Swarm Precise Orbit Determination

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Low Earth Orbiters (LEOs)

**CHAMP**

CHAllenging Minisatellite Payload

**GRACE**

Gravity Recovery And Climate Experiment

**GOCE**

Gravity and steady-state Ocean Circulation Explorer

But there are many more missions equipped with GPS receivers …

Jason

Jason-2

MetOp-A

Icesat

COSMIC

LEO Constellations

TanDEM-X  Swarm  Sentinel

and of course, in the future

GRACE-FO Mission

LISA Technology
Sheds Light on Climate Change

Global Navigation Satellite Systems (GNSS)

GPS

visible sat = 12

Galileo

Empfangene Satelliten: 10

Other GNSS are already existing (GLONASS) or being built up (Galileo, Beidou), but there are no multi-GNSS spaceborne receivers (with open data policy) in LEO orbit yet.
Introduction to GPS

GPS: Global Positioning System

Characteristics:

- Satellite system for (real-time) **Positioning** and **Navigation**

- **Global** (everywhere on Earth, up to altitudes of 5000km) and **at any time**

- **Unlimited** number of users

- **Weather-independent** (radio signals are passing through the atmosphere)

- 3-dimensional position, **velocity** and **time** information
Global Network of the International GNSS Service (IGS)

IGS stations used for computation of final orbits at CODE in June 2016 (Dach et al., 2009)
Parameters of a Global IGS Solution

- A large parameter estimation problem needs to be solved to determine GPS satellite orbits together with many other parameters on a routine basis.

- Thanks to this continuous effort performed by the Analysis Centers (ACs) of the International GNSS Service (IGS) many scientific applications are enabled.

![Parameters types diagram]

- Station coordinates: 50%
- Site-specific troposphere parameters: 5%
- Scaling factor for APL model: 5%
- Orbital elements: 27%
- Stochastic orbit parameters: 5%
- Earth rotation parameters: 5%
- Geocenter coordinates: 5%
- Satellite antenna offset parameters: 1%
- Satellite antenna pattern: 1%
- Scaling factor for higher-order ionosphere: 1%
- Ambiguity parameters: 1%
Performance of IGS Final Orbits

Final Orbits (AC solutions compared to IGS Final)

(smoothed)

Weighted RMS [mm]

Time [GPS weeks]

The final clock product with 5 min sampling is based on undifferenced GPS data of typically 120 stations of the IGS network.

The IGS 1 Hz network is finally used for clock densification to 5 sec.

The 5 sec clocks are interpolated to 1 sec as needed for 1 Hz LEO GPS data.
GPS Signals

Signals driven by an **atomic clock**

Two **carrier signals** (sine waves):

- \( L_1 \): \( f = 1575.43 \text{ MHz} \), \( \lambda = 19 \text{ cm} \)
- \( L_2 \): \( f = 1227.60 \text{ MHz} \), \( \lambda = 24 \text{ cm} \)

(e.g. Blewitt, 1997)

- **C/A-code** (Clear Access / Coarse Acquisition)
- **P-code** (Protected / Precise)
- Broadcast/Navigation Message
**Improved Observation Equation**

\[ L_i^k = \rho_i^k - c \cdot \Delta t_i^k + c \cdot \Delta t_i + \chi_i^k + \chi_i^k + \lambda \cdot N_i^k \]
\[ + \Delta_{rel} - c \cdot b_i^k + c \cdot b_i + m_i^k + \epsilon_i^k \]

- **\( \rho_i^k \)**: Distance between satellite and receiver
- **\( \Delta t_i^k \)**: Satellite clock offset wrt GPS time
- **\( \Delta t_i \)**: Receiver clock offset wrt GPS time
- **\( T_i^k \)**: Tropospheric delay
- **\( I_i^k \)**: Ionospheric delay
- **\( N_i^k \)**: Phase ambiguity
- **\( \Delta_{rel} \)**: Relativistic corrections
- **\( b_i^k \)**: Delays in satellite (cables, electronics)
- **\( b_i \)**: Delays in receiver and antenna
- **\( m_i^k \)**: Multipath, scattering, bending effects
- **\( \epsilon_i^k \)**: Measurement error

Satellite positions and clocks are known from ACs or IGS

Not existent for LEOs

Cancels out (first order only) when forming the ionosphere-free linear combination:

\[ L_c = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 \]
Geometric Distance

