^{\prime}}^{2} \lambda_{2}+v_{\phi}^{2} \lambda_{4}+u^{2} \lambda_{6}\right)  \tag{A.9}\\
\mathcal{M}_{A_{\phi^{\prime}} A_{\Phi}}^{2} & =-\frac{1}{2} v_{\phi} \mu \\
\mathcal{M}_{A_{\Phi} A_{\Phi}}^{2} & =m_{\Phi}^{2}+\frac{1}{2}\left(v_{\phi}^{2} \lambda_{5}+v_{\phi^{\prime}}^{2} \lambda_{6}+u^{2} \lambda_{3}\right)
\end{align*}
$$

After application of the tadpole equations in eq. (A.2), it is straightforward to show that the matrix $\mathcal{M}_{\mathcal{P}}^{2}$ has two vanishing eigenvalues. These correspond to the Goldstone bosons that constitute the longitudinal modes for the massive $Z$ and $Z^{\prime}$ bosons. Finally, the mass matrix for the charged scalars in the Landau gauge $(\xi=0)$ is given by

$$
\mathcal{M}_{\mathcal{H}^{ \pm}}^{2}=\left(\begin{array}{ccc}
\mathcal{M}_{\varphi^{+} \varphi^{+}}^{2} & \mathcal{M}_{\varphi^{+} \varphi^{\prime+}}^{2} & \mathcal{M}_{\varphi^{+} \Phi^{+}}^{2}  \tag{A.10}\\
\mathcal{M}_{\varphi^{+} \varphi^{\prime+}}^{2} & \mathcal{M}_{\varphi^{\prime+}}^{2} \varphi^{\prime+} & \mathcal{M}_{\varphi^{\prime+} \Phi^{+}}^{2} \\
\mathcal{M}_{\varphi^{+} \Phi^{+}}^{2} & \mathcal{M}_{\varphi^{\prime+} \Phi^{+}}^{2} & \mathcal{M}_{\Phi^{+} \Phi^{+}}^{2}
\end{array}\right)
$$

with

$$
\begin{align*}
& \mathcal{M}_{\varphi^{+} \varphi^{+}}^{2}=m_{\phi}^{2}+\frac{1}{2}\left(v_{\phi}^{2} \lambda_{1}+v_{\phi^{\prime}}^{2} \lambda_{4}+u^{2} \lambda_{5}\right) \\
& \mathcal{M}_{\varphi^{+} \varphi^{\prime}}^{2}=\frac{1}{2} u \mu, \\
& \mathcal{M}_{\varphi^{+} \Phi^{+}}^{2}=-\frac{1}{2} v_{\phi^{\prime}} \mu  \tag{A.11}\\
& \mathcal{M}_{\varphi^{\prime+}}^{2} \\
& \mathcal{M}_{\varphi^{\prime+}}^{2}=m_{\phi^{\prime} \Phi^{+}}^{2}+\frac{1}{2}=\frac{1}{2} v_{\phi} \mu \\
& \mathcal{M}_{\Phi^{+} \Phi^{+}}^{2}=m_{\Phi}^{2}+\frac{1}{2}\left(v_{\phi^{\prime}}^{2} \lambda_{2}+v_{\phi}^{2} \lambda_{4}+u^{2} \lambda_{6}\right) \\
&\left.\phi^{\prime} \lambda_{6}+u^{2} \lambda_{3}\right) .
\end{align*}
$$

Again, one can find two vanishing eigenvalues in $\mathcal{M}_{\mathcal{H}^{ \pm}}^{2}$ after applying the tadpole equations in eqs. (A.2). These correspond to the Goldstone bosons eaten-up by the $W$ and $W^{\prime}$ gauge bosons.

## B Pseudo-observables for $Z$ - and $W$-pole observables

In our model, the pseudo-observables considered in ref. [84] are given by:

$$
\begin{aligned}
\delta m & =-\delta v \frac{g^{\prime 2}}{g^{2}-g^{\prime 2}}, \\
\delta g_{L}^{W \ell_{i}} & =-\zeta \epsilon^{2} \frac{g_{2}^{4}}{n_{1}^{4}} \Delta_{i i}^{\ell}+f(1 / 2,0)-f(-1 / 2,-1),
\end{aligned}
$$

$$
\begin{align*}
\delta g_{L}^{Z \ell_{i}} & =\zeta \epsilon^{2} \frac{g_{2}^{4}}{2 n_{1}^{4}} \Delta_{i i}^{\ell}+f(-1 / 2,-1), \\
\delta g_{R}^{Z \ell_{i}} & =f(0,-1),  \tag{B.1}\\
\delta g_{L}^{Z u_{i}} & =-\zeta \epsilon^{2} \frac{g_{2}^{4}}{2 n_{1}^{4}}\left(V_{\mathrm{CKM}} \Delta^{q} V_{\mathrm{CKM}}^{\dagger}\right)_{i i}+f(1 / 2,2 / 3), \\
\delta g_{R}^{Z u_{i}} & =f(0,2 / 3), \\
\delta g_{L}^{Z d_{i}} & =\zeta \epsilon^{2} \frac{g_{2}^{4}}{2 n_{1}^{4}} \Delta_{i i}^{q}+f(-1 / 2,-1 / 3), \\
\delta g_{R}^{Z d_{i}} & =f(0,-1 / 3),
\end{align*}
$$

where

$$
\begin{equation*}
\delta v=-\zeta \epsilon^{2} \frac{1}{2} \frac{g_{2}^{4}}{n_{1}^{4}} \Delta_{22}^{\ell} \quad \text { and } \quad f\left(T^{3}, Q\right)=-\delta v\left(T^{3}+Q \frac{g^{\prime 2}}{g^{2}-g^{\prime 2}}\right) . \tag{B.2}
\end{equation*}
$$

The family index $i$ for these shifts covers the three fermion generations except for $\delta g_{R}^{Z u_{i}}$, for which $i=1,2$. We neglect corrections to the right-handed $Z$ and $W$ couplings that are suppressed by the fermion masses, see section 3 . We also neglect loop contributions, which we estimate to be comparable to the tree-level contributions for $\zeta \lesssim 0.02$. However, the resulting $\delta g$ 's in that case would be below the limits quoted in [84].

