GRAIL Gravity Field Determination Using the Celestial Mechanics Approach

First results from Doppler and KBRR data

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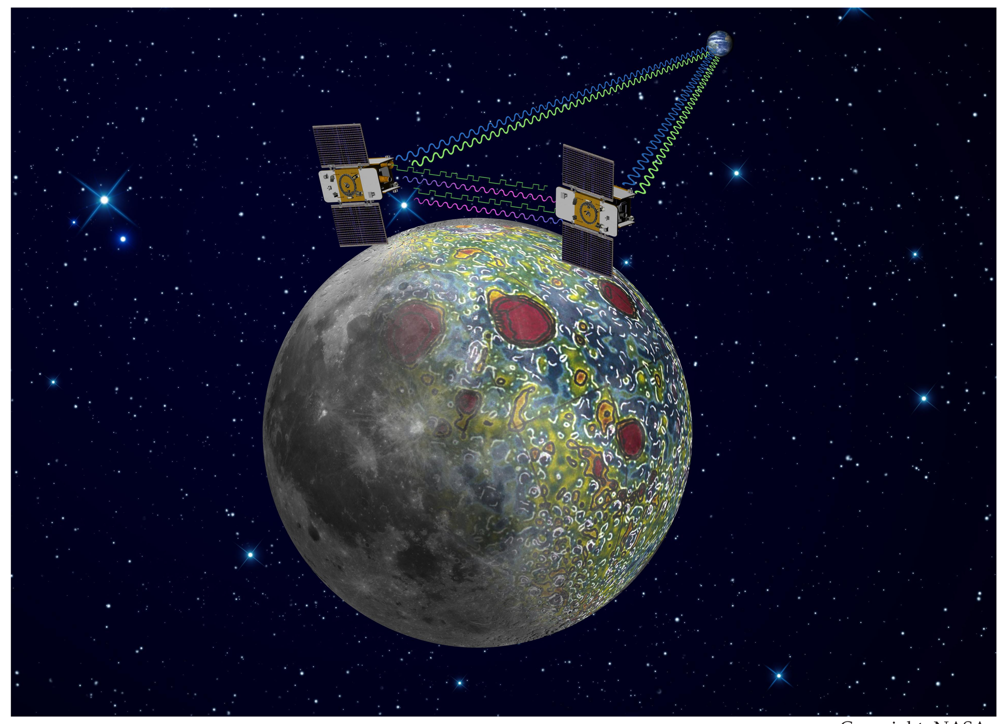
Introduction

To determine the gravity field of the Moon, the two satellites of the NASA mission GRAIL (Gravity Recovery and Interior Laboratory) were launched on September 10, 2011 and reached their lunar orbits in the beginning of 2012 (Zuber et al., 2013). The concept of the mission was inherited from the Earth-orbiting mission GRACE (Gravity Recovery and Climate Experiment) in that the key observations consisted of ultra-precise inter-satellite Ka-band range measurements. Together with the one- and two-way Doppler observations from the NASA Deep Space Network (DSN), the GRAIL data allows for a determination of the lunar gravity field with an unprecedented accuracy for both the near- and the far-side of the Moon. The latest official GRAIL gravity field models contain spherical harmonic (SH) coefficients up to degree and order 900 (Konopliv et al., 2014, Lemoine et al., 2014).



Based on our experience in GRACE data processing, we have adapted our approach for gravity field recovery, the Celestial Mechanics Approach (CMA, Beutler et al., 2010), to the GRAIL mission within the Bernese GNSS software. We use the level 1b Ka-band range-rate (KBRR) data as well as two-way Doppler observations from the DSN (relative weighting $10^8 : 1$). Earlier results using KBRR data along with JPL-provided GNI1B position data (Arnold et al., 2015) are also presented. The following results are based on the release 4 data of the primary mission phase (PM, 1 March to 29 May 2012).