**Geometric distance** $\rho_{leo}^k$ is given by:

$$\rho_{leo}^k = |\mathbf{r}_{leo}(t_{leo}) - \mathbf{r}^k(t_{leo} - \tau_{leo}^k)|$$

- $\mathbf{r}_{leo}$ Inertial position of LEO antenna phase center at reception time
- $\mathbf{r}^k$ Inertial position of GPS antenna phase center of satellite $k$ at emission time
- $\tau_{leo}^k$ Signal traveling time between the two phase center positions

Different ways to represent $\mathbf{r}_{leo}$:

- **Kinematic** orbit representation
- **Dynamic** or **reduced-dynamic** orbit representation
Satellite position $r_{leo}(t_{leo})$ (in inertial frame) is given by:

$$r_{leo}(t_{leo}) = R(t_{leo}) \cdot (r_{leo,e,0}(t_{leo}) + \delta r_{leo,e,ant}(t_{leo}))$$

- $R$ Transformation matrix from Earth-fixed to inertial frame
- $r_{leo,e,0}$ LEO center of mass position in Earth-fixed frame
- $\delta r_{leo,e,ant}$ LEO antenna phase center offset in Earth-fixed frame

**Kinematic positions** $r_{leo,e,0}$ are estimated for each **measurement epoch**:

- Measurement epochs **need not** to be identical with nominal epochs
- Positions are **independent** of models describing the LEO dynamics.
  Velocities cannot be provided
A kinematic orbit is an ephemeris at **discrete** measurement epochs.

Kinematic positions are **fully independent** on the force models used for LEO orbit determination (Švehla and Rothacher, 2004)
### Kinematic Orbit Representation (3)

**Excerpt of kinematic Swarm-C positions at begin of 1 June, 2016**

The kinematic orbits may be downloaded at ftp://ftp.unibe.ch/aiub/LEO_ORBITS/

<table>
<thead>
<tr>
<th>Measurement epochs</th>
<th>Clock correction to nominal epoch (μs), e.g., to epoch 00:00:20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Positions (km)</td>
<td>(Earth-fixed)</td>
</tr>
<tr>
<td><strong>PL49</strong></td>
<td></td>
</tr>
<tr>
<td>2023.517746</td>
<td>3061.130332 5742.844473 0.262691</td>
</tr>
<tr>
<td>2026.734429</td>
<td>3066.793833 5738.746569 0.111976</td>
</tr>
<tr>
<td>2029.949393</td>
<td>3072.273033 5734.641440 0.261262</td>
</tr>
<tr>
<td>2032.162630</td>
<td>3077.837924 5730.529099 0.210548</td>
</tr>
<tr>
<td>2036.374136</td>
<td>3083.398504 5726.409546 0.159835</td>
</tr>
<tr>
<td>2039.583909</td>
<td>3088.954755 5722.282787 0.209119</td>
</tr>
<tr>
<td>2042.791949</td>
<td>3094.506686 5718.148843 0.158440</td>
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<tr>
<td>2045.998248</td>
<td>3100.054291 5714.007709 0.207778</td>
</tr>
<tr>
<td>2049.202791</td>
<td>3105.597545 5709.859370 0.257064</td>
</tr>
<tr>
<td>2052.405584</td>
<td>3111.136450 5705.703844 0.206352</td>
</tr>
</tbody>
</table>
Dynamic Orbit Representation (1)

Satellite position $r_{leo}(t_{leo})$ (in inertial frame) is given by:

$$r_{leo}(t_{leo}) = r_{leo,0}(t_{leo}; a, e, i, \Omega, \omega, u_0; Q_1, \ldots, Q_d) + \delta r_{leo,ant}(t_{leo})$$

- $r_{leo,0}$ LEO center of mass position
- $\delta r_{leo,ant}$ LEO antenna phase center offset
- $a, e, i, \Omega, \omega, u_0$ LEO initial osculating orbital elements
- $Q_1, \ldots, Q_d$ LEO dynamical parameters

Satellite trajectory $r_{leo,0}$ is a particular solution of an equation of motion

- One set of initial conditions (orbital elements) is estimated per arc.
  Dynamical parameters of the force model on request
Dynamic Orbit Representation (2)

Equation of motion (in inertial frame) is given by:

\[
\ddot{r} = -GM \frac{r}{r^3} + f_1(t, r, \dot{r}, Q_1, ..., Q_d)
\]

with initial conditions

\[
\begin{align*}
  r(t_0) &= r(a, e, i, \Omega, \omega, u_0; t_0) \\
  \dot{r}(t_0) &= \dot{r}(a, e, i, \Omega, \omega, u_0; t_0)
\end{align*}
\]