Open Access. This article is distributed under the terms of the Creative Commons Attribution License (CC-BY 4.0), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

## References

[1] R.N. Mohapatra and J.C. Pati, Left-right gauge symmetry and an isoconjugate model of CP-violation, Phys. Rev. D 11 (1975) 566 [inSPIRE].
[2] H. Georgi, E.E. Jenkins and E.H. Simmons, Ununifying the Standard Model, Phys. Rev. Lett. 62 (1989) 2789 [Erratum ibid. 63 (1989) 1540] [INSPIRE].
[3] K. Hsieh, K. Schmitz, J.-H. Yu and C.P. Yuan, Global analysis of general $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ models with precision data, Phys. Rev. D 82 (2010) 035011 [arXiv:1003.3482] [INSPIRE].
[4] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, Non-Abelian gauge extensions for B-decay anomalies, Phys. Lett. B 760 (2016) 214 [arXiv:1604.03088] [INSPIRE].
[5] Heavy Flavor Averaging Group (HFAG) collaboration, Y. Amhis et al., Averages of b-hadron, c-hadron and $\tau$-lepton properties as of summer 2014, arXiv:1412.7515 [INSPIRE].
[6] LHCb collaboration, Test of lepton universality using $B^{+} \rightarrow K^{+} \ell^{+} \ell^{-}$decays, Phys. Rev. Lett. 113 (2014) 151601 [arXiv:1406.6482] [INSPIRE].
[7] G. Hiller and F. Krüger, More model independent analysis of $b \rightarrow s$ processes, Phys. Rev. D 69 (2004) 074020 [hep-ph/0310219] [inSPIRE].
[8] A. Guevara, G. López Castro, P. Roig and S.L. Tostado, Long-distance weak annihilation contribution to the $B^{ \pm} \rightarrow\left(\pi^{ \pm}, K^{ \pm}\right) \ell^{+} \ell^{-}$decays, Phys. Rev. D 92 (2015) 054035 [arXiv:1503.06890] [INSPIRE].
[9] M. Bordone, G. Isidori and A. Pattori, On the Standard Model predictions for $R_{K}$ and $R_{K^{*}}$, Eur. Phys. J. C 76 (2016) 440 [arXiv:1605.07633] [inSPIRE].
[10] S. Descotes-Genon, J. Matias and J. Virto, Understanding the $B \rightarrow K^{*} \mu^{+} \mu^{-}$anomaly, Phys. Rev. D 88 (2013) 074002 [arXiv:1307.5683] [inSPIRE].
[11] R.R. Horgan, Z. Liu, S. Meinel and M. Wingate, Calculation of $B^{0} \rightarrow K^{* 0} \mu^{+} \mu^{-}$and $B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}$observables using form factors from lattice QCD, Phys. Rev. Lett. 112 (2014) 212003 [arXiv:1310.3887] [InSPIRE].
[12] D. Ghosh, M. Nardecchia and S.A. Renner, Hint of lepton flavour non-universality in $B$ meson decays, JHEP 12 (2014) 131 [arXiv:1408.4097] [inSPIRE].
[13] T. Hurth, F. Mahmoudi and S. Neshatpour, Global fits to $b \rightarrow$ sll data and signs for lepton non-universality, JHEP 12 (2014) 053 [arXiv:1410.4545] [INSPIRE].
[14] W. Altmannshofer and D.M. Straub, New physics in $b \rightarrow s$ transitions after LHC run 1, Eur. Phys. J. C 75 (2015) 382 [arXiv:1411.3161] [InSPIRE].
[15] W. Altmannshofer and D.M. Straub, Implications of $b \rightarrow s$ measurements, arXiv:1503.06199 [INSPIRE].
[16] S. Descotes-Genon, L. Hofer, J. Matias and J. Virto, Global analysis of $b \rightarrow$ slौ anomalies, JHEP 06 (2016) 092 [arXiv:1510.04239] [inSPIRE].
[17] T. Hurth, F. Mahmoudi and S. Neshatpour, On the anomalies in the latest LHCb data, Nucl. Phys. B 909 (2016) 737 [arXiv:1603.00865] [inSPIRE].
[18] F. Beaujean, C. Bobeth and D. van Dyk, Comprehensive Bayesian analysis of rare (semi)leptonic and radiative B decays, Eur. Phys. J. C 74 (2014) 2897 [Erratum ibid. C 74 (2014) 3179] [arXiv:1310.2478] [INSPIRE].
[19] S. Descotes-Genon, L. Hofer, J. Matias and J. Virto, On the impact of power corrections in the prediction of $B \rightarrow K^{*} \mu^{+} \mu^{-}$observables, JHEP 12 (2014) 125 [arXiv:1407.8526] [InSPIRE].
[20] S. Jäger and J. Martin Camalich, Reassessing the discovery potential of the $B \rightarrow K^{*} \ell^{+} \ell^{-}$ decays in the large-recoil region: SM challenges and BSM opportunities, Phys. Rev. D 93 (2016) 014028 [arXiv:1412.3183] [INSPIRE].
[21] T. Hurth and F. Mahmoudi, On the LHCb anomaly in $B \rightarrow K^{*} \ell^{+} \ell^{-}$, JHEP 04 (2014) 097 [arXiv: 1312. 5267] [INSPIRE].
[22] J. Lyon and R. Zwicky, Resonances gone topsy turvy - the charm of $Q C D$ or new physics in $b \rightarrow s \ell^{+} \ell^{-}$?, arXiv:1406.0566 [INSPIRE].
[23] M. Ciuchini et al., $B \rightarrow K^{*} \ell^{+} \ell^{-}$decays at large recoil in the Standard Model: a theoretical reappraisal, JHEP 06 (2016) 116 [arXiv:1512.07157] [INSPIRE].
[24] A. Crivellin, C. Greub and A. Kokulu, Explaining $B \rightarrow D \tau \nu, B \rightarrow D^{*} \tau \nu$ and $B \rightarrow \tau \nu$ in a $2 H D M$ of type-III, Phys. Rev. D 86 (2012) 054014 [arXiv:1206.2634] [InSPIRE].
[25] A. Celis, M. Jung, X.-Q. Li and A. Pich, Sensitivity to charged scalars in $B \rightarrow D^{(*)} \tau \nu_{\tau}$ and $B \rightarrow \tau \nu_{\tau}$ decays, JHEP 01 (2013) 054 [arXiv:1210.8443] [INSPIRE].
[26] J.A. Bailey et al., Refining new-physics searches in $B \rightarrow D \tau \nu$ decay with lattice $Q C D$, Phys. Rev. Lett. 109 (2012) 071802 [arXiv: 1206.4992] [INSPIRE].
[27] P. Ko, Y. Omura and C. Yu, $B \rightarrow D^{(*)} \tau \nu$ and $B \rightarrow \tau \nu$ in chiral $\mathrm{U}(1)^{\prime}$ models with flavored multi Higgs doublets, JHEP 03 (2013) 151 [arXiv:1212.4607] [INSPIRE].
[28] A. Crivellin, J. Heeck and P. Stoffer, A perturbed lepton-specific two-Higgs-doublet model facing experimental hints for physics beyond the Standard Model, Phys. Rev. Lett. 116 (2016) 081801 [arXiv:1507.07567] [inSPIRE].
[29] J.M. Cline, Scalar doublet models confront $\tau$ and b anomalies, Phys. Rev. D 93 (2016) 075017 [arXiv: 1512.02210] [INSPIRE].