The Celestial Mechanics Approach (CMA)

The idea of the CMA is to rigorously treat the gravity field recovery as an extended orbit determination problem. It is a dynamic approach allowing for appropriately constrained stochastic pulses (instantaneous changes in velocity) to compensate for inevitable model deficiencies. For each satellite, the equations of motion to be solved read as $\ddot{\mathbf{r}} = \mathbf{a}_G + \mathbf{a}_P$, where $\mathbf{a}_G = \nabla V$ denotes the acceleration due to the gravity potential V, which we parametrize in terms of the standard SH expansion, and $\mathbf{a}_P$ denotes the sum of all perturbing accelerations. We consider 3rd body perturbations according to JPL ephemerides DE421, forces due to the tidal deformation of the Moon and relativistic corrections. We do not yet model direct or indirect solar radiation pressure explicitly.
All observations contribute to one and the same set of parameters, which are simultaneously estimated. Depending on the setup, these are chosen amongst:

- Orbits: Initial conditions every 24h; constant and once-per-revolution (opr) accelerations in R,S,W (radial, along-track, out-of-plane); stochastic pulses in R,S,W estimated periodically. Their spacing has to be chosen as a compromise between making up for model deficiencies and not absorbing too much of the gravity signal..
- Static gravity field: The coefficients of the SH expansion up to the chosen degree and order.

Doppler data processing in the Bernese Software

Besides the inter-satellite KBRR link, GRAIL orbit and gravity field determination is based on its Doppler tracking by several Earth-based stations of the DSN for the absolute positioning of the probes. Both one-way X-band and two-way S-band are available with an accuracy of 0.03 mm/s (~ 2 mHz) and 0.2 mm/s (~ 6 mHz), respectively.
We process Doppler two-way observations using new implementations in the Bernese GNSS software. Our modeling is based on the reference (Moyer, 2000) guidebook and it includes:

- Earth-fixed coordinates of the tracking stations (Folkner W., 2013)
- Earth rotation and pole motion (IERS 2010)
- planetary ephemeris (e.g., DE421)
- Space-time frame transformations (IAU 2010)
- Relativistic effects on light propagation (Shapiro delay, ...)
- Atmospheric delay (troposphere only)

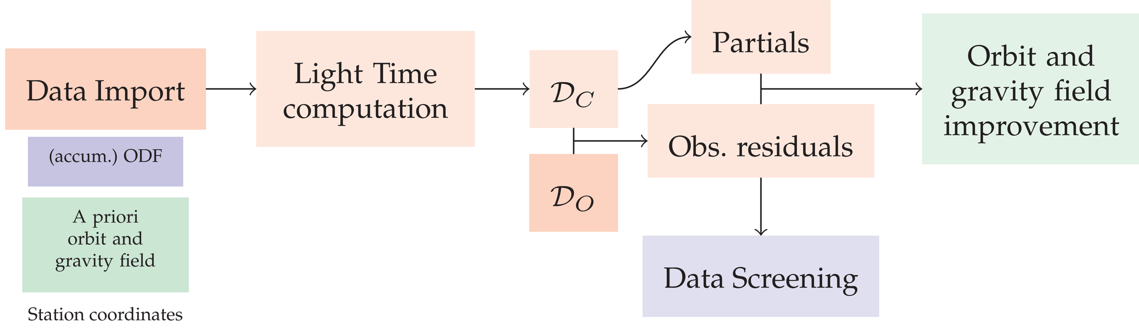


Figure 1: Processing flow of Doppler data, recently implemented in the Bernese (GNSS) Software. Doppler observations $\mathcal{D}_O$ from Orbit Determination Files (ODF) are imported to our internal format and eventually accumulated to the desired integration time. Orbit integration from a priori initial elements and parameters and an accurate modeling of light propagation are used to compute simulated Doppler $\mathcal{D}_C$ and hence Doppler residuals. The latter can be used to screen the observations or, along with the corresponding variational equations, to improve the "a priori" elements in an iterative orbit and gravity field improvement process.

We use the positions provided by the GRAIL navigation team as initial conditions for each daily arc and perform an orbit integration with the force model presented in the previous section. The initial orbital elements and, possibly, dynamical and stochastic parameters are then adjusted to the Doppler data (with an integration time of 10 s) using a classical least-square procedure. Observations are screened for outliers by setting a threshold on the residuals and by applying an elevation cutoff at 25°.