The acceleration \( f_1 \) consists of gravitational and non-gravitational perturbations taken into account to model the satellite trajectory. Unknown parameters \( Q_1, ..., Q_d \) of force models may appear in the equation of motion together with deterministic (known) accelerations given by analytical models.
Osculating Orbital Elements

(Beutler, 2005)
Perturbing Accelerations of a LEO Satellite

<table>
<thead>
<tr>
<th>Force</th>
<th>Acceleration (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central term of Earth’s gravity field</td>
<td>8.42</td>
</tr>
<tr>
<td>Oblateness of Earth’s gravity field</td>
<td>0.015</td>
</tr>
<tr>
<td>Atmospheric drag</td>
<td>0.00000079</td>
</tr>
<tr>
<td>Higher order terms of Earth’s gravity field</td>
<td>0.00025</td>
</tr>
<tr>
<td>Attraction from the Moon</td>
<td>0.0000054</td>
</tr>
<tr>
<td>Attraction from the Sun</td>
<td>0.0000005</td>
</tr>
<tr>
<td>Direct solar radiation pressure</td>
<td>0.000000097</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Dynamic orbit positions may be computed at any epoch within the arc.

Dynamic positions are fully dependent on the force models used, e.g., on the gravity field model.
Reduced-Dynamic Orbit Representation (1)

Equation of motion (in inertial frame) is given by:

$$\ddot{r} = -GM \frac{r}{r^3} + \mathbf{f}_1(t, r, \dot{r}, Q_1, \ldots, Q_d, P_1, \ldots, P_s)$$

$P_1, \ldots, P_s$ Pseudo-stochastic parameters

Pseudo-stochastic parameters are:

- additional empirical parameters characterized by a priori known statistical properties, e.g., by expectation values and a priori variances

- useful to compensate for deficiencies in dynamic models, e.g., deficiencies in models describing non-gravitational accelerations

- often set up as piecewise constant accelerations to ensure that satellite trajectories are continuous and differentiable at any epoch
Reduced-dynamic orbits are well suited to compute LEO orbits of highest quality (Jäggi et al., 2006; Jäggi, 2007)
Reduced-dynamic Orbit Representation (3)

Position epochs
(in GPS time)

| * 2016 6 1 0 0 0.00000000 |
| PL49 | -1965.328762 | -2960.079621 | 5815.366063 |
| VL49 | -32476.530949 | -56518.428574 | -39633.949261 |

* 2016 6 1 0 0 10.00000000

| PL49 | -1997.722965 | -3016.388318 | 5775.367094 |
| VL49 | -32311.097194 | -56097.834133 | -40363.154274 |

* 2016 6 1 0 0 20.00000000

| PL49 | -2029.949403 | -3072.273033 | 5734.641439 |
| VL49 | -32141.000143 | -55670.464832 | -41087.301898 |

* 2016 6 1 0 0 30.00000000

| PL49 | -2062.003415 | -3127.727011 | 5693.194205 |
| VL49 | -31966.250891 | -55236.380456 | -41806.300697 |

* 2016 6 1 0 0 40.00000000

| PL49 | -2093.880357 | -3182.743574 | 5651.030585 |
| VL49 | -31786.861194 | -54795.641569 | -42520.059999 |

* 2016 6 1 0 0 50.00000000

| PL49 | -2125.575594 | -3237.316095 | 5608.155863 |
| VL49 | -31602.843520 | -54348.309592 | -43228.489711 |

* 2016 6 1 0 1 0.00000000

| PL49 | -2157.084506 | -3291.438018 | 5564.575411 |
| VL49 | -31414.211010 | -53894.446726 | -43931.500489 |

Clock corrections are not provided

Excerpt of reduced-dynamic Swarm-C positions at begin of 1 June, 2016
Phase center offsets $\delta r_{\text{leo,ant}}$:

- are needed in the inertial or Earth-fixed frame and have to be transformed from the satellite frame using attitude data from the star-trackers.

- consist of a frequency-independent instrument offset, e.g., defined by the center of the instrument‘s mounting plane (CMP) in the satellite frame.

- consist of frequency-dependent phase center offsets (PCOs), e.g., defined wrt the center of the instrument‘s mounting plane in the antenna frame (ARF).