[30] M. Freytsis, Z. Ligeti and J.T. Ruderman, Flavor models for $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$, Phys. Rev. D 92 (2015) 054018 [arXiv:1506.08896] [INSPIRE].
[31] S. Nandi, S.K. Patra and A. Soni, Correlating new physics signals in $B \rightarrow D^{(*)} \tau \nu_{\tau}$ with $B \rightarrow \tau \nu_{\tau}$, arXiv:1605.07191 [INSPIRE].
[32] S. Fajfer, J.F. Kamenik, I. Nišandžić and J. Zupan, Implications of lepton flavor universality violations in $B$ decays, Phys. Rev. Lett. 109 (2012) 161801 [arXiv:1206.1872] [INSPIRE].
[33] N.G. Deshpande and A. Menon, Hints of R-parity violation in B decays into $\tau \nu$, JHEP 01 (2013) 025 [arXiv:1208.4134] [inSPIRE].
[34] M. Tanaka and R. Watanabe, New physics in the weak interaction of $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$, Phys. Rev. D 87 (2013) 034028 [arXiv:1212.1878] [inSPIRE].
[35] Y. Sakaki, M. Tanaka, A. Tayduganov and R. Watanabe, Testing leptoquark models in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$, Phys. Rev. D 88 (2013) 094012 [arXiv:1309.0301] [InSPIRE].
[36] I. Doršner, S. Fajfer, N. Košnik and I. Nišandžić, Minimally flavored colored scalar in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ and the mass matrices constraints, JHEP 11 (2013) 084 [arXiv:1306.6493] [INSPIRE].
[37] C. Hati, G. Kumar and N. Mahajan, $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ excesses in ALRSM constrained from $B$, $D$ decays and $D^{0}-\bar{D}^{0}$ mixing, JHEP 01 (2016) 117 [arXiv:1511.03290] [INSPIRE].
[38] J. Zhu, H.-M. Gan, R.-M. Wang, Y.-Y. Fan, Q. Chang and Y.-G. Xu, Probing the R-parity violating supersymmetric effects in the exclusive $b \rightarrow c \ell^{-} \bar{\nu}_{\ell}$ decays, Phys. Rev. D 93 (2016) 094023 [arXiv: 1602.06491] [inSPIRE].
[39] X.-Q. Li, Y.-D. Yang and X. Zhang, Revisiting the one leptoquark solution to the $R\left(D^{(*)}\right)$ anomalies and its phenomenological implications, JHEP 08 (2016) 054 [arXiv:1605.09308] [INSPIRE].
[40] X.-G. He and G. Valencia, B decays with $\tau$ leptons in nonuniversal left-right models, Phys. Rev. D 87 (2013) 014014 [arXiv:1211.0348] [InSPIRE].
[41] A. Abada, A.M. Teixeira, A. Vicente and C. Weiland, Sterile neutrinos in leptonic and semileptonic decays, JHEP 02 (2014) 091 [arXiv:1311.2830] [INSPIRE].
[42] G. Cvetič and C.S. Kim, Rare decays of $B$ mesons via on-shell sterile neutrinos, Phys. Rev. D 94 (2016) 053001 [arXiv:1606.04140] [inSPIRE].
[43] A.J. Buras, F. De Fazio and J. Girrbach, 331 models facing new $b \rightarrow s \mu^{+} \mu^{-}$data, JHEP 02 (2014) 112 [arXiv:1311.6729] [inSPIRE].
[44] A.J. Buras and J. Girrbach, Left-handed $Z^{\prime}$ and Z FCNC quark couplings facing new $b \rightarrow s \mu^{+} \mu^{-}$data, JHEP 12 (2013) 009 [arXiv:1309.2466] [inSPIRE].
[45] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, Quark flavor transitions in $L_{\mu}-L_{\tau}$ models, Phys. Rev. D 89 (2014) 095033 [arXiv:1403.1269] [inSPIRE].
[46] A. Crivellin, G. D'Ambrosio and J. Heeck, Addressing the LHC flavor anomalies with horizontal gauge symmetries, Phys. Rev. D 91 (2015) 075006 [arXiv:1503.03477] [INSPIRE].
[47] A. Crivellin, G. D'Ambrosio and J. Heeck, Explaining $h \rightarrow \mu^{ \pm} \tau^{\mp}, B \rightarrow K^{*} \mu^{+} \mu^{-}$and $B \rightarrow K \mu^{+} \mu^{-} / B \rightarrow K e^{+} e^{-}$in a two-Higgs-doublet model with gauged $L_{\mu^{-}} L_{\tau}$, Phys. Rev. Lett. 114 (2015) 151801 [arXiv:1501.00993] [INSPIRE].
[48] D. Aristizabal Sierra, F. Staub and A. Vicente, Shedding light on the $b \rightarrow s$ anomalies with a dark sector, Phys. Rev. D 92 (2015) 015001 [arXiv:1503.06077] [INSPIRE].
[49] A. Celis, J. Fuentes-Martin, M. Jung and H. Serodio, Family nonuniversal Z' models with protected flavor-changing interactions, Phys. Rev. D 92 (2015) 015007 [arXiv:1505.03079] [InSPIRE].
[50] G. Bélanger, C. Delaunay and S. Westhoff, A dark matter relic from muon anomalies, Phys. Rev. D 92 (2015) 055021 [arXiv:1507.06660] [INSPIRE].
[51] A. Celis, W.-Z. Feng and D. Lüst, Stringy explanation of $b \rightarrow s \ell^{+} \ell^{-}$anomalies, JHEP 02 (2016) 007 [arXiv: 1512.02218] [INSPIRE].
[52] A. Falkowski, M. Nardecchia and R. Ziegler, Lepton flavor non-universality in B-meson decays from a $\mathrm{U}(2)$ flavor model, JHEP 11 (2015) 173 [arXiv:1509.01249] [inSPIRE].
[53] B. Allanach, F.S. Queiroz, A. Strumia and S. Sun, $Z^{\prime}$ models for the LHCb and $g-2$ muon anomalies, Phys. Rev. D 93 (2016) 055045 [arXiv:1511.07447] [inSPIRE].
[54] C.-W. Chiang, X.-G. He and G. Valencia, $Z^{\prime}$ model for $b \rightarrow s \bar{\ell}$ flavor anomalies, Phys. Rev. D 93 (2016) 074003 [arXiv:1601.07328] [INSPIRE].
[55] C.S. Kim, X.-B. Yuan and Y.-J. Zheng, Constraints on a $Z^{\prime}$ boson within minimal flavor violation, Phys. Rev. D 93 (2016) 095009 [arXiv:1602.08107] [InSPIRE].
[56] G. Hiller and M. Schmaltz, $R_{K}$ and future $b \rightarrow$ sll physics beyond the Standard Model opportunities, Phys. Rev. D 90 (2014) 054014 [arXiv:1408.1627] [InSPIRE].
[57] S. Biswas, D. Chowdhury, S. Han and S.J. Lee, Explaining the lepton non-universality at the LHCb and CMS within a unified framework, JHEP 02 (2015) 142 [arXiv:1409.0882] [INSPIRE].
[58] B. Gripaios, M. Nardecchia and S.A. Renner, Composite leptoquarks and anomalies in $B$-meson decays, JHEP 05 (2015) 006 [arXiv:1412.1791] [inSPIRE].
[59] I. de Medeiros Varzielas and G. Hiller, Clues for flavor from rare lepton and quark decays, JHEP 06 (2015) 072 [arXiv:1503.01084] [inSPIRE].