Doppler orbit determination

Several tests were performed to show the impact of different background fields and parametrizations (dynamic or pseudo-stochastic) on the improved orbits. Fig. 2 (left) shows two-way Doppler residuals for GRAIL-A over days 70-72 of the PM phase as well as differences of the computed orbits w.r.t. GNI1B positions. Daily RMS of Doppler residuals and orbital differences over the PM are shown in Fig. 2 (right).

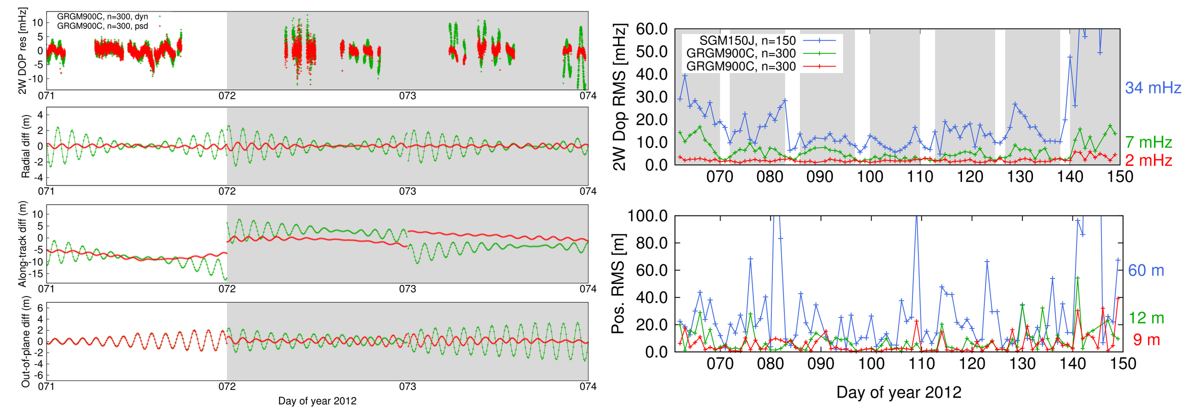


Figure 2: Left: (Top) Residuals from GRAIL-A Doppler fit using GRGM900C truncated to d/o 300 as background gravity field. Two different parametrizations are used: in green a purely dynamic orbit while in red we estimate a constant acceleration in S, a opr acceleration in R, and pulses in S and W directions every 30′. Shaded days represent geometries when less than 80% of the orbit is visible from Earth. (Bottom) Orbit differences w.r.t. GNI1B positions in the orbital frame.
Right: (Top) Daily RMS of GRAIL-A two-way Doppler residuals using GRGM900C (up to d/o 300) and SGM150J as background gravity fields and different parametrizations. (Bottom) Daily RMS of orbit differences w.r.t. GNI1B positions.

Combined orbit determination

Doppler and KBRR data are combined on the Normal EQuation (NEQ) level using a weighting appropriate to the relative accuracy ($1 : 10^8$). The resulting daily NEQs are then inverted to solve for the improved orbital parameters.