- consist of frequency-dependent phase center variations (PCVs) varying with the direction of the incoming signal, e.g., defined wrt the PCOs in the antenna frame.
Offset wrt satellite reference frame (SRF) is **constant**
Offset wrt center of mass (CoM) is **slowly varying**
Spaceborne GPS Antennas: GOCE

L1, L2, Lc phase center offsets

Measured from ground calibration in anechoic chamber

L2 PCO
L1 PCO
Lc PCO

CMP

Lc phase center variations

Empirically derived during orbit determination according to Jäggi et al. (2009)
Spaceborne GPS Antennas: Swarm

Swarm GPS antenna

Multipath shall be minimized by chokering

L_{if} phase center variations

Empirically derived during orbit determination according to Jäggi et al. (2009)

Visualization of Orbit Solutions

It is more instructive to look at differences between orbits in well suited coordinate systems …

Co-Rotating Orbital Frames

R, S, C unit vectors are pointing:
- into the radial direction
- normal to R in the orbital plane
- normal to the orbital plane (cross-track)

T, N, C unit vectors are pointing:
- into the tangential (along-track) direction
- normal to T in the orbital plane
- normal to the orbital plane (cross-track)

Small eccentricities: S~T (velocity direction)

(Beutler, 2005)
Orbit Differences KIN-RD (Swarm-C)

Differences at epochs of kin. positions

- **Radial**: $0.7 \pm 12.0$ mm
- **Along-track**: $1.0 \pm 9.7$ mm
- **Cross-track**: $-2.0 \pm 7.5$ mm

Hours of 1 June, 2016
Pseudo-Stochastic Accelerations (GOCE)

\[ 54.6 \pm 147.6 \text{ nm/s}^2 \]
\[ -74.5 \pm 221.8 \text{ nm/s}^2 \]
\[ -106.2 \pm 230.4 \text{ nm/s}^2 \]
Kinematic Orbit Validation with SLR

SLR statistics:
Mean ± RMS (cm)

- 0.27 ± 3.25 cm
- 0.10 ± 2.74 cm
- 0.06 ± 3.11 cm

(Jäggi et al., 2016)
Consequences of Ionospheric Effects in Orbits

For GOCE systematic effects around the geomagnetic equator were observed in the ionosphere-free GPS phase residuals \( \Rightarrow \text{affects kinematic positions} \)

Degradation of kinematic positions around the geomagnetic equator propagates into gravity field solutions.

Phase observation residuals (-2 mm … +2 mm) mapped to the ionosphere piercing point

Geoid height differences (-5 cm … 5 cm); R4 period

(Jäggi et al., 2015)
Systematic signatures along the geomagnetic equator may be efficiently reduced for static Swarm gravity field recovery when screening the raw RINEX GPS data files with the $\Delta L_{gf}$ criterion.

(Differences wrt GOCO05S, 400 km Gauss smoothing adopted)

(Jäggi et al., 2016)
Systematic signatures along the geomagnetic equator are not visible when using original L1B RINEX GPS data files from the GRACE mission. (Differences wrt GOCO05S, 400 km Gauss smoothing adopted) (Jäggi et al., 2016)
Significant amounts of data are missing in GRACE L1B RINEX files
=> problematic signatures cannot propagate into gravity field.

Swarm RINEX files are more complete (gaps only over the poles)
=> problematic signatures do propagate into the gravity field.

(Jäggi et al., 2016)
**Ionospheric Signatures in GPS Linear Combinations**

**South-pole pass**

\[ \Phi \]

\[ \begin{array}{ccc}
-60^\circ & -87.4^\circ & -60^\circ \\
\end{array} \]

-60°  -87.4°  -60°

\[ \begin{array}{c}
5 \\
4 \\
3 \\
2 \\
1 \\
0 \\
-1 \\
-2 \\
-3 \\
-4 \\
-5 \\
\end{array} \]

Minute of day 14/353

\[ \begin{array}{c}
-5 \\
-4 \\
-3 \\
-2 \\
-1 \\
0 \\
1 \\
2 \\
3 \\
4 \\
5 \\
\end{array} \]

\[ \begin{array}{c}
[\text{cm}, \text{cm/s}] \\
[\text{cm}, \text{cm/s}] \\
[\text{cm}, \text{cm/s}] \\
[\text{cm}, \text{cm/s}] \\
[\text{cm}, \text{cm/s}] \\
\end{array} \]

\[ L_{if} : \]
Phase residuals of ionosphere-free linear combination \( L_{if} \) from kinematic POD

\[ \Delta L_{gf} : \]
Epoch-to-epoch difference of geometry-free linear combination