[60] D. Bečirević, S. Fajfer and N. Košnik, Lepton flavor nonuniversality in $b \rightarrow s \ell^{+} \ell^{-}$ processes, Phys. Rev. D 92 (2015) 014016 [arXiv:1503.09024] [inSPIRE].
[61] S. Sahoo and R. Mohanta, Scalar leptoquarks and the rare $B$ meson decays, Phys. Rev. D 91 (2015) 094019 [arXiv:1501.05193] [inSPIRE].
[62] S. Sahoo and R. Mohanta, Study of the rare semileptonic decays $B_{d}^{0} \rightarrow K^{*} \ell^{+} \ell^{-}$in scalar leptoquark model, Phys. Rev. D 93 (2016) 034018 [arXiv:1507.02070] [InSPIRE].
[63] S. Sahoo and R. Mohanta, Leptoquark effects on $b \rightarrow s \nu \bar{\nu}$ and $B \rightarrow K \ell^{+} \ell^{-}$decay processes, New J. Phys. 18 (2016) 013032 [arXiv:1509.06248] [inSPIRE].
[64] H. Päs and E. Schumacher, Common origin of $R_{K}$ and neutrino masses, Phys. Rev. D 92 (2015) 114025 [arXiv:1510.08757] [inSPIRE].
[65] W. Huang and Y.-L. Tang, Flavor anomalies at the LHC and the R-parity violating supersymmetric model extended with vectorlike particles, Phys. Rev. D 92 (2015) 094015 [arXiv:1509.08599] [INSPIRE].
[66] C.-H. Chen, T. Nomura and H. Okada, Explanation of $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$and muon $g-2$ and implications at the LHC, Phys. Rev. D 94 (2016) 115005 [arXiv:1607.04857] [InSPIRE].
[67] C. Niehoff, P. Stangl and D.M. Straub, Violation of lepton flavour universality in composite Higgs models, Phys. Lett. B 747 (2015) 182 [arXiv:1503.03865] [inSPIRE].
[68] C. Niehoff, P. Stangl and D.M. Straub, Direct and indirect signals of natural composite Higgs models, JHEP 01 (2016) 119 [arXiv:1508.00569] [INSPIRE].
[69] A. Carmona and F. Goertz, Lepton flavor and nonuniversality from minimal composite Higgs setups, Phys. Rev. Lett. 116 (2016) 251801 [arXiv:1510.07658] [InSPIRE].
[70] B. Gripaios, M. Nardecchia and S.A. Renner, Linear flavour violation and anomalies in $B$ physics, JHEP 06 (2016) 083 [arXiv:1509.05020] [INSPIRE].
[71] F. Mahmoudi, S. Neshatpour and J. Virto, $B \rightarrow K^{*} \mu^{+} \mu^{-}$optimised observables in the MSSM, Eur. Phys. J. C 74 (2014) 2927 [arXiv:1401.2145] [InSPIRE].
[72] R. Alonso, B. Grinstein and J. Martin Camalich, Lepton universality violation and lepton flavor conservation in B-meson decays, JHEP 10 (2015) 184 [arXiv:1505.05164] [INSPIRE].
[73] M. Bauer and M. Neubert, Minimal leptoquark explanation for the $R_{D^{(*)}}, R_{K}$ and $(g-2)_{g}$ anomalies, Phys. Rev. Lett. 116 (2016) 141802 [arXiv:1511.01900] [InSPIRE].
[74] S. Fajfer and N. Košnik, Vector leptoquark resolution of $R_{K}$ and $R_{D^{(*)}}$ puzzles, Phys. Lett. B 755 (2016) 270 [arXiv:1511.06024] [inSPIRE].
[75] R. Barbieri, G. Isidori, A. Pattori and F. Senia, Anomalies in B-decays and U(2) flavour symmetry, Eur. Phys. J. C 76 (2016) 67 [arXiv:1512.01560] [inSPIRE].
[76] C. Hati, Explaining the diphoton excess in alternative left-right symmetric model, Phys. Rev. D 93 (2016) 075002 [arXiv:1601.02457] [INSPIRE].
[77] F.F. Deppisch, S. Kulkarni, H. Päs and E. Schumacher, Leptoquark patterns unifying neutrino masses, flavor anomalies and the diphoton excess, Phys. Rev. D 94 (2016) 013003 [arXiv:1603.07672] [INSPIRE].
[78] D. Das, C. Hati, G. Kumar and N. Mahajan, Towards a unified explanation of $R_{D^{(*)}}, R_{K}$ and $(g-2)_{\mu}$ anomalies in a left-right model with leptoquarks, Phys. Rev. D 94 (2016) 055034 [arXiv: 1605.06313] [INSPIRE].
[79] D. Buttazzo, A. Greljo, G. Isidori and D. Marzocca, Toward a coherent solution of diphoton and flavor anomalies, JHEP 08 (2016) 035 [arXiv:1604.03940] [INSPIRE].
[80] B. Bhattacharya, A. Datta, D. London and S. Shivashankara, Simultaneous explanation of the $R_{K}$ and $R\left(D^{(*)}\right)$ puzzles, Phys. Lett. B 742 (2015) 370 [arXiv:1412.7164] [inSPIRE].
[81] A. Greljo, G. Isidori and D. Marzocca, On the breaking of lepton flavor universality in $B$ decays, JHEP 07 (2015) 142 [arXiv:1506.01705] [inSPIRE].
[82] L. Calibbi, A. Crivellin and T. Ota, Effective field theory approach to $b \rightarrow$ sl申( ${ }^{(\prime)}$, $B \rightarrow K^{(*)} \nu \bar{\nu}$ and $B \rightarrow D^{(*)} \tau \nu$ with third generation couplings, Phys. Rev. Lett. 115 (2015) 181801 [arXiv:1506.02661] [INSPIRE].
[83] F. Feruglio, P. Paradisi and A. Pattori, Revisiting lepton flavour universality in $B$ decays, arXiv:1606.00524 [INSPIRE].
[84] A. Efrati, A. Falkowski and Y. Soreq, Electroweak constraints on flavorful effective theories, JHEP 07 (2015) 018 [arXiv:1503.07872] [inSPIRE].
[85] A. Pich, Precision tau physics, Prog. Part. Nucl. Phys. 75 (2014) 41 [arXiv:1310.7922] [inSPIRE].
[86] Particle Data Group collaboration, K.A. Olive et al., Review of particle physics, Chin. Phys. C 38 (2014) 090001 [inSPIRE].
[87] T. Kinoshita and A. Sirlin, Radiative corrections to Fermi interactions, Phys. Rev. 113 (1959) 1652 [InSPIRE].
[88] S.M. Berman, Radiative corrections to muon and neutron decay, Phys. Rev. 112 (1958) 267 [inSPIRE].
[89] W.J. Marciano and A. Sirlin, Electroweak radiative corrections to tau decay, Phys. Rev. Lett. 61 (1988) 1815 [INSPIRE].
[90] W.J. Marciano, Fermi constants and 'new physics', Phys. Rev. D 60 (1999) 093006 [hep-ph/9903451] [INSPIRE].
[91] V. Cirigliano and I. Rosell, $\pi / K \rightarrow e \bar{\nu}_{e}$ branching ratios to $O\left(e^{2} p^{4}\right)$ in chiral perturbation theory, JHEP 10 (2007) 005 [arXiv:0707.4464] [inSPIRE].
[92] R. Decker and M. Finkemeier, Short and long distance effects in the decay $\tau \rightarrow \pi \nu_{\tau}(\gamma)$, Nucl. Phys. B 438 (1995) 17 [hep-ph/9403385] [INSPIRE].
[93] V. Cirigliano, M. Giannotti and H. Neufeld, Electromagnetic effects in $K_{\ell 3}$ decays, JHEP 11 (2008) 006 [arXiv:0807.4507] [INSPIRE].