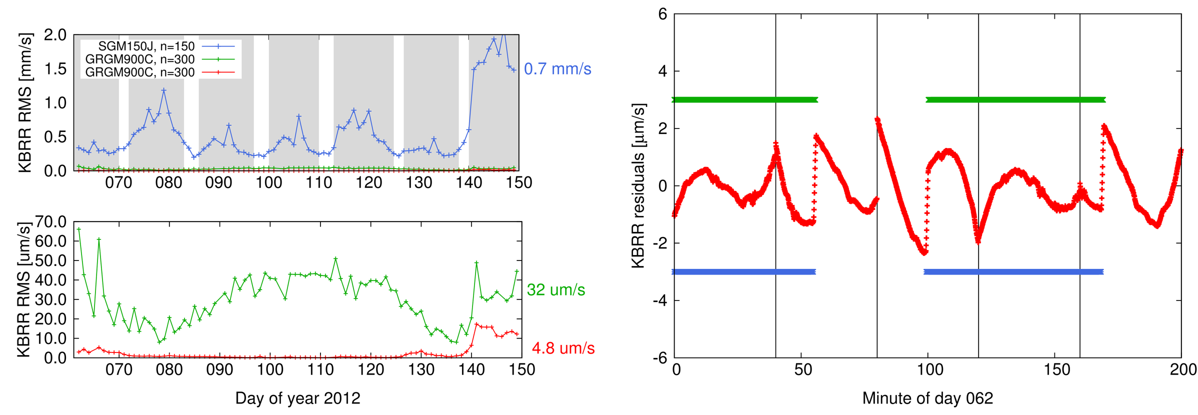


Figure 3: Daily RMS values of the KBRR residuals in the combined (Doppler and KBRR) orbit solution. Lower plots are zooms of upper ones. The fits are relatively bad when using the SELENE (SGM150J) gravity field and become better (more consistent) when introducing NASA's official GRAIL field GRGM900C (Lemoine et al., 2014), truncated at d/o 300. Right: KBRR residuals and time spans for which GRAIL-A (green) and GRAIL-B (blue) are in sunlight. Vertical black lines indicate locations of pseudo-stochastic pulses.

Fig. 3 (left) shows the global RMS of KBRR residuals over the PM phase. Residuals over several hours of day 062 when using the gravity field GRGM900C up to degree and order 660 as background field are shown in Fig. 3 (right). Compared to the expected noise level of around 0.05 μm/s, the residuals are still relatively large and clearly show the occurrence of pseudo-stochastic pulses. The green and blue bars indicate the time spans during which each satellite is in sunlight. The obvious correlation between these time spans and the large discontinuities suggests that radiation pressure modeling is crucial since the chosen parametrization is not able to fully compensate the deficiency.

Gravity field from Doppler and KBRR data (d/o 120)

The orbits determined in the first step serve as a priori information for a common orbit and gravity field estimation based on daily arcs. A classical least-squares adjustment is used. The daily normal equation systems (NEQs) are stacked to weekly, monthly and finally three-monthly NEQs, which are then inverted.
Fig. 4 shows several d/o 120 solutions computed from different a priori gravity fields. The solution represented by the red curve has been computed using GRGM900C (up to d/o 660) as background field, 30′ pulses in S and W directions, a constant acceleration in S and opr accelerations in R. A solution obtained from GNI1B and KBRR data by using a similar setup is shown (in blue) as comparison. Some first tests using SGM150J as background field and a purely dynamic solution are presented (yellow curve). They show an improvement of the higher degrees but a degradation up to degree 30, possibly due to a not-yet-optimal parametrization of the solution.

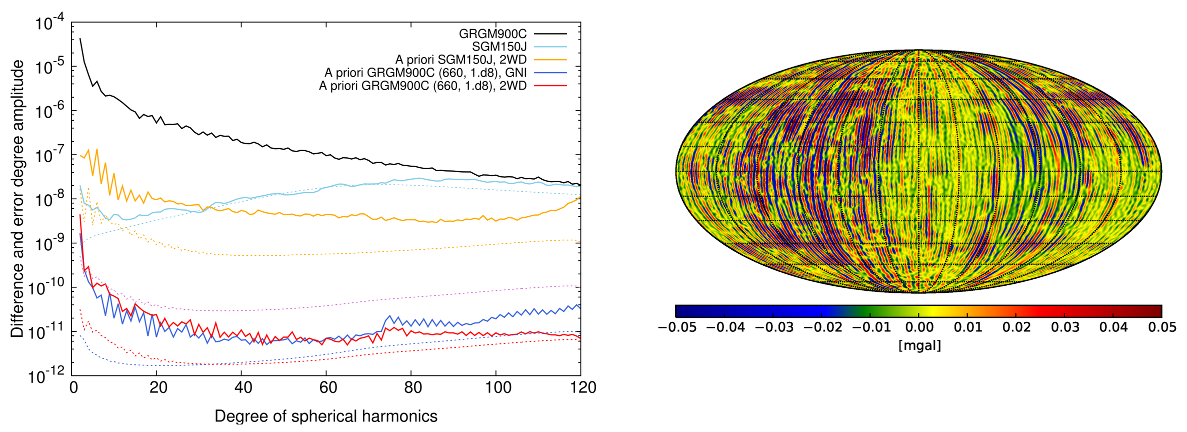


Figure 4: Left: Difference degree amplitudes (solid) and formal errors (dashed) of degree-120 solutions based on the a priori field GRGM900C (up to d/o 660 and using Doppler - red - or GNI1B data - blue) and SGM150J (yellow) compared to the SELENE solution. Right: differences of free-air gravity anomalies of the "red" solution w.r.t. GRGM900C.