**Equatorial pass**

30°  0°  -30°

\[ \begin{array}{c}
8 \\
6 \\
4 \\
2 \\
0 \\
\end{array} \]

Minute of day 14/305

\[ \begin{array}{c}
8 \\
6 \\
4 \\
2 \\
0 \\
\end{array} \]

\[ \begin{array}{c}
\text{Number of satellites} \\
\text{Number of satellites} \\
\text{Number of satellites} \\
\text{Number of satellites} \\
\text{Number of satellites} \\
\end{array} \]

Number of satellites used for kinematic positioning

Radial difference between kinematic and reduced-dynamic orbit

(Arnold et al., 2016)
Global Ionosphere behavior

RMS of $\Delta L_{gf}$ (full signal)

Equatorial regions are mainly governed by “deterministic” features
$=>$ Systematic gravity field errors

RMS of $\Delta L_{gf}$ (high-pass)

Polar regions are mainly governed by “scintillation-like” features
$=>$ Gravity field is hardly affected

(Arnold et al., 2016)
Upgrades in the Swarm GPS Receiver Settings

A wider L2 carrier loop bandwidth increases the robustness of the L2 carrier phase tracking. In an attempt to improve the performance of the Swarm GPS receivers, the L2 carrier loop bandwidth was e.g. increased several times:

<table>
<thead>
<tr>
<th>Date</th>
<th>Swarm A</th>
<th>Swarm B</th>
<th>Swarm C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before 6 May 2015</td>
<td>0.25Hz</td>
<td>0.25Hz</td>
<td>0.25Hz</td>
</tr>
<tr>
<td>6 May 2015</td>
<td></td>
<td></td>
<td>0.25Hz → 0.5Hz</td>
</tr>
<tr>
<td>8 October 2015</td>
<td>0.25Hz</td>
<td>0.25Hz</td>
<td>0.25Hz → 0.5Hz</td>
</tr>
<tr>
<td>10 October 2015</td>
<td>0.25Hz</td>
<td></td>
<td>0.5Hz → 0.75Hz</td>
</tr>
<tr>
<td>23 June 2016</td>
<td>0.5Hz</td>
<td></td>
<td>0.75Hz → 1.0Hz</td>
</tr>
<tr>
<td>11 August 2016</td>
<td>0.5Hz</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

One of the three settings (0.5 Hz, 0.75 Hz, 1.0 Hz) is expected to be optimal and shall eventually be implemented on all Swarm satellites.
Impact on Gravity Field Solutions (June 2015)

Screened GPS data ($\Delta L_{gf} > 2\text{cm/s excl.}$)

Swarm-A

Swarm-C

Original GPS data

Swarm-A

Swarm-C

750km Gauss

500km Gauss

400km Gauss

(Dahle et al., 2016)

No obvious gaps for Swarm-C along geomagnetic equator.

Reduction of artefacts in gravity field solutions is therefore not due to data gaps along geomagnetic equator.

This indicates that the equatorial GPS data were indeed “corrupted” before the tracking loop changes. With improved settings of the tracking loop the problem seems to be largely mitigated.

(Dahle et al., 2016)
**Comparison to RINEX Screening**

- RINEX screening is useful for gravity field recovery, but rejects a lot (too much) of GPS data, at least in the way as implemented so far.

- Improved tracking loop settings are most promising to use the full amount of GPS data while significantly reducing the observed artefacts in the gravity field recovery.

(Dahle et al., 2016)
Outlook: Time-Variable Gravity from Swarm

“True” signal:
- GFZ-RL05a (DDK5-filtered)

“Comparison” signal:
- GFZ-RL05a (500km Gauss)

Swarm signal:
- 90x90 solutions (Gauss-filtered)

Result:
- Best agreement for Swarm-C
  (Jäggi et al., 2016)
Solutions based on AIUB orbits show a very good performance. This is probably mainly related to the quality of the underlying kinematic orbits.

Combination of solutions from different groups (using different orbits and approaches for gravity field recovery) show a further reduced noise.

(Teixeira da Encarnação et al., 2016)
Outlook: Time-Variable Gravity from Non-Dedicated Satellites

Combination of hl-SST solutions with SLR reduces the variations over oceans and some spurious signals.

(Sośnica et al., 2014)
Thank you for your attention


Literature (2)


Literature (3)