[94] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich and J. Portoles, Kaon decays in the Standard Model, Rev. Mod. Phys. 84 (2012) 399 [arXiv:1107.6001] [InSPIRE].
[95] A. Kastner and H. Neufeld, The $K_{\ell 3}$ scalar form factors in the Standard Model, Eur. Phys. J. C 57 (2008) 541 [arXiv:0805.2222] [INSPIRE].
[96] S. Fajfer, I. Nišandžić and U. Rojec, Discerning new physics in charm meson leptonic and semileptonic decays, Phys. Rev. D 91 (2015) 094009 [arXiv:1502.07488] [InSPIRE].
[97] A. Lenz et al., Anatomy of new physics in $B-\bar{B}$ mixing, Phys. Rev. D 83 (2011) 036004 [arXiv:1008.1593] [INSPIRE].
[98] Fermilab Lattice and MILC collaborations, A. Bazavov et al., $B_{(s)}^{0}$-mixing matrix elements from lattice QCD for the Standard Model and beyond, Phys. Rev. D 93 (2016) 113016 [arXiv:1602.03560] [INSPIRE].
[99] BaBar collaboration, B. Aubert et al., Measurement of the $B \rightarrow X_{s} \ell^{+} \ell^{-}$branching fraction with a sum over exclusive modes, Phys. Rev. Lett. 93 (2004) 081802 [hep-ex/0404006] [INSPIRE].
[100] Belle collaboration, M. Iwasaki et al., Improved measurement of the electroweak penguin process $B \rightarrow X_{s} \ell^{+} \ell^{-}$, Phys. Rev. D 72 (2005) 092005 [hep-ex/0503044] [INSPIRE].
[101] T. Huber, T. Hurth and E. Lunghi, Inclusive $\bar{B} \rightarrow X_{s} \ell^{+} \ell^{-}$: complete angular analysis and a thorough study of collinear photons, JHEP 06 (2015) 176 [arXiv:1503.04849] [InSPIRE].
[102] LHCb and CMS collaborations, Observation of the rare $B_{s}^{0} \rightarrow \mu^{+} \mu^{-}$decay from the combined analysis of CMS and LHCb data, Nature 522 (2015) 68 [arXiv:1411.4413] [inSPIRE].
[103] C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, E. Stamou and M. Steinhauser, $B_{s, d} \rightarrow \ell^{+} \ell^{-}$in the Standard Model with reduced theoretical uncertainty, Phys. Rev. Lett. 112 (2014) 101801 [arXiv:1311.0903] [INSPIRE].
[104] LHCb collaboration, Angular analysis of charged and neutral $B \rightarrow K \mu^{+} \mu^{-}$decays, JHEP 05 (2014) 082 [arXiv:1403.8045] [inSPIRE].
[105] J. Matias, F. Mescia, M. Ramon and J. Virto, Complete anatomy of $\bar{B}_{d} \rightarrow \bar{K}^{* 0}(\rightarrow K \pi) \ell^{+} \ell^{-}$ and its angular distribution, JHEP 04 (2012) 104 [arXiv:1202.4266] [INSPIRE].
[106] S. Descotes-Genon, J. Matias, M. Ramon and J. Virto, Implications from clean observables for the binned analysis of $B \rightarrow K^{*} \mu^{+} \mu^{-}$at large recoil, JHEP 01 (2013) 048 [arXiv:1207.2753] [inSPIRE].
[107] S. Descotes-Genon, T. Hurth, J. Matias and J. Virto, Optimizing the basis of $B \rightarrow K^{*} \ell \ell$ observables in the full kinematic range, JHEP 05 (2013) 137 [arXiv:1303.5794] [InSPIRE].
[108] LHCb collaboration, Differential branching fraction and angular analysis of the decay $B^{0} \rightarrow K^{* 0} \mu^{+} \mu^{-}$, JHEP 08 (2013) 131 [arXiv:1304.6325] [INSPIRE].
[109] LHCb collaboration, Measurement of the $B^{0} \rightarrow K^{* 0} e^{+} e^{-}$branching fraction at low dilepton mass, JHEP 05 (2013) 159 [arXiv:1304.3035] [INSPIRE].
[110] LHCb collaboration, Differential branching fractions and isospin asymmetries of $B \rightarrow K^{(*)} \mu^{+} \mu^{-}$decays, JHEP 06 (2014) 133 [arXiv:1403.8044] [INSPIRE].
[111] LHCb collaboration, Angular analysis of the $B^{0} \rightarrow K^{* 0} \mu^{+} \mu^{-}$decay using $3 \mathrm{fb}^{-1}$ of integrated luminosity, JHEP 02 (2016) 104 [arXiv:1512.04442] [INSPIRE].
[112] LHCb collaboration, Differential branching fraction and angular analysis of the decay $B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}$, JHEP 07 (2013) 084 [arXiv:1305.2168] [INSPIRE].
[113] LHCb collaboration, Angular analysis and differential branching fraction of the decay $B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}$, JHEP 09 (2015) 179 [arXiv:1506.08777] [INSPIRE].
[114] LHCb collaboration, Angular analysis of the $B^{0} \rightarrow K^{* 0} e^{+} e^{-}$decay in the low- $q^{2}$ region, JHEP 04 (2015) 064 [arXiv:1501.03038] [InSPIRE].
[115] S. Descotes-Genon, J. Matias and J. Virto, An analysis of $B_{d, s}$ mixing angles in presence of new physics and an update of $B_{s} \rightarrow K^{0 *} \bar{K}^{0 *}$, Phys. Rev. D 85 (2012) 034010 [arXiv:1111.4882] [INSPIRE].
[116] K. De Bruyn et al., Probing new physics via the $B_{s}^{0} \rightarrow \mu^{+} \mu^{-}$effective lifetime, Phys. Rev. Lett. 109 (2012) 041801 [arXiv:1204.1737] [inSPIRE].
[117] S. Descotes-Genon and J. Virto, Time dependence in $B \rightarrow$ V€८ decays, JHEP 04 (2015) 045 [Erratum ibid. 07 (2015) 049] [arXiv:1502.05509] [INSPIRE].
[118] BaBar collaboration, B. Aubert et al., Measurements of the semileptonic decays $\bar{B} \rightarrow D \ell \bar{\nu}$ and $\bar{B} \rightarrow D^{*} \ell \bar{\nu}$ using a global fit to $D X \ell \bar{\nu}$ final states, Phys. Rev. D 79 (2009) 012002 [arXiv:0809.0828] [INSPIRE].
[119] BaBar collaboration, J.P. Lees et al., Evidence for an excess of $\bar{B} \rightarrow D^{(*)} \tau^{-} \bar{\nu}_{\tau}$ decays, Phys. Rev. Lett. 109 (2012) 101802 [arXiv:1205.5442] [inSPIRE].
[120] Belle collaboration, M. Huschle et al., Measurement of the branching ratio of $\bar{B} \rightarrow D^{(*)} \tau^{-} \bar{\nu}_{\tau}$ relative to $\bar{B} \rightarrow D^{(*)} \ell^{-} \bar{\nu}_{\ell}$ decays with hadronic tagging at Belle, Phys. Rev. D 92 (2015) 072014 [arXiv:1507.03233] [inSPIRE].