More sophisticated parametrizations including pulses are being tested in order to start the iteration process which should finally lead to a fully independent solution of d/o 200 or larger.

Gravity field from GNI1B and KBRR data (d/o 200)

We also present our latest solutions up to d/o 200 using the GNI1B and KBRR combination from Arnold et al. (2015). Fig. 5 (left) shows the difference degree amplitudes of solutions AIUB-GRL200A and AIUB-GRL200B, which use GRGM900C as a priori field up to d/o 200 and 660, respectively.

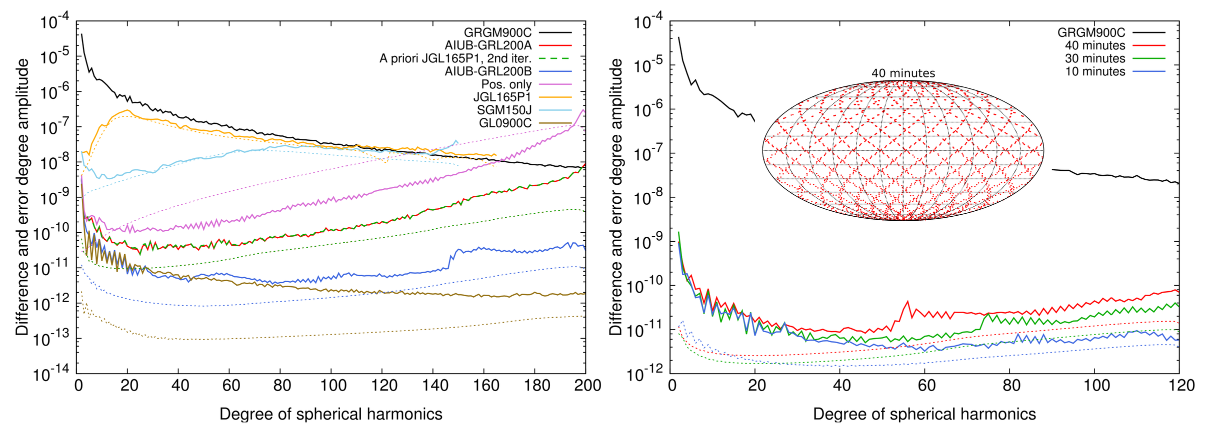


Figure 5: Left: Difference degree amplitudes (solid) and formal errors (dashed) of degree-200 solutions based on the a priori field GRGM900C (up to d/o 200, red, and 660, green) compared to pre-GRAIL solutions and GL0900C. The brown curve represents a position-only solution while the green dotted curve is the second iteration solution using JGL165P1 as a priori field. Right: d/o 120 solutions used to test the relation between degradation of particular degrees and pulses spacing and their geographical position.

For AIUB-GRL200A, we set up stochastic pulses every 40 minutes. AIUB-GRL200B illustrates the impact of the omission error on our solutions. The consistency between AIUB-GRL200B and GRGM900C markedly drops around degree 150. A thorough analysis revealed that the coefficients of order $\sim$55 (as well as the zonal terms) are degraded, and that this degradation shows a correlation with the spacing of the pulses (see Fig. 5, right). A possible explanation was identified in the geographical location of the pulses, showing a very regular pattern depending on their spacing. Finally, a less problematic spacing of 15′ was chosen for AIUB-GRL200B.

Conclusions

- The adaption of the CMA from GRACE to GRAIL allows for good-quality lunar gravity fields obtained entirely within the Bernese GNSS software.
- We present our first independent solution for GRAIL gravity field computed from original Doppler and KBRR data, hence showing our ability to extend our activities to the analysis of planetary missions data.
- Our gravity field solutions are so far computed without explicitely modeling non-gravitational forces and demonstrate the potential of pseudo-stochastic orbit parametrization. However, to fully exploit the precision of the Ka-band observations, we recently started to address an explicit modeling of solar radiation pressure in our modeling.

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