[121] LHCb collaboration, Measurement of the ratio of branching fractions $\mathcal{B}\left(\bar{B}^{0} \rightarrow D^{*+} \tau^{-} \bar{\nu}_{\tau}\right) / \mathcal{B}\left(\bar{B}^{0} \rightarrow D^{*+} \mu^{-} \bar{\nu}_{\mu}\right)$, Phys. Rev. Lett. 115 (2015) 111803 [Addendum ibid. 115 (2015) 159901] [arXiv:1506.08614] [INSPIRE].
[122] Belle collaboration, A. Abdesselam et al., Measurement of the branching ratio of $\bar{B}^{0} \rightarrow D^{*+} \tau^{-} \bar{\nu}_{\tau}$ relative to $\bar{B}^{0} \rightarrow D^{*+} \ell^{-} \bar{\nu}_{\ell}$ decays with a semileptonic tagging method, arXiv:1603.06711 [INSPIRE].
[123] A. Abdesselam et al., Measurement of the $\tau$ lepton polarization in the decay $\bar{B} \rightarrow D^{*} \tau^{-} \bar{\nu}_{\tau}$, arXiv:1608.06391 [INSPIRE].
[124] J.F. Kamenik and F. Mescia, $B \rightarrow D \tau \nu$ branching ratios: opportunity for lattice $Q C D$ and hadron colliders, Phys. Rev. D 78 (2008) 014003 [arXiv:0802.3790] [INSPIRE].
[125] S. Fajfer, J.F. Kamenik and I. Nišandžić, On the $B \rightarrow D^{*} \tau \bar{\nu}_{\tau}$ sensitivity to new physics, Phys. Rev. D 85 (2012) 094025 [arXiv:1203.2654] [inSPIRE].
[126] MILC collaboration, J.A. Bailey et al., $B \rightarrow$ D $\nu$ form factors at nonzero recoil and $\left|V_{c b}\right|$ from $2+1$-flavor lattice $Q C D$, Phys. Rev. D 92 (2015) 034506 [arXiv:1503.07237] [INSPIRE].
[127] HPQCD collaboration, H. Na, C.M. Bouchard, G.P. Lepage, C. Monahan and J. Shigemitsu, $B \rightarrow D \ell \nu$ form factors at nonzero recoil and extraction of $\left|V_{c b}\right|$, Phys. Rev. D 92 (2015) 054510 [Erratum ibid. D 93 (2016) 119906] [arXiv:1505.03925] [inSPIRE].
[128] Z. Ligeti and F.J. Tackmann, Precise predictions for $B \rightarrow X_{c} \tau \bar{\nu}$ decay distributions, Phys. Rev. D 90 (2014) 034021 [arXiv:1406.7013] [InSPIRE].
[129] I. Caprini, L. Lellouch and M. Neubert, Dispersive bounds on the shape of $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}$ form-factors, Nucl. Phys. B 530 (1998) 153 [hep-ph/9712417] [InSPIRE].
[130] CKMfitter Group collaboration, J. Charles et al., CP violation and the CKM matrix: assessing the impact of the asymmetric B factories, Eur. Phys. J. C 41 (2005) 1 [hep-ph/0406184] [INSPIRE].
[131] D. Foreman-Mackey, D.W. Hogg, D. Lang and J. Goodman, emcee: the MCMC hammer, Publ. Astron. Soc. Pac. 125 (2013) 306 [arXiv:1202.3665] [inSPIRE].
[132] A. Datta, M. Duraisamy and D. Ghosh, Diagnosing new physics in $b \rightarrow c \tau \nu_{\tau}$ decays in the light of the recent BaBar result, Phys. Rev. D 86 (2012) 034027 [arXiv:1206.3760] [INSPIRE].
[133] Y. Sakaki and H. Tanaka, Constraints on the charged scalar effects using the forward-backward asymmetry on $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}_{\tau}$, Phys. Rev. D 87 (2013) 054002 [arXiv:1205.4908] [INSPIRE].
[134] P. Biancofiore, P. Colangelo and F. De Fazio, On the anomalous enhancement observed in $B \rightarrow D^{(*)} \tau \bar{\nu}_{\tau}$ decays, Phys. Rev. D 87 (2013) 074010 [arXiv:1302.1042] [INSPIRE].
[135] Y. Sakaki, M. Tanaka, A. Tayduganov and R. Watanabe, Probing new physics with $q^{2}$ distributions in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$, Phys. Rev. D 91 (2015) 114028 [arXiv:1412.3761] [InSPIRE].
[136] M. Duraisamy, P. Sharma and A. Datta, Azimuthal $B \rightarrow D^{*} \tau^{-} \overline{\nu_{\tau}}$ angular distribution with tensor operators, Phys. Rev. D 90 (2014) 074013 [arXiv:1405.3719] [InSPIRE].
[137] S. Shivashankara, W. Wu and A. Datta, $\Lambda_{b} \rightarrow \Lambda_{c} \tau \bar{\nu}_{\tau}$ decay in the Standard Model and with new physics, Phys. Rev. D 91 (2015) 115003 [arXiv:1502.07230] [INSPIRE].
[138] S. Bhattacharya, S. Nandi and S.K. Patra, Optimal-observable analysis of possible new physics in $B \rightarrow D^{(*)} \tau \nu_{\tau}$, Phys. Rev. D 93 (2016) 034011 [arXiv:1509.07259] [InSPIRE].
[139] D. Bečirević, S. Fajfer, I. Nišandžić and A. Tayduganov, Angular distributions of $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_{\ell}$ decays and search of new physics, arXiv:1602.03030 [INSPIRE].
[140] S. Sahoo and R. Mohanta, Effects of scalar leptoquark on semileptonic $\Lambda_{b}$ decays, New J. Phys. 18 (2016) 093051 [arXiv:1607.04449] [INSPIRE].
[141] R. Alonso, A. Kobach and J. Martin Camalich, New physics in the kinematic distributions of $\bar{B} \rightarrow D^{(*)} \tau^{-}\left(\rightarrow \ell^{-} \bar{\nu}_{\ell} \nu_{\tau}\right) \bar{\nu}_{\tau}$, Phys. Rev. D 94 (2016) 094021 [arXiv:1602.07671] [INSPIRE].
[142] A.K. Alok, D. Kumar, S. Kumbhakar and S.U. Sankar, D* polarization as a probe to discriminate new physics in $B \rightarrow D^{*} \tau \bar{\nu}$, arXiv:1606.03164 [INSPIRE].
[143] F.U. Bernlochner and Z. Ligeti, Semileptonic $B_{(s)}$ decays to excited charmed mesons with $e, \mu, \tau$ and searching for new physics with $R\left(D^{* *}\right)$, arXiv:1606. 09300 [INSPIRE].
[144] Belle-II collaboration, T. Abe et al., Belle II technical design report, arXiv:1011.0352 [INSPIRE].
[145] G. Hiller and M. Schmaltz, Diagnosing lepton-nonuniversality in $b \rightarrow$ sौौ, JHEP 02 (2015) 055 [arXiv:1411.4773] [inSPIRE].
[146] W. Altmannshofer and I. Yavin, Predictions for lepton flavor universality violation in rare $B$ decays in models with gauged $L_{\mu}-L_{\tau}$, Phys. Rev. D 92 (2015) 075022 [arXiv:1508.07009] [INSPIRE].
[147] B. Capdevila, S. Descotes-Genon, J. Matias and J. Virto, Assessing lepton-flavour non-universality from $B \rightarrow K^{*} \ell \ell$ angular analyses, JHEP 10 (2016) 075 [arXiv:1605.03156] [INSPIRE].
[148] S.L. Glashow, D. Guadagnoli and K. Lane, Lepton flavor violation in B decays?, Phys. Rev. Lett. 114 (2015) 091801 [arXiv:1411.0565] [inSPIRE].
[149] S.M. Boucenna, J.W.F. Valle and A. Vicente, Are the B decay anomalies related to neutrino oscillations?, Phys. Lett. B 750 (2015) 367 [arXiv:1503.07099] [InSPIRE].
[150] A. Crivellin, L. Hofer, J. Matias, U. Nierste, S. Pokorski and J. Rosiek, Lepton-flavour violating $B$ decays in generic $Z^{\prime}$ models, Phys. Rev. D 92 (2015) 054013 [arXiv:1504.07928] [INSPIRE].
[151] D. Guadagnoli and K. Lane, Charged-lepton mixing and lepton flavor violation, Phys. Lett. B 751 (2015) 54 [arXiv:1507.01412] [INSPIRE].
[152] S. Sahoo and R. Mohanta, Lepton flavor violating B meson decays via a scalar leptoquark, Phys. Rev. D 93 (2016) 114001 [arXiv:1512.04657] [INSPIRE].
[153] A. Crivellin, G. D'Ambrosio, M. Hoferichter and L.C. Tunstall, Violation of lepton flavor and lepton flavor universality in rare kaon decays, Phys. Rev. D 93 (2016) 074038 [arXiv:1601.00970] [INSPIRE].
[154] D. Bečirević, O. Sumensari and R. Zukanovich Funchal, Lepton flavor violation in exclusive $b \rightarrow s$ decays, Eur. Phys. J. C 76 (2016) 134 [arXiv:1602.00881] [INSPIRE].
[155] G. Kumar, Constraints on a scalar leptoquark from the kaon sector, Phys. Rev. D 94 (2016) 014022 [arXiv:1603.00346] [INSPIRE].
[156] D. Guadagnoli, D. Melikhov and M. Reboud, More lepton flavor violating observables for LHCb's run 2, Phys. Lett. B 760 (2016) 442 [arXiv:1605.05718] [INSPIRE].
[157] A. Celis, V. Cirigliano and E. Passemar, Model-discriminating power of lepton flavor violating $\tau$ decays, Phys. Rev. D 89 (2014) 095014 [arXiv:1403.5781] [InSPIRE].
[158] Belle collaboration, Y. Miyazaki et al., Search for lepton flavor violating $\tau^{-}$decays into $\ell^{-} \eta, \ell^{-} \eta^{\prime}$ and $\ell^{-} \pi^{0}$, Phys. Lett. B 648 (2007) 341 [hep-ex/0703009] [INSPIRE].
[159] A. Falkowski, D.M. Straub and A. Vicente, Vector-like leptons: Higgs decays and collider phenomenology, JHEP 05 (2014) 092 [arXiv:1312.5329] [INSPIRE].
[160] ATLAS collaboration, Search for pair and single production of new heavy quarks that decay to a $Z$ boson and a third-generation quark in pp collisions at $\sqrt{s}=8$ TeV with the ATLAS detector, JHEP 11 (2014) 104 [arXiv:1409.5500] [INSPIRE].
[161] ATLAS collaboration, Search for production of vector-like quark pairs and of four top quarks in the lepton-plus-jets final state in pp collisions at $\sqrt{s}=8 \mathrm{TeV}$ with the ATLAS detector, JHEP 08 (2015) 105 [arXiv:1505.04306] [INSPIRE].
[162] ATLAS collaboration, Analysis of events with b-jets and a pair of leptons of the same charge in pp collisions at $\sqrt{s}=8$ TeV with the ATLAS detector, JHEP 10 (2015) 150 [arXiv:1504.04605] [INSPIRE].
[163] ATLAS collaboration, Search for single production of vector-like quarks decaying into Wb in pp collisions at $\sqrt{s}=8$ TeV with the ATLAS detector, Eur. Phys. J. C 76 (2016) 442 [arXiv:1602.05606] [INSPIRE].
[164] CMS collaboration, Search for vector-like T quarks decaying to top quarks and Higgs bosons in the all-hadronic channel using jet substructure, JHEP 06 (2015) 080 [arXiv:1503.01952] [INSPIRE].
[165] CMS collaboration, Search for vector-like charge 2/3T quarks in proton-proton collisions at $\sqrt{s}=8 \mathrm{TeV}$, Phys. Rev. D 93 (2016) 012003 [arXiv:1509.04177] [InSPIRE].
[166] ATLAS collaboration, A search for high-mass resonances decaying to $\tau^{+} \tau^{-}$in pp collisions at $\sqrt{s}=8$ TeV with the ATLAS detector, JHEP 07 (2015) 157 [arXiv:1502.07177] [INSPIRE].
[167] ATLAS collaboration, Search for high-mass dilepton resonances in pp collisions at $\sqrt{s}=8$ TeV with the ATLAS detector, Phys. Rev. D 90 (2014) 052005 [arXiv:1405.4123] [inSPIRE].
[168] CMS collaboration, Search for high-mass resonances and large extra dimensions with $\tau$-lepton pairs decaying into final states with an electron and a muon at $\sqrt{s}=8 \mathrm{Te} V$, CMS-PAS-EXO-12-046, CERN, Geneva Switzerland (2012).
[169] CMS collaboration, Search for physics beyond the Standard Model in dilepton mass spectra in proton-proton collisions at $\sqrt{s}=8 \mathrm{TeV}$, JHEP 04 (2015) 025 [arXiv:1412.6302] [inSPIRE].
[170] J. Alwall et al., The automated computation of tree-level and next-to-leading order differential cross sections and their matching to parton shower simulations, JHEP 07 (2014) 079 [arXiv:1405.0301] [INSPIRE].
[171] A.D. Martin, W.J. Stirling, R.S. Thorne and G. Watt, Parton distributions for the LHC, Eur. Phys. J. C 63 (2009) 189 [arXiv:0901.0002] [inSPIRE].
[172] ATLAS collaboration, Search for new phenomena in dijet mass and angular distributions from pp collisions at $\sqrt{s}=13$ TeV with the ATLAS detector, Phys. Lett. B 754 (2016) 302 [arXiv:1512.01530] [INSPIRE].
[173] CMS collaboration, Search for narrow resonances decaying to dijets in proton-proton collisions at $\sqrt{s}=13$ TeV, Phys. Rev. Lett. 116 (2016) 071801 [arXiv:1512.01224] [INSPIRE].
[174] F. Staub, SARAH 4: a tool for (not only SUSY) model builders, Comput. Phys. Commun. 185 (2014) 1773 [arXiv:1309.7223] [INSPIRE].
[175] W. Porod, SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at $e^{+} e^{-}$colliders, Comput. Phys. Commun. 153 (2003) 275 [hep-ph/0301101] [INSPIRE].
[176] W. Porod and F. Staub, SPheno 3.1: extensions including flavour, CP-phases and models beyond the MSSM, Comput. Phys. Commun. 183 (2012) 2458 [arXiv:1104.1573] [INSPIRE].
[177] J.D. Hunter, Matplotlib: a $2 D$ graphics environment, Comput. Sci. Eng. 9 (2007) 90.


[^0]:    ${ }^{1}$ We will assume that $\mu$ is of the same order of the largest scale in the scalar potential, i.e. $\mu \sim u$.

[^1]:    ${ }^{2}$ Note however that the free parameters have to satisfy the condition $\left(1-g^{2} / g_{2}^{2}\right)\left(\Delta_{s, \mu}^{2}+\Delta_{b, \tau}^{2}\right) \leq 1$ for consistency with eq. (3.7).

[^2]:    ${ }^{3}$ The fit in ref. [16] includes $b \rightarrow s \gamma$ observables. These observables are not included in our fit.
    ${ }^{4}$ Contributions to the primed operators $Q_{9,10}^{\prime}$ are found to be negligible since the right-handed flavour changing $Z^{(\prime)}$ couplings to down-type quarks are suppressed by $m_{f}^{2} / u^{2}$, see section 3 .

[^3]:    ${ }^{5}$ New results for $R\left(D^{*}\right)$ and the tau polarization asymmetry in $B \rightarrow D^{*} \tau \nu$ decays $\left(P_{\tau}\right)$ using a hadronic tag have been presented by the Belle collaboration in ref. [123]. The reported measurements are $R\left(D^{*}\right)=$ $0.276 \pm 0.034_{-0.026}^{+0.029}$ and $P_{\tau}=-0.44 \pm 0.47_{-0.17}^{+0.20}$ [123]. These measurements are not included in our analysis but would have a negligible impact if added given that the weighted average for $R\left(D^{*}\right)$ remains basically the same and the experimental uncertainty in $P_{\tau}$ is still very large. Note that the measured tau polarization asymmetry is well compatible with the SM prediction $P_{\tau}=-0.502_{-0.005}^{+0.006} \pm 0.017$ [25].

[^4]:    ${ }^{6}$ These CKM inputs are obtained from a fit by the CKMFITTER group with only tree-level processes [130], as used in ref. [98].

[^5]:    ${ }^{7}$ See ref. [147] for other observables in $B \rightarrow K^{*} \ell \ell$ testing lepton-flavour non-universality.

[^6]:    ${ }^{8}$ We find our main conclusions in this regard to agree with those posed previously by the authors of ref. [81] while analysing a similar new physics case.

